# Nuachúrsa Céimseatan

Máirtín Ó Tnuthail

Facsimile text of Ó Tnuthail,
tentative transcription of the text,
rough translation of the transcription,
and comments and observations,
assembled by Anthony G. O'Farrell

Draft Edition
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# Contents

# Editor's Foreword

Máirtín Ó Tnúthail, Martin Joseph Newell (1910-1985), henceforth MOT, was a distinguished academic mathematician, and served as lecturer, professor, and president at University College, Galway, with some earlier experience teaching secondary school in Tralee. He wrote three books, of which two, both undergraduate textbooks, were published in his lifetime. This book, whose title means "A New Course in Geometry", and which was completed in the mid-fifties, was never published, as the financial support that was needed was not forthcoming from the state. The book was intended for use by teachers and pupils in the early years of secondary schools, and there was in place an adequately-funded government scheme to assist with the publication of textbooks written in Irish, so one might wonder why this text was not deemed worthy of support. To make a long story short, the basic reason appears to be that the book was considered too original, and hence unlikely to be adopted by many schoolteachers. In fact, the text appears to have no parallel in any language.

Logic Press is pleased, with the consent of MOT's heirs, to make a facsimile of this manuscript text available to the general public, not only because the content will be of interest to mathematics teachers, but also because it preserves the living Irish of someone whose fluency and style were much admired. Like the Irish of most speakers of the period, it does not adhere to An Caighdeán (the official standard), and is all the more interesting for that.

The online facsimile (in pdf format) is freely downloadable at the book's home page:

`https://www.logicpress.ie/2019-3.`

The eight chapters of the text are in separate pdf files, named caibidil1.pdf to caibidil8.pdf, with a link to each on the home page. Each is a megabyte or two in size.

In the present document we include four elements:

- A facsimile scan of the original manuscript, the original and unique copy of which is in the possession of the Newell family. The scanned pages appear in sequence, scattered through the document.
- The tentative transcription of MOT's text. This should become more accurate in later editions. The transcriptions appear somewhere after their corresponding facsimile pages. We'll eventually aim to place the facsimiles on even-numbered (left-facing) pages, so that each faces (at least the beginning of) its transcription.

- A stab at an English translation of some of the text. We are more concerned with mathematical accuracy than faithfulness to the language of the original. This English text is printed in a distinct colour, currently red. It appears in the same order as the Irish text, and somewhere after the text it translates. In the meantime, any Irish text in red is awaiting translation.

- Commentary and opinions of the editor, provoked by his reading of the text. This is printed in a third colour, currently blue.

The possibility of issuing a printed facsimile edition is under consideration, and it would be helpful to hear from anyone who might consider purchasing such a thing. It would probably be expensive. If anyone is both competent and keen enough to help (on a voluntary basis!) with the task of correcting the transcription of the text, or of translating it into some other language, then the Press would be happy to proceed with an inexpensive printed edition.

For the avoidance of doubt, I want to make it absolutely clear that neither MOT nor any of his family had any hand, act, or part in any of the text in this document, apart from the text included in the facsimile reproduction of MOT's manuscript. Any errors or opinions that appear are entirely my own.

Anthony G. O'Farrell
6 December 2024

# Caibidil I

Mar is léir ón ainm féin geométracht (.i. an talamh a thomhas) is ó cheisteanna cheárdúla a shíolruigh an t-ábhar léinn sin. 'Sé a threoraíos captaen na loinge ar a chúrsa tríd an teiscinne mhór. Gheobhaidh an siúinéir treoir ann chun clár adhmaid a ghearradh de réir deilbhe áirithe sa gcaoi is tíobhusaí. Airíonn gach aoinne siúinéireacht aoibhinn, chórach i ndeilbh ard-teampuill mhaordha, agus feicfimid go bhfuil an tsiúinéireacht sin bunaithe ar phrionnsabail geométrachta a tharraing an foirgnteoir chuige á leagan amach dó.

De réir mar mhéaduigh an t-eolas geométrach le trialacha, fritheadh amach gur féidir a fhírinní uilig a ghnóthú le réasúnaíocht as an oiread seo bun-phrionnsabal. Tréith an-aoibhinn san geométracht í sin, a mheall chuige meas agus dúthracht na n-intleachta ba mhó cáil ó ghlúin go glúin. Ní fhágann sin áfach nach ionánn uair pléisiúir agus sólás intinne a thug sí d'intleachta ab uirísle i bhfad ná iad.

## Dluthóga

Tá na gnáth-rudaí a ndéanann ar gcéadfaí eolas dúinn orthu, suite i spás trí-mhiosúrdha. Ar thréithre spásúla na neithe sin aithnímid ionad, méad, agus deilbh. Athraíonn an t-ionad má aistrítear an rud ó áit go h-áit, ach más buan, seasmhach dá mhéad is dá dheilbh tugtar dlúthóg air. E.G. Dluthóga is ea cathaoir, bord, bosca na cailce etc.

D'fhéadfadh dhá dhluthóg a bheith cóimhionann le chéile ina méad is ina ndeilbh ionnus gur macasamhla dá chéile iad ina dtréithre spásúla. Is minic freisin a citear dhá dhluthóg atá ar cómhdheilbh gan a bheith ar chómhméad.

## Dromchlaí

Tá teora le gach dluthóg atá cuimseach ina méad agus tugtar dromchla na dluthóige ar an teorainn

# Caibidil 1

Mar is léir ón ainm féin geómétracht (.i. talamh a thómhas) is ó ceisteanna cheardúla a shíolruigh an t-ábhar léine sin. 'Sé a threoraíos captaen na loinge ar a chúrsa tríd an teiscinne mhór. Ghiobhaidh an súinéir treoir ann chun clár adhmaid a ghearradh de réir deilbhe áirithe sa chaoi is tíobhaisí. Airíonn gach aoinne suiméatracht aoibhinn, chórach i ndeilbh árd-teampaill mhaordha, agus feicfimid go bhfuil an tsuiméitreacht sin bunaithe ar phrionsabail geómétrachta a tharraing an foirginteoir chuige á leagan amach dó.

The origin in matters of trade or craft of our subject *geometry* is plain to see from its very name: geo-metry means *land-measurement*. It guides the captain of a ship in finding his course on the open sea. The carpenter finds guidance there for cutting a wooden plank into some particular shape in the most economical way. Everyone recognises beautiful symmetry in a majestic cathedral, and we shall see that such symmetry is founded on the geometrical principles employed by the designer.

What is the difference between an *art* and a *craft*? In fact, there is no difference. There is a lot of preciousness about "Art", these days. There is a community of self-styled artists who set themselves apart from the rest of us, and indeed set themselves above us. Some are painters or sculptors, novelists or poets, playwrights or actors, composers or performers. They see their work as radically different from that of craftsmen, tradesmen. But at its root, the word art just means *skill*, exactly the same as craft (but coming to English from a different direction, East instead of South.) The Faculty of Arts at the University of Paris was a faculty of skills. The skills were those of the trivium and quadrivium. The skills of jewellers and watchmakers, carpenters and joiners, plumbers and fitters, electricians and programmers, engineers and surveyers are not essentially different from those of architects and film directors. These are all *human skills, human art*. The earliest relics of human activity studied by archaeologists are immediately recognisable as human. What one recognises is the range of invention and ingenuity, and the combination of utility and beauty.

The subjects of the quadrivium were geometry, astronomy, arithmetic and music, so geometry has a claim to be queen and sovereign of the arts. This claim is supported by the reported priority given it in the assessment of matriculands: At Plato's Academy those ignorant of geometry were denied entry, and the *pons asinorum* (Euclid I.v) was required at Paris.

De réir mar mhéadaigh an t-eolas geómétrach le trialacha, fritheadh amach gur féidir a fhírinní uilig a ghnothú le réasúnaíocht as an oiread seo bun-phronsabal. tréith an-aoibhinn san geómétríocht í sin, a mheall chuige meas agus dúthracht na n-intleochta ba mhó cáil ó ghlúin go glún. Ní fhágann sin áfach nach iomaí uair pléisiúir agus sólás intinne a thug sí d'intleachta ab uirísle i bhfad ná iad.

As mankind's knowledge of geometry grew, it was discovered that all its truths can be worked out by reason from a number of basic principles. This is a sweet feature of geometry, which has drawn to it the admiration and energy of the greatest intellects of generation after generation. It has to be said, however, that geometry has also given hours of pleasure and mental consolation to far more modest intellects.

## Dlúthóga

Tá an-chuid gnáth-rudaí a ndéanann ár gcéadfaí eolas dúinn orthu, suite i spás trí-mhiosúrdha. Ar thréithre spásúla na neithe sin aithnímid *ionad, méad, agus deilbh.* Athraítear an t-ionad má aistrítear an rud o áit go h-áit, ach más buan, seasmhach dá mhéad is dá deilbh tugtar *dlúthóg* air. E.g. dlúthóga is ea cathaoir, bord, bosca na cailce, etc.

D'féadfadh dhá dhlúthóg a bheith cóimhionann le chéile ina méad is ina dheilbh ionas gur macasamhla dár chéile iad ina dtréithe spásúla. Is minic freisin a cítear dhá dhlúthóg atá ar cómhdheilbh gan a bheith ar chómhméad.

## Solids

Our senses make known to us many ordinary things situated in three-dimensional space. Among the spatial properties of those things, we recognise their location, their size, their shape. The location is changed if the thing is moved from place to place, but if its size and shape remain unchanged then we call it a solid. For instance, a chair, a table, the chalk-box are solids.

Two solids might be identical to one another in size and shape, so that they are exact copies of one another in their spatial properties. One often sees, also, two solids that have the same shape but are not the same size.

He's skating on thin ice, here. But this is to be read not as formal mathematics, but as an informal introduction. We are getting the pupils into the way of thinking that is used in geometry. This requires us to relate the formal concepts of geometry to their sense-experience to date.

Siad na dromchlaí is mó a bhfuil taithí againn orthu, agus is iad a chinneas deilbh na dluthóige.

Nuair a bhíos teach á chur suas is mór an spéis a chuireas an saor cloiche i dtiús an bhalla, de bhrí gurb é a rialaíos daingneacht an tí. Níl suim ar bith ag an bpéintéir sa tiús, áfach; is ar dhromchla an bhalla a oibríos seisean.

Ar na dromchlaí is simplí dá bhfuil ar eolas againn aithnímid (i) dromchla leibhéal balla réidh, (ii) dromchla bhosca na cailce, (iii) an sféir (iv) an roithleoir, (v) an cón ciorcalach, etc.

## Línte, Pointí

Is línte nó lúba iad teoranna dromchla ar bith e.g. ciumhsa an bhalla, faobhar scine. Má leagtar cupán ar a bhéal faoi ar an mbord, tagann an dá dhromchla le chéile i lúib chruinn.

Mar an gcéanna, sé an pointe a chríochnaíos, nó a chuireas teora le líne. Is i bpointe a theagmhaíos dhá líne le chéile.

## Sonraithe, Abstraicsiúin

Níl aon chuimse lena bhfuil de dhealbha dhifriúla ar na línte agus ar na dromchlaí a fheicimid thar timpeall orainn, agus ba cheist ró-aimhréiteach í, tabhairt fá na tréithe spásúla a mhíniú mura simplítí an obair roimh ré sa méad is féidir é. Is é mó na geoiméadrachta, na línte is na dromchlaí is simplí a thoghadh i dtosach, féachaint an féidir na dealbha eile go léir a mhíniú leo. Ní mór freisin na téarmaí bunúsacha a shonrú sa gcaoi nach baol ar bith nach é an chiall chéanna a bhainfeas chuile dhuine astu.

Chomh mór is is féidir

Nuair a deir duine go bhfuil Gaillimh 130 míle bealaigh ó Bhaile Átha Cliath, tuigfear láithreach é cé gur fiú linne an smaoineadh a bhreithniú tuille.

## Dromchlaí

Tá teora le gach dlúthóg atá cuimseach ina méad agus tugtar *dromchla* na dlúthóige ar an teorainn.

## Surfaces

Each solid has an exact boundary (perimeter, limits, edges), and that boundary is called the *surface* of the solid.

Tosach leathanach 2 sa LSS.

Siad na dromchlaí is mó a bhfuil taithí againn orthu, agus is iad a chinneas deilbh na dlúthóige.

Nuair a bhíonn teach á chur suas is mór an spéis a chuireas an saor cloiche i dtús an bhalla, de bhrí gurb é a riailíos daingneacht at tí. Níl suim ar bith ag an bpéinteoir sa tiús, áfach; is ar dhromchla an bhalla a oibríos seisean.

Ar na dromchlaí is símplí a bhfuil ar eolas againn aithnímid (i) dromchla leibhéal balla réidh, (ii) dromchla bhosca na cailce, (iii) an sféir, (iv) an rothleoir, (v) an cón ciorcalach, etc.

It is mainly the surfaces that we observe, and they determine the shape of the solid.

When a house is being built, the mason takes great care with the foundation of the wall, because that determines the soundness of the building. The painter is not at all interested in the thickness, however; he works on the surface.

Among the simplest surfaces we know about are: (i) the flat surface of a smooth wall, (ii) the surface of the chalk-box, (iii) the sphere, (iv) the cylinder, (v) the right circular cone, etc.

There are variations in the spelling of words. The manuscript shows evidence of work aimed at achieving uniformity. The evidence consists of letters struck out, corrections in red ink, and so on. For example *drompla* is altered to *dromchla*. There are a few marginal comments on the mathematical content. I have no information as to whether the corrections and comments were the work of MOT or of other editors. The variations may be of interest to linguists, but in making the transcription I shall assume that the final state of the MSS reflects the intent of MOT, and I shall attempt to respect that intent. Thus, where a spelling is usually corrected, I shall correct any remaining instance I find. For example, *isea* is almost always corrected to *is ea* in the MSS, and I make the same change to any remaining *isea*.

I would appreciate the assistance of any reader who notices an error in transcription, and I would welcome notice of such errors. This document is in continual revision, and each release carries a 'Draft Edition Number' on the title verso, the fourth page of the pdf file. please quote the draft edition number and the page number (usually top right, sometimes bottom centre) and line number of the offending text. Page numbers of the original MSS (where present) appear at top right, and may be used for reference, or you may use the pdf document page number as you please.

## Línte, Pointí

Is línte nó lúba iad teorann dromchla ar bith. e.g. ciúsa an bhalla, faobhar sgine. Má leagtar cupán ar a bhéal faoi ar an mbord, tagann an dá dhromchla le chéile i lúib chruinn.

Mar an gcéanna, sé an pointe a chríochnaíos, nó a chuireas teora le líne. Is i bpointe a theagmhaíos dhá líne le chéile.

## Lines, Points

The boundary (edges) of any surface are lines or curves. e.g. the edges of the wall, the blade of a knife. If one places a cup upside-down on the table, then the two surfaces meet in a sharp curve.

In the same way, points are what end a line, or constitute its boundary. Two lines meet in a point.

## Sonraithe, Abstraicsún

Níl aon chuimse lena bhfuil de dhealbha dhifriúla ar na línte agus ar na dromchlaí a fheicimid thar timpeall orainn, agus ba cheist ró-aimhréiteach í , tabhairt faoi na tréithe spásúla a mhíniú mura simplítí an obair roimhré chómh móir is is féidir. Is í módh na geómétrachta, na línte is na dromchlaí is simplí a thoghadh i dtosach, féachaint an féidir na dealbha eile go léir a mhíniú leo. Ní mór freisin na téarmaí bunúsacha a shonraí sa gcaoi nach baol ar bith nach é an chiall chéanna a bhaineas chuile dhuine astu.

Nuair adeir éoinne go bhfuil Gaillimh 130 míle bealaigh ó Bhaile Átha Cliath, tuigfear láithreach é cé gur fiú linne an smaoineamh a bhreithniú tuille.

## Definitions and Abstractions

There is no end to the variety of shapes that the lines and surfaces we see about us have, and it would be a hopeless task to try to explain all their spatial properties, unless the job were simplified as much as possible to start with. The method in geometry is to start with the simplest lines and surfaces, and then use them to explain all the others. It is important, too, to define all the basic terms, to avoid the danger that not everyone will understand their meaning in the same way.

When someone says that Galway is 130 miles from Dublin, we understand straight away that it is worth pursuing that thought a little further.

Má fiafraítear de cén áit i nGaillimh (agus i mBaile Átha Cliath) atá i gceist aige, déarfaidh sé go bhfuil Faiche Mhór na Gaillimhe 130 míle ó Shráid Uí Chonaill i mBaile Átha Cliath. Ach tig linn a fhiafraí arís cén áit i Sráid Uí Chonaill etc. Má leantar de sin tuigeann sé go mbeidh sé i sáinn againn ar deire, agus is dócha go n-abradh sé gur cuma cén áit sa Faiche Mhór (nó sa geathair sa geás sin de) a toghtar, mar is rí-bheag an difríocht a dhéanas sé sin ar ghualainn an 130 míle.

Sé sin le rá, samhlaítear an dá chathair mar áiteacha a bhfuil ionad cinnte acu, ach feileann sé dúinn amafinta neamhshuim a dhéanamh dá méad nuair nach fiú linn an difríocht a dheanfhadh sé sin a áireamh.

Ag machnamh dúinn ar thréith ruda ar bith, is minic gur mian linn tús áite in ár gcuid smaointe a thabhairt do thréith áirithe amháin seachas na cinn eile go léir, gur cuma ann nó as iadsan sa réasúnaíocht. Ba mhór an simpliú gan áird ar bith a thabhairt orthu. Is gnás linn ina leithéid de chás, neamhshuim a dhéanamh de na tréithe breise sin in aon turas, agus a ligean orainn nach bhfuil siad ann.

Tugtar abstraicsiún ar an modh smaointe sin. Tearacht an ghnáth-duine is mór an leas a bhaineas an geoméatracht as d'fhonn simplithe agus soiléireachta. e.g. Nuair deirtear go bhfuil teach áirithe leath-bealaigh go díreach idir an stáisiún agus oifig an phoist, déantar neamhshuim in aon turas de thoisc na dtrí bhfoirgintí sin.

## An Pointe

**Tá ionad ag pointe geoméatrach ach níl méad ar bith ann.**

Dá réir sin abstraicsiún is ea an pointe geoméatrach agus níl aon taithí ag ár gcéadfaí air. Nuair déantar marc an-bheag ar an bpáipéar le peann luaidhe, ní miste pointe a thabhairt air, cé nach fíor-phointe é de bhrí go bhfuil méad áirithe ann. Dá laghad é an marc

Tosach leathanach 3 sa LSS.

Má fiafraítear de cé'n áit i nGaillimh (agus i mBaile atha Cliath) atá i gceist aige, déarfadh sé go bhfuil Faiche Mhór na Gaillimhe 130 míle ó Shráid Uí Chonaill i mBaile Átha Cliath. Ach tig linn a fhiafraí arís cé'n áit i Sráid Uí Chonaill, etc. Má leantar de sin tuigeann sé go mbeidh sé i sáinn againn ar deire, agus is dócha go n-abródh sé gur cuma cé'n áit sa bhFaiche Mhór (nó sa gcathair sa chás sin de) a thoghtar, mar is rí-bheag an difríocht a dhéanas sé sin ar ghualainn 130 míle.

Sé sin le rá, samhlaítear an dá chathair mar áiteacha a bhfuil *ionad* cinnte acu, ach feileann sé dúinn amanta neamhshuim a dhéanamh d'á *méad* nuair nach fiú linn an difríocht a déanfhadh sé sin a áireamh.

Ag machnamh dúinn ar thréithre ruda ar bith, is minic gur mian linn tús áite in ár gcuid smaointe a thabhairt do thréith áirithe amháin seachas na cinn eile go léir, gur cuma ann nó as iadsan sa réasúnacht. Ba mhór an simpliú gan ord ar bith a thabhairt orthu. Is gnás linn ina leithéad de chás, neashuim a déanamhde na tréithre breise sin in aon turas, agus a ligean orainn nach bhfuil siad ann.

Tugtar abstracsiún ar an módh smaointe sin. Fearacht an ghnáth-duine is mór an leas a bhaineas an geómétracht as d'fhonn simplithe agus soiléireachta. e.g. Nuair deirtear go bhfuil teach áirithe leath-bhealaigh go díreach idir an stáisiún agus oifig an phoist, déantar neamhshuim in aon turas de thoisí na dtrí bhfoirgnintí sin.

If he were asked which place in Galway (or in Dublin) he is talking about, he could say that it is 130 miles from Ayre Square to O'Connell Street in Dublin. But then we could ask, which place in O'Connell Street, etc. If we carry on like that, he will realise that he is eventually going to be stuck, and he may say that it does not matter which exact place in the square (or in the city) are chosen, because that is going to make very little difference on top of the 130 miles.

In other words, one thinks of the two cities as things that have a definite location, but sometimes it suits us to ignore their size, when that is not going to make a significant difference in the calculation.

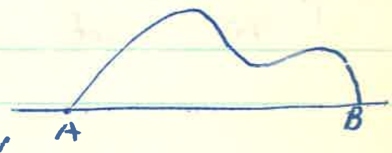
When we think about the properties of any thing, it often happens that it suits us to give first place to some particular property, over all the others which are of no importance for the question at hand. It greatly simplifies matters if we ignore them completely. Our usual practice in that kind of situation, is to take no interest in those other properties, and pretend they don't even exist.

This way of thinking is called *abstraction*. In contrast to the ordinary way of thinking, geometry makes a great deal of use of it in order to achieve simplicity and clarity. For instance, when one says that some house is halfway between the Post Office and the Station, one is ignoring the sizes of those three buildings.

'Sé:
~~Ata~~ is goire don fhointe é.

De bhrí gur abstraiseiún é tá sé feirsach againn a bheith ag súil le sampla beacht. Ní ró-dheacair afach a leithead a shamhlú agus feicfear gur mór an gar é sin chun réasúnaíocht gheométrach a shimpliú.

## Líntí; An Dronlíne



Sna gnáthlíntí a chímid thar timpeall bíonn fad éigin, cé go mbíonn a bheag nó a mhór de leithead agus de thiús ionntu freisin. De bharr neamhshuim a dhéanamh den leithead is den tiús, faightear abstraic-siún d'a ngoirtear líne gheométrach.

Tá fad i líne, ach níl tomhas ar bith eile innte.

Níl aon chuimse lena bhfuil de líntí dhifriúla a ghabhann tré dhá phointe A agus B, ach ní féidir gan ~~suntas~~ sondas a thabhairt do líne amháin acu thar na líntí eile go léir, tá sé chomh simplí, follasach sin .i. an dronlíne AB.

Tá tuairim ag gach duine roimh ré fá céard is dronlíne ann. Chun threithe na dronlíne a mheas, breith-nimís i dtosach ar chaoi a n-aimsíonn an foirgintéoir (nó an garradóir) í.

Tré na "pointí" A agus B, cóiríonn sé tiósa corda, agus más córda caol é is geall gur lúb gheométrach idir A agus B a lúbfios sé ansin. Tarraingíonn sé ~~ar~~ an gcórda sin go mbí sé teann chun an líne is giorra idir A agus B a fháil, agus cuirtear sa gceap sin an dronlíne idir A is B.

.i. (1) 'Sí an dronlíne an líne is giorra idir dhá phointe.

Chun na dronlíne AB a shíneadh, tógann an foirgintéoir pointe ar bith C san dronlíne, agus cóiríonn sé

## An Pointe

*Tá ionad ag pointe geómétrach ach níl méad a bith ann.*

Dá réir sin abstracsiún is ea an pointe geómétrach agus níl aon taithí ag ár gcéadfaí air. Nuair déantar marc an-bheag ar an bpáipéar le peann luaí, ní miste pointe a thabhairt air, cé nach fíor-phointe é de brí go bhfuil méad áirithe ann. Dá laighead é an marc

Tosach leathanach 4 sa LSS.

is ea is goire don phointe é.

De bhrí gur abstracsiún é tá sé fánach againn a bheith ag súil le sampla beacht. Ní ró-dheachair áfach a leithéad a shamhlú agus feicfear gur mór an gar é sin chun réasúnaíocht geómétrach a shímpliú.

## The Point

*A geometric point has position but has no size.*

It follows that a geometric point is an abstraction, but our senses cannot experience it. If one makes a small mark on paper, we might as well call it a point, even though it is not a real point because it has some size. The smaller the size, the closer it comes to a point.

Since it is an abstraction, it is futile for us to ask for a real actual example. It is not too hard, however, to imagine such a thing, and it will be seen how much that helps to simplify geometric reasoning.

## Línte; An Dronlíne

'Sna gnáthlínte a chímid thar timpeall bíonn fad éigin, cé go mbíonn a bheag nó a mhór de leithead agus de thiús iontu freisin. De bharr neamhshuim a dhéanamh den leithead is den tiús, faightear abstracsiún d'á ngoirtear *líne geómétrach*.

*Tá fad i líne, ach níl tomhas ar bith eile inti.*

Níl aon chuimse lena bhfuil de líte dhifrúla a ghabhann trí dhá phointe $A$ agus $B$, ach ní féidir gan sondas a thabhairt do líne amháin acu thar na línte eile go léir, tá sí chomh simplí , follasach sin .i. an dronlíne $AB$.

Tá tuairim ag gach duine roimhré fá casadhis dronlíne ann. Chun thréithre na dronlíne a mheas, breithnimís i dtosach cé'n chaoi a n-aimsíonn an foirginteoir (nó an garradóir) é.

Tré na "pointí " $A$ agus $B$, cóiríonn sé píosa corda, agus más corda caol é is geall gur lúb geómétrach idir $A$ agus $B$ a léiríos sé ansin. Tarraingíonn sé an corda sin go mbí sí teann chun an líne is giorra idir $A$ agus $B$ a fháil, agus cinntear sa chás sin an *dronlíne idir $A$ agus $B$*.

*(1) 'Sí an dronlíne an líne is giorra idir dhá phointe.*

## Curves. The Straight Line

The ordinary curves that we see about us have length, but they also have width and thickness. By ignoring the width and thickness, we get the abstraction called the *geometric curve*:

*A curve has length, but has no other dimensions*

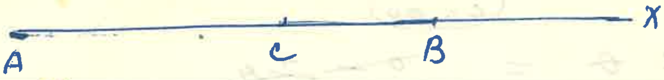
There is no limit to the number of different curves that pass through two points $A$ and $B$, but one has to give pride of place to one curve over all the others, because it is so simple and important, the straight line.

Everyone has some idea to start with of what a straight line actually is. To examine the properties of the straight line, let's look at how a builder (or a gardiner) finds one:

He passes a string through the points $A$ and $B$, and if it is a narrow string, then it is something like a curve between $A$ and $B$. He pulls the string tight to find the shortest curve between $A$ and $B$, and that determines the straight line between $A$ and $B$.

*(1) The straight line is the shortest line between two points.*

The fourth page of the manuscript has the first diagram. It shows what we usually call (the image of) a (parametrised) *curve*. From Euclid's day to MOT's, it was called a line. In mathematical writing nowadays, a line is usually understood to be straight, but when MOT wrote, a straight line needed its adjective. It was also called a *right line*, and in Irish *dronlíne*.




córda teann eile CX ó C, a ghabhas tré B freisin. Is eol dó go dteagmhaíonn an dá chórda le chéile ar fead CB, agus nuair déantar neamhshuim den tiús, deir sé gur aon dronlíne amháin iad CX agus AB a théighas tré C agus B.

(2) Tré dhá phointe ar bith, ní théigheann ach dronlíne chinnteach amháin.
Is ionann é sin agus:-
(2)' Is aon dronlíne amháin, dronlínte a bhfuil dhá phointe dhifriúla orthu san am chéanna.

Tréith eile den dronlíne é gur féidir ceann acu a shleamhnú ar cheann eile. e.g. an dá chórda AB, CX thuas. De bhrí go ndéantar neamhshuim den tiús is cuite rá go sleamhnaíonn an dronlíne geómatreach uirthi féin. Is minic a tugtar líne dhíreach ar dhronlíne, agus feicfear i gcáib go bhfuil baint ag díre na dronlíne leis an tréith deiridh sin.

Dromchlaí. An Plána

De bharr neamhshuim a dhéanamh de thiús na ngnáth-dhromchlaí gnóthaítear abstraicsiún d'á ngoirtear dromchla geómatreach.

Orthu siúd tá dromchla slim réidh a dtugtar plána air: e.g. dromchla balla mín, dromchla locha chiúin.

Nuair a bhíos urlár stroighne dá leagan síos ag foirgínteoir, leagann sé clár fána fhaobhar dhíreach anuas ar an stroighin, ionnus go bhfuil dhá cheann an chláir ar an leibhéal ceart. Leasaíonn sé dromchla na stroighne ansin má's gá é, chun go luighfidh an faobhar díreach uilig air go cruinn. Bogann sé an clár anonn is anall agus is eol dó nach urlár leibhéal é, go luighfidh an faobhar díreach ar a fhad air i ngach ionad.

Dromchla is ea an plána a chrioslaíonn go h-iomlán gach dronlíne a theagmhaíos leis i ndhá phointe dhifriúla

Chun an dronlíne $AB$ a shíneadh, tógann an foirginteoir pointe ar bith $C$ san dronlíne, agus cóiríonn

Tosach leathanach 5 sa LSS.

Tá Fíoghair anseo sa LSS, leathanach 5.

sé córda teann $CX$ ó $C$, a ghabhas tré $B$ freisin. Is eol dó go dteagmhaíonn an dá chórda le chéile ar feadh $CB$, agus nuair déantar neamhshuim den tiús, deir sé gur aon dronlíne amháin iad $CX$ agus $AB$ a théigheas trí $C$ agus $B$.

*(2) Tré dhá phointe ar bit, ní théigheann ach dronlíne chinnteach amháin.*

Is ionann é sin agus:—

*(2)' Is aon dronlíne amháin, dronlínte a bhfuil dhá phointe dhifriúla orthu san am céanna.*

Tréith eile den dronlíne é gur féidir ceann acu a shleamhnú ar cheann eile. e.g. an dá córda $AB$, $CX$ thuas. De bhrí go ndéantar neamhshuim den tiús is cirte a rá go sleamhnaíonn an dronlíne geómétrach uirthi féin. Is minic a tugtar líne dhíreach ar dhronlíne, agus feicfear i gCaibidil IV? go bhfuil baint ag dirí na dronlíne leis antréith deiridh sin.

To extend the straight line $AB$, the builder takes any point $C$ on the line, and he stretches a cord $CX$ from $C$, that passes through $B$ as well. Obviously, the two cords lie together along $CB$, and when one ignores the thickness, one says that $AB$ and $CX$ form (part of) a single straight line that passes through $C$ and $B$.

*(2) Through any two points goes just one definite straight line.*

That amounts to the same thing as saying:

*(2)' Two straight lines coincide if they share two different common points.*

Another property of straight lines is that one can slide one along another, e.g. the two cords $AB$,$CX$ above. Since one ignores the thickness, it is more correct to say that one can slide a geometric straight line along itself. A right line is often called a straight (direct) line, and it will be seen in Chapter IV? that the straightness (direction?) of the line has a connection to that last property.

The straight line or right line is defined as a geodesic, and the uniqueness of the segment between two points and the whole extended geodesic is assumed axiomatically. Length is an undefined term.

The same term “straight line" is used for the line segments and the whole unbounded lines, but MOT carefully draws a distinction bewteen them. One could formalise what he does by defining the whole line as an equivalence class of segments.

## Dromchlaí: An Plána

De bhárr neamhshuim a dhéanamh de thiús na ngnáth-dhromchlaí gnóthaítear abstracsiún dá ngoirtear dromchla geómétrach.

Orthu siúd tá dromchla *slinn* réidh a dtugtar *plána* air: e.g. dromchla balla mín, dromchla locha chiúin.

Nuair a bhíos urlár stroighne dá leagan síos ag foirginteoir, leagann sé clár fána faobhar dhíreach anuas ar an stroighin, ionas go bhfuil dhá cheann an chláir ar an leibhéal ceart. Deasaíonn sé dromchla na stroighne más gá é, chun go luíonn an faobhar díreach uilig air go cruinn. Bogann sé an clár anonn is anall agus is eol dó nach urlár leibhéal é, go luíi onn an faobhar díreach ar a fhad air i ngach ionad. .i.

*Dromchla is ea an plána a chrioslaíonn go h-iomlán gach dronlíne a theagmhaíos leis i ndhá phointe dhifriúla.*

Is í an dronlíne a thagas dhá phlána le chéile.
Mar, má's pointí ~~iad~~ A, B atá ar an dá phlána, is léir go luíonn an dronlíne AB in iomlán san dá phlána.
e.g. balla agus urlár an tseomra; dhá leathanach leabhair.

## Ceisteanna

1) Cé mhéad aighthe i mbrosca cailce? Cé mhéad faobhar? Cé mhéad cúinne? Má's fíor-phlánaí iad na haighthe, dronlínte iseadh na faobhair. Tuige? Cé mhéad aighthe a ghabhas tré fhaobhar ar bith? Cé mhéad faobhar a théigheas tré chúinne ar bith?

2) Ainmnigh trí plánaí agus trí dronlínte sa seomra. Tabhair sompla de (i) phlána agus dronlíne a luíos ann; (ii) de phlána agus dronlíne a thagas le chéile in aon phointe amháin.

3) Leaghain trí pointí sa seomra a ctear duit a bheith in aon dronlíne amháin (i) nuair atá péire acu ar bhalla den tseomra, agus (ii) nuair nach bhfuil ach ceann acu ar an mballa.

4) Tabhair somplaí de thrí phointí agus plána a ghabhas thríothu (i) nuair atá péire acu ar bhalla amháin agus an ceann eile ar an mballa ós a chóir.

5) Faigh roithleoir ciorcalach agus deimhnigh le faobhar dhíreach (i) go bhféadfadh dronlíne a luighe san dromchla sin agus (ii) go bhfuil dronlínte ann a ghearras i ndá phointe dhifriúla é gan a bheith ina luighe san dromchla. (cones?)

## Dromchla ag sleamhnú air féin

(6) Faigh roithleoir cuasach agus dlú-roithleoir eile a thoilleann go beacht ann. Fíoruigh gur féidir dromchla an dlú-roithleora a shleamhnú ar an dromchla a theagmhaíos leis amuigh ar thrí mhódh.
(i) Nuair is síos suas go díreach, gan chasadh ar bith, a ghluaiseas an dlú-roithleoir (oibriú loinithe i roithleoir)
(ii) Nuair a chasann an dlú-roithleoir timpeall gan barr an roithleora a dhul ar aghaidh. (oibriú acastóra)
(iii) Nuair a chasann agus a gluaiseann an roithleoir ar aghaidh san am chéanna (gluaiseacht seriú).

Tosach leathanach 6 sa LSS.

Is i ndronlíne a thagas dhá phlána le chéile. Mar, má's pointí $A$,$B$ atá ar an dá phlána, is léir gi luíonn an dronlíne $AB$ go h-iomlań san dá phlána. e.g. balla agus urlár an tseomra; dhá leathanach leabhair.

## Surfaces. The Plane

By ignoring the thickness of an ordinary surface, one obtains the abstraction known as a geometrical surface.

Among these is the smooth, even surface called a plane: e.g. the surface of a wall, the surface of a placid lake.

When a builder is laying down a concrete floor, he lays a plank with a straight edge on it, so that the two ends of the plank are at the right level. He touches up the surface of the cement as need be, so that the edge lies directly on it all along its length. He moves the plank to and fro, and he knows that he has a level floor when the edge lies directly on it in every position. i.e.

*A plane is a surface that contains the whole of any straight line that meets it in at least two points.*

Two planes meet in a straight line. For, if $A$ and $B$ are points on the plane, then the whole straight line $AB$ lies in both planes. e.g. the wall and floor of the room, two pages of a book.

## Ceisteanna

1. Cé mhéid aighthe i mbosca na cailce? Cé mhéid faobhar? Cé mhéid cúinne? Más fíor-phlánaí iad na aighthe, dronlínte is ea na faobhair. 'Tuige? Cé méid aighthe a ghabhas trí fhaobhar ar bith? Cé mhéid faobhar a théigheas trí cúinne ar bith?

2. Ainmnigh trí plánaí agus trí dronlínte sa seomra. Tabhair sampla de (i) phlána agus dronlíne a luíos ann; (ii) de phlána agus dronlíne a thagas le chéile in aon phointe amháin.

3. Teaspaáin trí pointí sa seomra a cítear duit a bheith in aon dronlíne amháin (i) nuair atá péire acu ar bhalla den tseomra, agus (ii) nuair nach bhfuil ach ceann acu ar an mballa.

4. Tabhair sample de trí pointí agus phlána a ghabhas tríothu. nuair atá péire acu ar bhalla amháin agus an ceann eile ar an mballa ós a chóir.

5. Faigh roithleor ciorcalach agus deimhnigh le faobhar díreach (i) go bhféadfadh dronlíne a luighe san dromchla sin agus (ii) go bhfuil dronlínte ann a ghearas i ndhá phointe dhifriúla é gan a bheith ina luighe ar an dromchla.

   **Dromchla ag sleamhnú air féin:**

6. Faigh roithleoir cuasach agus dlú-roithleoir eile a thoileann go beacht ann. Fíoruigh gur féidir dromchla an dlú-roithleoir a shleamhnú ar an dromchla a theangmhníos leis amuigh ar thrí mhódh:
   (i) Nuair is síos suas go díreach, gan casadh ar bith, a ghluaiseas an dlú-roithleoir (oibriú loinithe i roithleoir)
   (ii) Nuair a chasann an dlú-roithleoir timpeall gan barr an roithleoira a dhul ar aghaidh (oibriú acastóra)
   (iii) Nuair a chasann agus a ghluaiseann an roithleoir ar aghaidh san am chéanna (gluaiseach scriú).

7.) Má castar sféir timpeall fearsaide ar bith a ghabhas tré na cheart-lár, deimhnigh nach n-athraitear ionad na sféire iomláine sa spás, cé go mbogann na pointí difriúla den sféir féin.

Sin é an fáth a n-oibríonn liathróid sféireach go réidh i gcuasán sféireach eile a dtoilleann an liathróid go cruinn ann (Siúnta ~~liathróideach~~ liathróideach). Baintear úis as i gcorp an duine.

Is follasach go sleamhnionn plána ar phlána freisin, rud a mbeidh gnotha againn leis amach anseo.

Congrúacht.

Má cuirtear beirt bhuachaill ina seasamh taobh le taobh, is féidir a innseacht ar an gcaoi sin cé acu is airde. De bhrí nach gcuirtear i bhfáth ach an airde, ní miste na buachaillí sin a shamhlú mar dhoigh de gur dírlínte a bhí ionntu. Nuair nach caoithiúil dúinn dhá dhírlíne a chur taobh le taobh, tig linn slat díreach (riail) atá chomh h-árd le líne acu a iompar go dtí an líne eile agus iad a chur i gcoimhneas sa gcaoi sin. Is follasach, dar linn, go gcaithfidh an t-slat a bheith chomh h-árd leis an gcéad líne san ionad nua freisin, ionnus gur mar a chéile dóibh an t-aon tomhas spásúil amháin atá ionntu viz. an fhad. D'fhéadfadh an dara líne a bheith níos faide nó níos giorra ná an t-slat. Má's comhfhada leis atá sé, deirtear go bhfuil an dá líne (agus an t-slat) congrúach. Sa gcás sin macasamhla dá chéile iad ina dtréithe spásúla.

Mar an gcéanna is soiléir dúinn, ar an gcuma sin, gur mó pingin ná raol, ach go bhfuil dhá phingin congrúach, ionnus gurb ionann méad agus deilbh dóibh. Is ar an gcuma sin a cuirtear dhá fhíoghair phlánacha i gcoimhneas lena chéile.

Maidir le dhá dhlúthóig A agus B, thar éis dúinn A a chur in áit ar bith eile, má's féidir B a chur go cruinn, beacht san ionad mar a raibh A i dtosach, dlúthóga congrúacha iad A agus B.

Tuigfidh an léitheoir anois nach féidir dhá fhíoghair

Tosach leathanach 7 sa LSS.

7. Má castar sféir timpeall fearsaide ar bith a ghabhas tríd na cheartlár, deimhnigh nach n-athraítear iodad na sféire iomláine sa spás, cé go mbogann na pointí difriúla den sféir féin.

   Sin é an fá go n-oibríonn liathróid sféireach go réidh i gcuasán sféireach eile a dtoileann an liathróid go cruinn ann (sciúta? liathróideach). Baintear úsaid as i gcorp an duine.

Is follasach go sleamhníonn plána ar phlána freisin, rud a mbeidh gnotha againn leis amach anseo.

## Questions

1. How many faces does the chalk-box have? How many edges? How many corners? If the faces are true plances, then the edges are straight lines. Why? How many faces pass through any edge? How many edges pass through any corner?

2. Name three planes and three straight lines in the room. Give examples of (i) a plane and a straight line that lies in it, (ii) a plane and a straight line that meet in a single point.

3. Show three points in the room that appear to you to be in a single straight line (i) with two of them on a wall of the room; (ii) with only one of them on the wall.

4. Give an example of three points and a plane that goes through them, where two of them are on one wall and the other one is on the opposite wall.

5. get hold of a circular cylinder and verify with a straight edge (i) that a straight line could lie on the surface, and (ii) that there are straight lines that cut it in two different points without lying on the surface.

   **A surface sliding on itself**

6. Get a cyclidrical shell and a solid cylinder that fits snugly into it. Verify that the solid cylinder can slide against the shell in three ways:
   When the solid cyclinder slides straight up and down inside the shell (the motion of a piston in a cyclinder);
   When the solid cylinder spins around without its end moving up or down (the motion of an axle in its housing)
   When the solid cylinder twists and moves up at the same time (the motion of a screw).

7. If a sphere turns about any axis through its centre, verify that the sphere as a whole does not alter its position in space, although the individual points of the sphere do move.

> That is why a spherical ball can move smoothly in a spherical socket that fits it precisely (a ball joint). This is used in the human body.

Of course, a plane can slide on a plane as well, a matter with which we shall have more to do later.

## Congrúacht

Má cuirtear beirt bhuachaill ina seasamh taobh le taobh, is féidir a innseacht sa gcaoi sin cé acu is áirde. De bhrí nach gcuirtear i bhfác ach an áirde, ní miste na buachaillí sin a shamhlú mar doingh de gur dronlínte a bhí ionntu. Nuair nach caoithiúil dúinn dhá dhronlínte a chur taobh le taobh, tig linn slat dhíreach (riail) atá chomh h-ard le líne acu a iompar go dtí an líne eile agus iad a chur i gcoibhneas sa gcaoi sin. Is follasach, dar linn, go gcaithfidh an tslat a bheithchomh h-árd leis an gcéad líne san ionad nua freisin, ionnus gur mar a chéile dóibh an t-aon tomhas spásúil amháin atá ionntu viz. an fhad. D'fhéadfadh an dara líne a bheith níos faide nó níos giorra ná an tslat. Má's comhfhada leis atá sé, deirtear go bhfuil an dá líne (agus an tslat) *congrúach*. Sa gcás sin macasamhla dá chéile is ea iad ina dtréithe spásúla.

Mar an gcéanna is soiléar dúinn-ne, ar an gcuma sin, gur mó pingin ná raol, ach go bhfuil dhá phingin congrúach, ionnus gurb ioann méad agus deilbh dóibh. Is ar an gcuma sin a gcuirtear dhá fhiogair plánacha i gcoibhneas lena chéile.

Maidir le dhá dhlúthóg $A$ agus $B$, tar éis dúinn $A$ a chur in áit ar bith eile, más féidir$B$ a chur go cruinn, beacht san ionad mar a raibh $A$ i dtosach, dlúthóga congrúacha is ea $A$ agus $B$.

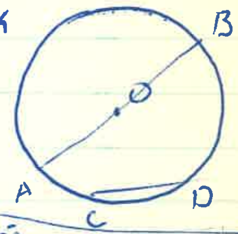
Tuigfidh an léitheoir anois nach féidir dhá fhíohair geómétrachta gíométrachta a chur i gcoimhmeas le chéile gan gluaiseacht éigin. Nuair tugtar dúinn go bhfuil dhá dhronlíne (nó dhá phlónacha) congrúach, is ionann é sin is a rá gurbféidir ceann acu a leagan anuas go cruinn ar an gceann eile de bharr gluaiseachta áirithe.

## Téarmaí

Tugtar ciorcal ar an líne chruinn a ghearras pointe sóinséálach ar phláná, a fhanas an fhad céanna amach ó phointe shocair (lár an chiorcail) agus é ag dul timpeall. 'Sé ga an chiorcail an fhad atá idir an lár agus an pointe gluaisteach.

Le compás a línítear ciorcail san geoiméadracht.

? Amanta is mar chaidhte (.i. idir taobh istigh agus teora) a samhlaítear an ciorcal, agus sa gcás sin tugtar imlíne an chiorcail ar an líne teorann ACDB.




Tugtar lárlíne ar dhronlíne ar bith trí na lár e.g. AB. bórda nó líne eile a ghearras i ndá phointe é; e.g. CD. Tugtar stua den chiorcal ar phíosa den imlíne e.g. CD. Amanta scríobhtar an rud ⊙ in áit "chiorcail".

## Cleachtaithe ar Líníocht le Compás agus Riail

1) Tarraing ciorcal ar lár dó pointe áirithe A a ghabhas trí phointe áirithe eile X. Tarraing ciorcal eile trí X gur lár dó pointe áirithe B. Má's i bpointe Y a thagann an dá ⊙ le chéile arís, faigh an pointe M ina ngearrann na dronlínte AB agus XY a chéile. Deimhnigh leis an riail go bhfuil na línte MX agus MY comhfhada.

2) Tagann aon dhá dhronlíne ar bith le chéile i bpointe A. Ar dhronlíne acu geartar míreanna comhfhada AX is XB, agus mar an gcéanna déantar AY, YC ar an líne eile comhfhada le, ~~áit nach gá go ...~~

Deimhnigh gur faide faoi a dhó BC ná XY.

Tosach leathanach 8 sa LSS.

a chur i gcoibhneas le chéile gan gluaiseacht éigin. Nuair tugtar dúinn go bhfuil dhá dhromchla (nó dá fíoghair phlánacha) congrúach, is ionann é sin is a rá gur féidir ceann acu a leagan anuas go cruinn ar an gceann eile de bharr gluaiseacht áirithe.

## Congruence

If two boys are placed side by side, one can then tell which is taller. Since only their height is in question, you might as well imagine the boys as straight lines. If it is not convenient to place the lines side by side, one could carry a straight rod that is as long as one of the lines over to the other and compare them in that way. It is obvious to us that the rod will have the same height in its new location, because it shares with the first line the one spatial dimension it has: its length. The second line might be greater or less than the rod. It it has the same length, one says that the two lines (and the rod) are *congruent*. In that case they are copies of one another as far as their spatial properties ar concerned.

In the same way it is clear that a penny is bigger than a sixpence, but that two pennies are congruent, so that they have the same size and shape. That is the way one relates plane figures to one another.

As for two solids $A$ and $B$, if, after moving $A$ away we can move $B$ so that it exactly fills the space vacated by $A$, then $A$ and $B$ are congruent solids.

The reader will now understand that it is impossible to compare geometric figures without some movement. When we are told that two surfaces (or two planar figures) are congruent, that is the same as saying that one of them can be laid down exactly on the other as a result of some movement.

Length is undefined, but “it is obvious to us" that it is a property of objects that remains invariant when they are moved about. The underlying level one theory must have the axiom that the metric on space admits a reasonably rich group of isometries.

## Téarmaí

Tugtar *ciorcal* ar an lúib chruinn a ghineas pointe sóinséalach ar phlána, a fhanas an fhad chéanna amach ó phointe shocair (lár an chiorcail) agus é ag dul timpeall.

Tá Fíoghair anseo sa LSS, leathanach 8.

’Sé *ga* an chiorcail an fad buan atá idir an lár agus an pointe gluaisteach.

Le compás a línítear ciorcail san geómétracht. Amanta is mar *chaidh’te* (.i. idir taobh istigh agus teora) a samhlaítear an ciorcal, agus sa chás sin tugtar *imlíne* an chiorcail ar an líne teorann $ACDB$.

Tugtar lárlíne ar dhronlíne ar bith tréna lár e.g $AB$. *Córda* is ea líne eile a i ndhá phointe é; e.g. $CD$. Tugtar *stua* den chiorcail ar phíosa den imlíne e.g. $CD$.

Amanta scríobhtar an nod ⊙ in áit “chiorcail".

The name *circle* is given to the curve generated by a variable point of the plane that stays the same distance from a fixed point (the *centre* of the circle) as it moves around.

The *radius* of the circle is the constant distance between the centre and the moving point.

In geometry, one draws a circle with a compass

Tá Fíoghair anseo sa LSS, leathanach 8.

Sometimes people regard a circle as a *disc* (.i. having both inside and boundary), and in that case the boundary curve $ACDB$ is called the *perimeter* of the circle.

Any straight line through the centre is called a diameter; e.g. $AB$.

A *chord* is another line that cuts it in two points; e.g. $CD$.

A piece of the perimeter is called an *arc*; e.g. $CD$.

Sometimes one writes the symbol $\odot$ in place of “circle".

3 Trí pointí ar bith iséa A, B, C. Tógtar pointe ar bith X in BC agus pointe ar bith Y in CA. Má's in O a theagmhann na línte AX agus BY le chéile, faigh an pointe Z ina dteagmhaíonn CO le AB.

Deimhnigh gur pointí in aon dronlíne amháin iad, pointí teagmhála BC le YZ, CA le ZX agus AB le XY.

4) Dronlínte teagmhálacha iséa l agus m. Tógtar trí pointí ar bith A, B, C in ord a chéile ar l; agus trí pointí in ord a chéile ar m iséa ~~X, Y, Z~~. L, M, N.

Faigh pointí teagmhála BL le AM, BN le MC, AN le LC agus deimhnigh gur pointí in aon dronlíne amháin iad.

5) Sé pointí in ord a chéile ar imlíne chiorcail iséa A, B, C, D, E, F. Faigh pointí teagmhála AC le BF, AD le BE, CE le DF, agus deimhnigh gur pointí in aon dronlíne amháin iad.

## Cleachtaithe ar Líníocht le Compás agus Riail

1. Tarraing ciorcal ar lár dó pointe áirithe $A$ a ghabhas trí phointe áirithe eile $X$. Tarraing ciorcal eile trí $X$ gur laár dó pointe áirithe $B$. Más i bpointe $Y$ a thagann an dá ciorcail le lhéile arís, faigh an pointe $M$ ina ngearann na dronlínte $AB$ agus $XY$ a chéile. Deimhnigh leis an riail go bhfuil na línte $MX$ agus $MY$ comhfhada.

2. Tagann dhá dronlíne le céile i bpointe $A$. Ar dhronlíne acu gearrtar míreanna cómhfhada $AX$ is $XB$, agus mar an céanna déantar $AY$ cómhfhada le $YC$ ar an líne eile. Deimhnigh gur faide faoi dhó $BC$ ná $XY$.

   Tosach leathanach 9 sa LSS.

3. Trí pointí ar bith is ea $A, B, C$. Tógtar pointe ar bith $X$ in $BC$ agus pointe ar bith $Y$ in $CA$. Más in $O$ a thagann na línte $AX$ agus $BY$ le chéile, faigh an pointe $Z$ ina dteagmhaíonn $CO$ le $AB$.

   Deimhnigh gur pointí in aon dronlí ne amháin iad, pointí teagmhála $BC$ le $YZ$, $CA$ le $ZX$ agus $AB$ le $XY$.

4. Dronlínte teangmhálacha is ea $\ell$ agus $m$. Tógtar trí pointí are bith $A, B, C$ in ord a chéile ar $\ell$; agus trí pointí $L, M, N$ in ord a chéilee ar $m$.

   Faigh pointí teagmhála $BL$ le $AM$, $BN$ le $MC$, $AN$ le $LC$ agus deimhnigh gur pointí in aon dronlíne amháin iad.

5. Sé pointí in ord a chéile ar imlíne chiorcail is ea $A, B, C, D, E, F$.

   Faigh pointí teagmhála $AC$ le $BF$, $AD$ le $BE$, $CE$ le $DF$, agus deimhnigh gur pointí in aon dronlíne amháin iad.

## Exercises on Drawing with Ruler and Compass

1. Draw a circle with centre at some point $A$ that goes through some other point $X$. Draw another circle through $X$ with centre at some other point $B$. If the two circles meet at the point $Y$, find the point $M$ in which the the straight lines $AB$ and $XY$ meet. Check with the ruler that the lines $MX$ and $MY$ have the same length.

2. A pair of straight lines meet at a point $A$. On one of those straight lines cut equal segments $AX$ and $XB$, and in the same way make $AY$ the same length as $YC$ on the other line. Verify that $BC$ is twice as long as $XY$.

3. Take three points $A, B, C$. Pick any point $X$ on $BC$ and any point $Y$ on $CA$. If the lines $AX$ and $BY$ meet at the point $O$, find the point $Z$ at which $CO$ meets $AB$.

   Verify that the points where $BC$ meets $YZ$, $CA$ meets $ZX$, and $AB$ meets $XY$ lie on a single straight line.

4. The straight lines $\ell$ and $m$ meet. Take any three points $A, B, C$ in order on $\ell$; and any three points $L, M, N$ in order on $m$.

   Find the points where $BL$ meets $AM$, $BN$ meets $MC$, and $AN$ meets $LC$, and verify that they all lie on a single straight line.

5. Take six points $A, B, C, D, E, F$ in order on a circle. Find the points where $AC$ meets $BF$, $AD$ meets $BE$, and $CE$ meets $DF$, and verify that they all lie on a single straight line.

For more on these exercises, see the last chapter.

## Caibidil II

# Casadh an Phlána.

### 1. Casadh

Nuair a castar roth timpeall ar acastóir shocair, ní athraíonn ionad an rotha iomláin sa spás cé go ngluaiseann na pointí difriúla den roth sin. Is amhlaidh go nglacann gach pointe P, de bharr casta áirithe, an t-ionad ina raibh pointe éigin eile Q den roth roimhe sin. Nuair nach miste neamhshuim a dhéanamh den tiús agus an roth a shamhlú mar dhoigh dhe gur fíoghair phlánach a bhí ann, tig linn a rá go sleamhnaíonn plána an rotha air féin.

Sé lár an rotha an t-aon phointe amháin nach mbogann

Mar an gcéanna, is féidir plána geoiméadrach a shleamhnú air féin ionnas go gcoinnítear pointe amháin O (agus gan ach an pointe sin) socair. Tugtar casadh an phlána timpeall O ar an ngluaiseacht sin.

Tig linn an ghluaiseacht úd a léiriú mar seo a leanas. Leag páipéar trí-shoillseach anuas ar pháipéar eile atá ar an mbord, agus sáith biorán thríothu ag pointe O. Línigh fíoghair ar bith ar an bpáipéar íochtarach agus rianuigh an fhíoghair chéanna ar an bpáipéar thrí-shoillseach. Má coinnítear an páipéar thíos socair le linn dúinn an ceann eile a shleamhnú air, timpeall O, léiríonn sé sin cé'n chaoi a n-athraítear ionad na fíoghaire de bharr chasta áirithe.

Cé go bhfuil dhá pháipéar san léiriú, is mar aon phlána amháin a samhlaítear dúinn iad, mar déantear neamhshuim de thiús na bpáipéar. Comhairlítear don léitheoir an modh sin a chleachtadh go mbí eolas maith aige ar chasadh an phlána. Is ceart dó go h-áirithe a dó nó a trí de (i) phointí, (ii) de dhronlínte trí O, (iii) de chiorcail ar lár dóibh O, (iv) de dhronlínte nach ngabhann trí O, a ghrinniú, féachaint céard a thaireas dóibh de bharr na gcasadh difriúil timpeall O.

# Caibidil 2

# Casadh an Phlána

Tosach leathanach 10 sa LSS.

# Rotating the Plane

## 2.1 Casadh

Nuair a castar roth timpeall ar acastóir shocair, ní athríonn ionad an rotha iomláin sa spás cé go ngluaiseann na pointí difriúla den roth sin. Is amhlaidh go nglacann gach pointe $P$, de bharr casta áirithe, an t-ionad ina raibh pointe éigin eile $Q$ den roth roimhe sin. 'Sé lár an rotha an t-aon pointe amháin nach mbogann. Nuair nach miste neamhshuim a dhéanamh den tiús agus an roth a shamhlú mar doigh de gur fíoghair phlánach a bhí ann, tig linn a rá go sleamhnaíonn plána an rotha air féin.

Mar an gcéanna, is féidir plána geómétrach a shleamhnú air féin ionas go gcoinnítear pointe amháin $O$ (agus gan ach an pointe sin) socair. Tugtar *casadh an phlána timpeall* $O$ ar an ngluaiseacht sin.

Tig linn an ghluaiseacht úd a léiriú mar sea leanas. Leag páipéar *trí-shoillseach* anuas ar pháipéar eile atá ar an mbord, agus sáth biorán thríothu ag pointe $O$. Línigh fíogair ar bith ar an bpáipéar íochtarach agus rianuigh an fhíi ogair céanna ar an bpáipéar thríshoillseach. Má coinnítear an páipéar thíos socair le linn dúinn an ceann eile a shleamhnúair, timpeall $O$, léiríonn sé sin cé'n chaoi a n-aithrítear ionad na fíi ogaire de bharr chasta áirithe.

Cé go bhfuil dhá pháipéar sn léiri'u, is mar aon phlána amháin a samhlaítear dúinne iad, mar déantar neaspás de thiús na bpáipéar. Comhairlítear don léitheoir an modh sin a chleachtadh go mbí eolas maith aige ar chasadh an phlána. Is ceart dó go h-áirithe a dó nó a trí de (i) phointí , (ii) de dhronlínte trí $O$, (iii) de chiorcail ar lár dóibh $O$, (iv) de dhronlínte nach ngabhann trí $O$, a ghrinniú, féachaint céard a bhaineas dóibh de bharr na gcasadh ndrifriúl timpeall $O$.

## 2.2 Rotation (Turning)

When a wheel rotates on a fixed axle, the position of the wheel as a whole in space does not vary, even though the various individual points of the wheel do move. In fact, as a result of the rotation, each point $P$ of the wheel occupies the position that was previously occupied by some other point $Q$. The centre of the wheel is the only point that does not move. When we ignore its thickness, and think of the wheel as a plane figure, we may say that the plane of the wheel slides round upon itself.

In the same way, it is possible for a whole geometrical plane to slide around on itself in such a way that some point $O$ (and only that point) remains fixed. That kind of motion is called a *rotation of the plane about O*.

We can illustrate that motion as follows. Place a sheet of transparent paper down flat on another sheet of paper on the table, and stick a pin through them at the point $O$. Draw any figure at all on the lower sheet, and trace the same figure on the transparent paper. If you keep the lower paper fixed while sliding the other sheet round, that shows how the position of the figure varies as a result of particular rotations.

Even though this trial involves two sheets of paper, we think of them as a single plane, because we ignore the thickness of the paper. The reader is advised to try out this method until he has a good understanding of the idea of a plane rotation. In particular, he should try drawing two or three of (i) points, (ii) straight lines though $O$, (iii) straight lines that do not pass through $O$, and see what happens to them as a result of different rotations around $O$.

## 2. Dhá Threo an Chasta. Casadh Iomlán.

Ag trácht dúinn ar chasadh áirithe timpeall O, tá dhá chaoi ina bhféadtar pointe (nó dronlíne, etc.) a shamhlú; (i) mar ionad tosaigh an phointe (nó na dronlíne) sin féin, nó (ii) mar ionad deiridh pointe (nó dronlíne) éigin eile de bharr an chasta. Beidh sé soiléir i gcomhnaí cé acu den dá chiall sin a bheas i gceist.

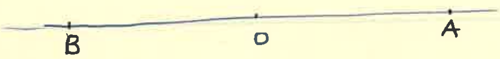
tosaigh?

Tá dhá threo chontrárdha ann chun plána a chasadh ionntu; (a) mar is in aghaidh treo shnáthaidí an chlog a castar an plána, agus (b) casadh i gcomhthreo leo. D'fhonn idirdhealú idir an dá threo sin tugtar casadh deimhneach ar (a), agus casadh diúltach ar (b).

(a) i n-aghaidh threo shn. an chloig
(b) i gcomhthreo leo.

Is soiléir gur féidir an plána a chasadh timpeall ar phointe ar bith O, ionnas go gcuirtear gach pointe (agus gach fíoghair) ar ais mar a raibh sé i dtosach. Tugtar casadh iomlán ar an gcasadh is lú a dhéanfadh sin.

Tá casadh eile ann a chuireas OA ar OB (agus a chuireas OB ar OA) áit gur dronlíne ar bith tré O í AOB, ionnas




nach n-athraíonn ionad na dronlíne iomláine dá bharr, cé go n-athraítear pointí éagsúla na dronlíne sin go léir, cé is moite de O. Is cuma deimhneach nó diúltach don chasadh sin, 'sé an cás céanna é maidir le pointí an phlána, agus má cuirtear i ngníomh in athuair é is ionann é sin agus casadh iomlán. Tugtar a leath de chasadh iomlán ar an gceann sin.

## 3. Uilleacha.

Chun dronlíne a h-aon (sa léaráid) a leagan anuas ar dhronlíne a dó, teastaíonn casadh áirithe timpeall O, agus 'sé a chontrárdha sin a chuirfeas dronlíne 2 ar dhronlíne 1:

Tosach leathanach 11 sa LSS.

## 2.3 Dhá Treo an Chasta. Casadh Iomlán

Ag trácht dúinn ar chasadh áirithe timpeall $O$, tá dhá chaoi ina bhféadtar pointe (nó dronlíne, etc) a shamhlú :
(i) mar ionad tosaigh an phointe (nó na dronlíne) sin féin, nó
(ii) mar ionad deiridh phointe (nó dronlíne) éigin eile de bharr an chasta.
Beidh sé soiléir i gcomhnaí cé acu den dá chiall sin a bhéas i gceist.

Tá dhá threo chontrárdha ann chun plána a chasadh ionntu:
(a) i n-aghaidh threo snáthaidí an cluig, nó
(b) i gcómhthreo leo.
D'fhonn idirdhealú idir an dá threo sin tugtar casadh *deimhneach* ar (a), agus casadh *diúltach* ar (b).

Is soiléir gur féidir an plána a chasadh timpeall ar phointe ar bith, ionas go gcuirtear gach pointe (agus gach fíogair) ar ais mar a raibh sé i dtosach. Tugtar *casadh iomlán* ar an gcasadh is lú a dhéanfadh é sin.

Tá casadh eile ann a chuireas $OA$ ar $OB$ (agus a chuireas $OB$ ar $OA$) áit gur dronlíne ar bith trí $O$ í $AB$, ionas nach nathríonn ionad na dronlíne iomláine dá bharr, cé go n-athrítear pointí éagsúla na dronlíne sin go léir, cé is moite de $O$. Is cuma deimhneach nó diúltach don chasadh sin, 'sé an chás céanna é maidir le pointí an phlána, agus má cuirtear i ngníomh in athuair é is ionann é sin agus casadh iomlán. Tugtar a leath de chasadh iomlán ar an gceann sin.

## 2.4 The two directions of rotation. Full rotations.

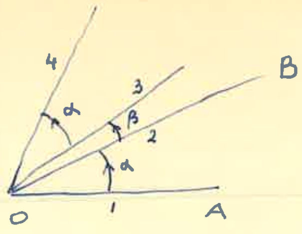
When we talk about a particular rotation about $O$, there are two ways to think of a point (or a straight line, etc):
(i) as the starting position of that point (or straight line), or
(ii) as the final position of some other point (or straight line) after the rotation.
It will always be clear which of the two ways is in question.

There are two opposite ways to turn a plane:
(a) counterclockwise or (b) clockwise.
In order to distinguish between those two ways, we call (a) a *positive* rotation, and (b) a *negative* rotation.

Obviously it is possible to turn the plane around any point in such a way as to bring each point (and each figure) back into its original position. We call the least rotation that does that a *full rotation*.

There is another rotation that puts $OA$ on $OB$ (and puts $OB$ on $OA$) when $AB$ is any straight line through $O$, so that the position of the whole straight ine is not changed, even though all the points of the straight line do move, apart from $O$. It makes no difference whether that rotation is positive or negative, it has the same effect on all points of the plane, and if the same rotation is repeated it has the same effect as a full rotation. This rotation is called half of a full rotation.

*AB* is not quite any line through *O*.

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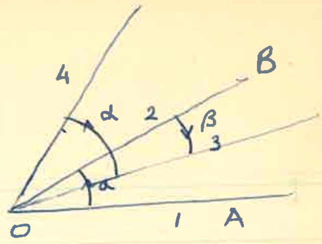



II

Sonrú — Is ionann an uille idir na dronlínte OA is OB agus méad an chasta (timpeall a bpointe teagmhála) a chuireas OA fan OB.

'Siad na línte OA, OB géaga na h-uilleann, agus 'sé O an rinn.

Tabhair fá deara nach bhfuil baint ar bith idir méad na h-uilleann agus fad na ngéag.

Má thárlaíonn go leagtar líne 3 ar líne 4 de bharr an chasta a chuireas líne 1 ar líne 2, deirtear go bhfuil an uille idir na línte 1 is 2 cothrom, nó ar comhméad, leis an uillinn idir na línte 3 is 4. [Scríobhtar an nod = in áit "cothrom le"]. Ciallaíonn $A\hat{O}B$ an uille idir OA is OB ach is é a chomharthú le litir gréigise $\alpha$ (nó, $\beta$, $\gamma$, etc.) is mó a déanfar sa leabhar seo:

1. $A\hat{O}B = \hat{\alpha}$ i bhfiog. I, II.

Chun dronlíne 1 a leagan anuas ar dhronlíne 3 níor mhór casadh $\hat{\alpha}$ i dtosach agus casadh eile $\hat{\beta}$ ina dhiaidh sin, gurb ionann i dteannta a chéile iad agus casadh singil áirithe timpeall O. Ní léir dúinn roimh ré céard a bhaineas do dhronlíne 2 de bharr na gluaiseachta sin, ach is féidir é sin a thriáil le páipéar trí-shoilleach. Gheofar amach gur anuas go cruinn ar dhronlíne 4 a leagtar í, rud a dteastaíonn casadh $\hat{\beta}$ i dtosach agus casadh $\hat{\alpha}$ ina dhiaidh sin chuige. 'Sé sin le rá, is cuma cé acu den dá chasadh $\hat{\alpha}$ is $\hat{\beta}$ a cuirtear i ngníomh i dtosach, is ionann meandí iad agus casadh singil áirithe.

Fágann sin nach miste casaidh timpeall an phointe chéanna O (agus na h-uilleacha a fhreagraíos dóibh) a shuimiú is a dhealú, agus beidh $\hat{\alpha}+\hat{\beta} = \hat{\beta}+\hat{\alpha}$ fearacht na ngnáth-uimhreacha. Léiríonn Fiog. II go bhfuil $\hat{\alpha} - \hat{\beta} = -\hat{\beta} + \hat{\alpha}$.

Na h-Uilleacha $2\hat{\alpha}$, $3\hat{\alpha}$, $\frac{1}{2}\hat{\alpha}$.

Nuair castar OA go dtí OB, abair gurb é OC ionad nua

## 2.5 Uilleacha

Chun dronlíne a h-aon (sa léaráid) a leagan anuas ar dhronlíne a dó , teastaíonn casadh áirithe timpeall $O$, agus 'sé a chontrárdha sin a chuirfeas dronlíne 2 ar dhronlíne 1:

Tosach leathanach 12 sa LSS.

Tá Fíoghair anseo sa LSS, leathanach 12.

Is ionann an uille idir na dronlínte $OA$ is $OB$ agus méad an chasta (timpeall a bpointe teagmhála) a chuireas $OA$ fan $OB$.

'Siad na línte $OA$,$OB$ *géaga* na huilleann, agus sí $O$ an *rinn*.

Tabhair faoin ndeara nach bhfuil baint ar bith idir méad na nuilleann agus fad na ngéag.

Má thálaíonn go leagtar líne 3 ar líne 4 de bharr an chasta a chuireas líne 1 ar líne 2, deirtear go bhfuil an uille idir na línte 1 is 2 cothrom, nó ar comhmhéad, leis an uilinn idir na línte 3 is 4. [Scríobhtar an nod = is áit “cothrom le"]. Ciallaíonn $\widehat{AOB}$ an uille idir $OA$ is $OB$ ach is é a chomarthú lelitir gréigise $\alpha$ (nó $\beta, \gamma$, etc) is mó a déanfar sa leabhar seo: .i. $\widehat{AOB} = \hat{\alpha}$ is bhfíog I,II.

Chun dron líne 1 a leagan anuas ar dhronlíne 3, níor mhór casadh $\hat{\alpha}$ is dtosach agus casadh eile $\hat{\beta}$ 'ina dhiaidh sin, gurb ionann i dteannta a chéile iad agus casadh singil áirithe timpeall $O$. Ní léir dúinn roimhré céard a bhaineas de Dhronlíne 2 de bharr na gluaiseachta sin, ach is féidir é sin a thriáil le páipéar trí-shoillseach. Gheófar amach gur anuas go cruinn ar dhronlíne 4 a leagtar í , rud a dteastaíonn casadh $\hat{\beta}$ agus casadh $\hat{\alpha}$ ina dhiaidh sin chuige. 'Sé sin le rá , is cuma cé acu den dá chasadh $\hat{\alpha}$ is $\hat{\beta}$ a cuirtear i ngníomh i dtosach, is ionann in éindí iad agus casadh singil áirithe.

Fágann sin nach miste casaidh timpeall an phointe chéanna $O$ (agus na huilleacha a fhreagraíos dóibh) a shuimiú is a dhealú , agus beidh $\hat{\alpha} + \hat{\beta} = \hat{\beta} + \hat{\alpha}$ fearacht na ngnáth-uimhreacha. Léiríonn Fiog II go bhfuil $\hat{\alpha} - \hat{\beta} = -\hat{\beta} + \hat{\alpha}$.

## 2.6 Angles

To lay the straight line 1 (in the figure) on top of straight line 2 requires a particular rotation around $O$, and the opposite rotation lays straight line 2 on straight line 1.

(Page 12 in MSS, Figure on page 12)

The angle between the straight lines $OA$ and $OB$ is the size of the rotation (about the point where they meet) that lays $OA$ along $OB$.

The lines $OA$ and $OB$ are the *arms* of the angle, and the point $O$ is its *vertex*.

Notice that there is no connection at all between the length of the arms and the size of the angle.

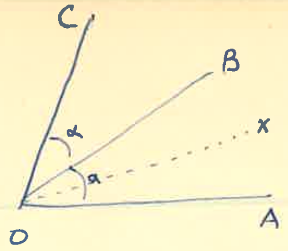
If it happens that line 3 is laid on line 4 by the rotation that puts line 1 on line 2, we say that the angle between the lines is equal, or the same size as, the angle between the lines 3 and 4.

[One writes the sign = instead of “equal to".]

$\widehat{AOB}$ means the angle between $OA$ and $OB$, but in this book we more often symbolise it with a greek letter $\alpha$ (or $\beta, \gamma$, etc) .i. $\widehat{AOB} = \hat{\alpha}$ in Figures I and II.

To move straight line 1 onto straight line 3 requires the rotation $\hat{\alpha}$ first, followed by the rotation $\hat{\beta}$, which combined together are equivalent to a certain single rotation about $O$. It is not obvious to us in advance what is the effect of this motion on straight line 2, but you can try it out using transparent tracing paper. One finds that line 2 lands exactly on straight line 4, a result that requires a rotation $\hat{\beta}$ to start followed by a rotation $\hat{\alpha}$. That is to say that it does not matter which of the two rotations $\hat{\alpha}$ and $\hat{\beta}$ one executes first, the have the same combined effect as a certain single rotation.

As a result we are free to add and subtract rotations (and the angles that correspond to them) about the same point $O$, and we have $\hat{\alpha} + \hat{\beta} = \hat{\beta} + \hat{\alpha}$ just as with ordinary numbers. Fig 2 shows that $\hat{\alpha} - \hat{\beta} = -\hat{\beta} + \hat{\alpha}$.

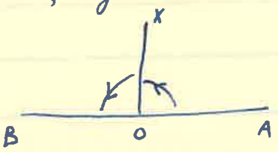



na dronlíne a bhí fan na líne OB i dtosach. Is léir nár mhór an casadh $\hat{\alpha}$ faoi dhó chun OA a leagan ar OC; .i. tá $O\hat{A}C = 2\hat{\alpha}$. Is féidir na h-uilleacha $3\hat{\alpha}$, $4\hat{\alpha}$ etc., a mhíniú ar an gcuma chéanna.

Tá dronlíne chinnteach OX ann freisin a ghníos an uille $\frac{1}{2}\hat{\alpha}$ le OA. Taspáinfear i gCaib. III cé'n chaoi a n-aimsítear ionad OX. Deirtear go gcomhroinneann sí an uille $A\hat{O}B$.

## 4. An Dronuille

Uilleacha cómhgaracha is ea uilleacha a bhfuil cómhghéag acu, agus uille acu ar gach aon taobh den chómhghéig sin.




Má sheasann dronlíne OX ar cheann eile AB ionnas gur cómhionann don dá uillinn chómhgaracha, is é an t-aon chasadh amháin a chuireas OA fan OX, nó a chuireas OX fan OB. Is léir gurb ionann an casadh sin faoi dhó agus a leath de chasadh iomlán. Tugtar dronuille ar gach uillinn den dá uillinn chothroma, chómhgaracha ionnas gur mar a chéile casadh iomlán agus casadh tré cheithre dronuilleacha. (Annamh tugtar uille dhíreach ar dhá dhronuillinn). Deirtear go bhfuil OX agus AB ingearach le chéile má's dronuilleacha iad $A\hat{O}X$ agus $X\hat{O}B$.

De bhrí go bhfuil dronuille cothrom lena cómhuillinn cómhgaraí tá suimeitreacht taithneamhach ag gabháil léithe agus is mór an leas a baintear aiste i bhfoirgníocht, líníocht, etc.

## 5 Bun-Phrionnsabal. Teoirmí

Fritheadh amach le trialacha (in Alt 3) gur féidir an uille $\hat{\alpha}$ idir na línte 1 is 2, a leagan anuas go cruinn

### Na hUilleacha $2\hat{\alpha}, 4\hat{\alpha}, \frac{1}{2}\hat{\alpha}$

Nuair castar $OA$ go ddí $OB$, abair gurb é $OC$ ionad nua

Tosach leathanach 13 sa LSS.

na dronlíne a bhí fan na líne $OB$ i dtosach. Is léir nach mór an casadh $\hat{\alpha}$ faoi dhó chun $OA$ a leagan ar $OC$; .i. tá $\widehat{AOC} = 2\hat{\alpha}$. Is féidir na huilleach $3\hat{\alpha}$, $4\hat{\alpha}$ etc, a mhíniú ar an gcuma chéanna.

Tá dronlíne cinnteach $OX$ ann freisin a ghníos an uille $\frac{1}{2}\hat{\alpha}$ le $OA$. Teaspáinfear i gCaibidil III cé'n chaoi a n-aimsítear ionad $OX$. Deirtear go hcómhroinneann sí an uille $\widehat{AOB}$.

### The Angles $2\hat{\alpha}, 4\hat{\alpha}, \frac{1}{2}\hat{\alpha}$

When $OA$ is rotated to $OB$, suppose that $OC$ is the new position of the straight line that was along $OB$ at the start.

Clearly it takes two goes of the rotation $\hat{\alpha}$ to lay $OA$ on $OC$; .i. $\widehat{AOC} = 2\hat{\alpha}$. The angles $3\hat{\alpha}$, $4\hat{\alpha}$ and so on can be explained in the same way.

There is also a definite straight line $OX$ that makes the angle $\frac{1}{2}\hat{\alpha}$ with $OA$. It will be shown in Chapter III how one finds the position of $OX$. One says that it *bisects* the angle $\widehat{AOB}$.

## 2.7 An dronuille

Uilleacha *cómhgoracha* is ea uilleacha a bhfuil cómhgéag acu, agus uille acu ar gach aon taobh den chómhgéig sin.

Tá Fíoghair anseo sa LSS, leathanach 13.

Má sheasann dronlíne $OX$ ar cheann eile $AB$ ionas gur cómhéad don dá uillinn chómhgoracha, is é an t-aon casadh amháin a chuireas $OA$ fan $OX$, nó a chuireas $OX$ fan $OB$. Is léir gurb ionann an casadh sin faoi dhó agus a leath de chasadh ionlán. Tugtar *dronuille* ar gach uilinn den dhá uilinn cothroma, chómhgoracha ionas gur mar a chéile casadh ionlán agus casadh tré cheithre dronuilleacha. (Amanta tugtar *uille dhíreach* ar dhá dhronuillinn.) Deirtear go bhfuil $OX$ agus $AB$ *ingearach* le chéile m'ás dronuilleacha iad $\widehat{AOX}$ agus $\widehat{XOB}$.

De bhrígo bhfuil dronuille cothrom lena cómhuillinn cómhgaraí tá suiméatracht taithneamhach ag gabháil léi agus is mór an leas a baintear aisti i bhfoirgníocht, líníocht, etc.

## 2.8 The right angle

*Adjacent angles* are angles that have have a common arm, and lie one on each side of that arm.

If one straight line $OX$ stands on another $AB$ in such a way that the two adjacent angles are equal, then the rotation that lays $OA$ along $OX$ is the same as the rotation that lays $OX$ along $OB$. Clearly two times that rotation is the same as half a full rotation. We call each of the two equal adjacent angles a *right angle*, so that a full rotation is the same as a rotation through four right angles. (Sometimes two right angles is called a *straight angle*.) One says that $OX$ agus $AB$ are *perpendicular* or *orthogonal* to one another if $\widehat{AOX}$ and $\widehat{XOB}$ are right angles.

The fact that a right angle is equal to its adjacent angle means that there is a pleasing symmetry to it, and this is much exploited in architecture, drawing, etc.

ar an uillinn chothrom eile ∠ idir na línte 3 is 4 le casadh an phlána timpeall O. Is féidir aon dá uillinn ag O a chur i gcóimheas le chéile ar an gcuma sin, agus uilleacha cothroma is ea iad má leagtar uille acu ar an gceann eile de bharr chasadh áirithe timpeall O. Má leagtar líne AB (bíodh sé díreach nó lúbtha) anuas go beacht ar líne eile CD, deirtear go bhfuil na línte sin congruach, nó comhfhada.

Níor mhiste dá réir an prionnsabal seo a chur ar bun.

Bun-Phrionnsabal I.

Fanann méad gach uilleann agus fad gach líne buan de bharr an phlána a chasadh timpeall ar phointe ar bith ann.

Maidir leis na fírinní geoméadracha gur féidir iad a ghnóthú ón mbun-phrionnsabal le réasúnaíocht, is gnáthach gur i bhfoirm teoirmí a cóirítear iad .i. ráiteas ginearálta fá fhíoghair gheoméadraigh agus é roinnte ina dhá chuid, viz. an hipotéis, ina gcuirtear in iúl dúinn roimh ré go bhfuil tréith áirithe ag an bhfíoghair, agus an tátall, a fhuagraíos gur féidir eolas ar thréith éigin eile a ghnóthú ón hipotéis le réasúnaíocht. Cuirtear an réasúnaíocht ar fáil sa gcruthúnas.

e.g. I dteoirim I seo a leanas, is dronlíne a sheasann ar dhronlíne eile (agus ní hé dhá chioreal, ná cioreal agus dronlíne, etc.) a ghníos an fhíoghair; sin é an hipotéis. 'Sé an tátall gurb é dhá dhronuillinn suim an dá uillinn comhgaraach.

De bharr an hipotéis agus an tátall a mhalairt ar a chéile, gheibhtear teoirim nua dá ngoirtear coinvéarsa na céad teoirme. Má's fíor bréagach don teoirim féin áfach, ní gá gur fírinne an coinvéarsa.

e.g. Má suimítear dhá uimhir réidh, is uimhir réidh í an tsuim.

Is fíor é sin; mar shampla tá, 6 + 4 = 10.

'Sé an choinvéarsa sin ná;

má's uimhir réidh í suim dhá uimhir, is uimhreacha réidhe a suimítear.

Is léir go bhfuil sé sin bréagach; mar shampla tá 7 + 3 = 10.

## 2.9 Bun-phroinseabal. Teormí

Fritheach amach le triáileacha (in Alt 2.5) gur féi dir an uille $\hat{\alpha}$ idir na línte 1 is 2, a leagan anuas go cruinn

Tosach leathanach 14 sa LSS.

ar an uillinn chothruim eile $\hat{\alpha}$ idir na línte 3 is 4 le casadh an phlána timpeall $O$. Is féidir aon dá uilinn ag $O$ a chur i gcoibhneas le chéile ar an gcuma seo, agus uilleacha cothroma is ea iad má leagtar uille acu ar an gceann eile de bharr chasta áirithe timpeall $O$. Má leagtar líne $AB$ (bíodh sí díreach nó lúbtha) anuas go beacht ar líne eile $CD$, deirtear go bhfuil na línte sin congrúach nó cómhfhada.

Níor mhiste dá réir an prionnsabal seo a chur ar bun.

### Bun-Phrionnsabal I

*Fanann méad gach uilleann agus fad gach líne buan de bharr an plána a chasadh timpeall ar phointe ar bith ann.*

Maidir leis na fírinní geómétracha gur féidir iad a ghnothú ón mbun-phroinnsabal le réasúnaíocht, is gnáthach gur i bhfoirm *teoirmí* a cóirítear iad .i. ráiteas geinearáltha fá fíoghair gheómétraigh, agus é roinnte ina dhá chuid, viz. an *hipotéis*, ina gcuirtear in iúl dúinn roimhré go bhfuil tréith áirithe ag an bhfíoghair, agus an *tátall*, a fhuagraíos gur féidir eolas ar thréith éigin eile a ghnothú ón hipotéis le réasúnaíocht. Cuirtear an réasúnaíocht ar fáil sa *gcruthúnas*.

e.g. i dteoirim I seo a leanas, is dronlíne a sheasann ar dhronlíne eile (agus ní hé dhá chiorcal, ná ciorcal agus dronlíne, etc) a ghníos an fhíoghair; sin é an hipotéis. 'Sé an tátall gurb dhá dhronuillinn suim an dá uillinn comhgarach.

De bharr an hipotéis agus an tátall a mhalairt ar a chéile, gintear teoirim nua dá ngoirtear coinvéarsa na céad teoirme. Má's fíor bréagach don teoirim féin, áfach, ní gá gur fírinne an coinvéarsa.

e.g. Má suimítear dhá uimhir réidh, is uimhir réidh í an tsuim. Is fíor é sin; mar shampla tá $6+4=10$. 'Sé an choinvéarsa sin ná :
Má's uimhir reidh í suim dhá uimhir, is uimhreacha réidhe a suimítear.
Is léir go bhfuil sésin bréagach; mar shampla tá $7+3=10$.

## 2.10 An Axiom. Theorems

We found experimentally (in section 2.6) that the angle $\hat{\alpha}$ between the lines 1 and 2 could be laid exactly on the other equal angle $\hat{\alpha}$ between the lines 3 and 4 by rotating the plance about $O$. In this way, any two angles at $O$ can be compared to one another, and they are equal angles if one can be laid on the other as a result of a rotation about $O$. If a curve $AB$ (straight line or not) is laid exactly on another curve, one says that those curves are are congruent or isometric.

It behoves us to lay down this principle:

## Axiom I

*The size of each angle and the length of each curve remain unchanged when the plane is rotated about any of its points.*

It is customary to set out the geometrical truths that can be deduced by reasoning from axioms (basic principles) in the form of *theorems*, .i. general statements about a geometric figure, divided into two parts, viz. the *hypothesis*, in which one specifies in advance that the figure has some specific properties, and the *conclusion*, which asserts that some other properties of the figure can be derived from the hypothesis by reasoning. The reasoning is provided in the *proof*.

e.g. in the following theorem, the figure involves a straight line standing on another straight line (as opposed to two circles, or a circle and a straight line, etc); that is the hypothesis. The conclusion is that the two adjacent angles add to two right angles.

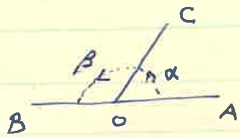
By interchanging the hypothesis and the conclusion one generates a new theorem called the converse of the original theorem. However, the truth of the original theorem does not guarantee the truth of the converse.

e.g. *If you sum two even numbers, you get an even number.* This is true; for instance, $6+4=10$.

The converse statement is:
*If the sum of two numbers is even, then the numbers are both even.*
Obviously that is FALSE: for instance $7+3=10$.

## Teoirim I

Nuair a sheasann dronlíne ar dhronlíne eile, is ionann suim an dá uillinn chómhgarach agus dhá dhronuillinn.




**Hipotéis**

Dronlíne is ea OC a sheasann ar an dronlíne AB.

**Tátall**

Tá $\hat{\alpha} + \hat{\beta}$ = dhá dhronuillinn.

**Cruthúnas**

De bharr an fléra a chasadh timpeall O tré'n uillinn $\hat{\alpha}$, cuirtear OA fan na líne OC.

De bharr chasta eile tré'n uillinn $\hat{\beta}$ ina dhiaidh sin, cuirtear OC fan na líne OB.

Fágann sin go leagtar OA fan OB de bharr an chasta $\hat{\alpha} + \hat{\beta}$.

Ach sin a leath de chasadh iomlán mar is aon dronlíne amháin iad AO, OB.

$\therefore \hat{\alpha} + \hat{\beta}$ = dhá dhronuillinn.

Q.E.D.

Is fíor é coinvéarsa teoirme I, mar atá:

## Teoirim I(a)

Má's ionann is dhá dhronuillinn suim dhá uillinn chómhgarach, tá dhá ghéig in aon dronlíne amháin.

**Hipotéis**

Tá $\hat{\alpha} + \hat{\beta}$ = dhá dhronuillinn, agus dronlínte is ea OA, OC, OB.

**Tátall**

Tá AO agus OB in aon dronlíne amháin.

**Cruthúnas**

De bharr an chasta $\hat{\alpha} + \hat{\beta}$ timpeall O, cuirtear OA fan OB. Sin a leath de chasadh iomlán (de réir na hipotéise) rud a leagann OA ar shíneadh na dronlíne AO.

$\therefore$ aon dronlíne amháin is ea AO agus OB.

Q.E.D.

Tosach leathanach 15 sa LSS.

## 2.11 Teoirim I

*Nuair a sheasann dronlíne ar dhronlíne eile, is ionann suim an dá uillinn chómhgarach agus dhá dhronuillinn.*

Tá Fíoghair anseo sa LSS, leathanach 15.

**Theorem 1.** *When a straight line stands on another, the sum of the two adjacent angles equals two right angles.*

*Hipotéis:*
Dronlíne is ea $OC$ a sheasann ar an dronlíne $AB$.

*Tátall:*
Tá $\hat{\alpha} + \hat{\beta}$ =dhá dhronuillinn.

*Cruthúnas:*
De bharr an plána a chasadh timpeall $O$ trén uilleann $\hat{\alpha}$, cuirtear $OA$ fan na líne $OC$.
De bharr casta eile trén uillean $\hat{\beta}$ ina dhiaidh sin, cuirtear $OC$ fan na líne $OB$.
Fágann sin go leagtar $OA$ fan $OB$ de bharr an chasta $\hat{\alpha} + \hat{\beta}$.
Ach sin a leath de chasadh iomlán mar is aon dronlíne amháin iad $OA, OB$.

$$\therefore\ \hat{\alpha} + \hat{\beta} = \text{dhá dhronuillinn.}$$

□

*Hypothesis:*
$OC$ is a straight line standing on the straight line $AB$.

*Conclusion:*
$\hat{\alpha} + \hat{\beta}$ =two right angles.

*Proof:*
If the plane is rotated about $O$ through the angle $\hat{\alpha}$, $OA$ is laid along the line $OC$.
If that is followed by another rotation through the angle $\hat{\beta}$, $OC$ is laid along $OB$.
Thus $OA$ is laid along $OB$ by the rotation $\hat{\alpha} + \hat{\beta}$.
But that is half of a full rotation, because $OA$ and $OB$ lie in a straight line.

$$\therefore\ \hat{\alpha} + \hat{\beta} = \text{two right angles.}$$

□

Is fíor é coinvéarsa Teoirme I, mar atá :

## Teoirim Ia

*Má's ionann is dhá dhronuillinn suim dhá uillinn chómhgarach, tá dhá ghéig in aon dronlíne amháin.*

*Hipotéis:*

Tá $\hat{\alpha} + \hat{\beta} =$dhá dhronuillinn agus dronlínte is ea $OA, OC, OB$.

*Tátall:*

Tá $AO$ agus $OB$ in aon dronlíne amháin.

*Cruthúnas:*

De bharr an chasta $\hat{\alpha} + \hat{\beta}$ timpeall $O$, cuirtear $OA$ fan $OB$. Sin a leath de chasadh iomlán (de réir na hipotéise) rud a leagann $OA$ ar síneadh na dronlíne $BO$.

∴ aon dronlíne amháin is ea $AO$ agus $OB$. □

The converse of Theorem 1 is true. It is:

**Theorem** (1a)**.** *If the sum of two adjacent angles is equal to two right angles, then the arms are in a single straight line.*

*Hypothesis:*

$\hat{\alpha} + \hat{\beta} =$two right angles and $OA, OC, OB$ are straight lines.

*Conclusion:*

$AO$ and $OB$ lie in a single straight line.

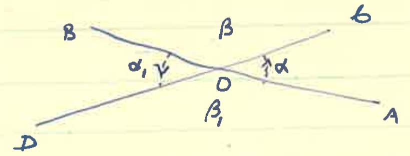
*Proof:*

The rotation through $\hat{\alpha} + \hat{\beta}$ around $O$ lays $OA$ along $OB$. That is half of a full rotation. (by hypothesis), which means that $OA$ lies on the extension of the straight line $BO$.

∴ $AO$ and $OB$ form a single straight line. □

## Teoirim II

Má theangmhaíonn dhá dhronlíne le chéile i bpointe O, is ionann gach uille ag O agus an uille atá ós a cóir anonn.




## Hipotéis

Dronlínte teangmhálacha iseadh AOB, bOD.

## Tátal

Tá $\hat{\alpha} = \hat{\alpha}_1$, agus tá $\hat{\beta} = \hat{\beta}_1$.

## Cruthúnas

cén plána?

Má castar an plána trí dhá dhronuillinn timpeall O, cuirfear OA ar an líne OB, agus cuirfear Ob ar an líne OD.

∴ Leagfar an uille $\hat{\alpha}$ anuas ar an uillinn $\hat{\alpha}_1$.

$\therefore \quad \hat{\alpha} = \hat{\alpha}_1$.

Is léir freisin go gcuirfear $\hat{\beta}$ ar $\hat{\beta}_1$, ionas go bhfuil $\hat{\beta} = \hat{\beta}_1$.

Q.E.D.

## Nóta

Maidir leis an téarma "an uille idir dhá dhronlíne" (e.g. idir na línte AB agus CD sa léaráid) tá éiginnteacht ag baint leis fiú amháin nuair nach gcuirtear uilleacha atá níos mó ná dhá dhronuillinn i bhfáth. Is féidir áfach idirdhealú idir $\hat{\alpha}$ is $\hat{\beta}$ mar seo; 'sí $\hat{\alpha}$ an uille dheimhneach idir AB is CD (nó an uille dhiúltach idir CD is AB) de bhrí nach foláir casadh sa treo deimhneach tríd an uillinn α, chun BA a leagan ar DC.

Is féidir an chéad líne acu a leagan ar an gceann eile le casadh sa treo diúltach freisin tríd an uillinn β.

Má's pointe ar bith é P ar chiorcal gurb é O a lár, cuirfear P ar phointe éigin eile den chiorcal sin de bharr a chastá timpeall O, de réir bun-phrionsabail I: ∴ is a shleamhnú fan a imlíne féin a dhéanas gach ciorcal ar lár dó O, nuair castar an plána timpeall O.

Tosach leathanach 16 sa LSS.

## 2.12 Teoirim II

*Má teagmhaíonn dá dhronlíne le chéile i bpointe O, is ionann gach uille ag O agus an uille atá ós a cóir amach.*

Tá Fíoghair anseo sa LSS, leathanach 16.

*Hipotéis:*

Dronlínte teagmhálacha is ea $AOB$,$COD$.

*Tátall:*

Tá $\hat{\alpha} = \hat{\alpha}_1$, agus tá $\hat{\beta} = \hat{\beta}_1$.

*Cruthúnas:*

Má castar an plána tré dhá dhronuillinn timpeall $O$, cuirfear $OA$ ar an líne $OB$, agus cuirfear $OC$ ar an líne $OD$. .i. Leagfar an uille $\hat{\alpha}$ anuas ar an uille $\hat{\alpha}_1$.

$$\therefore \hat{\alpha} = \hat{\alpha}_1.$$

Is léir freisin go gcuirfear $\hat{\beta}$ ar $\hat{\beta}_1$, ionas go bhfuil $\hat{\beta} = \hat{\beta}_1$. □

**Theorem 2.** *2 If two straight lines meet in a point O, each angle they make at O is equal to the angle opposite to it.*

*Hypothesis:*

$AOB$,$COD$ are straight lines that meet.

*Conclusion:*

We have $\hat{\alpha} = \hat{\alpha}_1$, and $\hat{\beta} = \hat{\beta}_1$.

*Proof:*

When the plane is rotated through two right angles about $O$, the line $OC$ lands on the line $OD$. .i. The angle $\hat{\alpha}$ is laid on the angle $\hat{\alpha}_1$.

$$\therefore \hat{\alpha} = \hat{\alpha}_1.$$

It is also clear that $\hat{\beta}$ is laid on $\hat{\beta}_1$, so that $\hat{\beta} = \hat{\beta}_1$. □

### Nótá

Maidir leais an téarma “an uille idir dhá dhronlíne" (e.g. idir na línte $AB$ agus $CD$ san léaráid) tá éigcinnteacht ag baint leis fiú amháin nuair nach gcuirtear uilleacha atá níos mó ná dhá dhonuillinn i bhfáth. Is féidir áfach idirdhealú idir $\hat{\alpha}$ is $\hat{\beta}$ mar seo: sé $\hat{\alpha}$ an uille *deimhneach* idir $AB$ is $CD$ (nó an uille dhiúltach idir $CD$ is $AB$) de bhrí nach foláir casadh deimhneach tríd an uille $\hat{\alpha}$, chun $BA$ a leagan ar $DC$.

Is féidir an chéad líne acu a leagan ar an gceann eile le casadh sa treo diúltach freisin tríd an uillinn $\hat{\beta}$.

Má's pointe ar bith é $P$ ar chiorcal gurb é $O$ a lár, cuirtear $P$ ar phointe éigin eile den chiorcal sin de bharr an chasta timpeall $O$, de réir bun-phrionnsabail I: .i. is a shleamhnú fan a imlíne féin a dheineas gach ciorcal ar lár dó $O$, nuair castar an plána timpeall $O$.

## Note

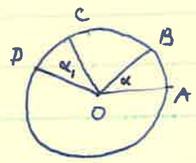
There is a lack of precision in the term "the angle between two straight lines" (e.g. between the lines $AB$ and $CD$ in the figure), even without taking into account the possibility that an angle might be greater than two right angles. However, it is possible to distinguish between $\hat{\alpha}$ and $\hat{\beta}$ as follows: $\hat{\alpha}$ is the (positive) angle between $AB$ and $CD$ (or the negative angle between $CD$ and $AB$) in the sense that you have to make a positive rotation through the angle $\hat{\alpha}$, to lay $BA$ on $DC$.

The first line may also be laid on the other one by a negative rotation through the angle $\hat{\beta}$.

If $P$ is any point on a circle with centre $O$, then $P$ is moved to some other point of that circle by a rotation about $O$, according to Axiom I: .i. each circle with centre $O$ slides along its own perimeter when the plane is rotated about $O$.

Teoirim III

I gciorcal ar bith, (i) gearrann uilleacha cothroma ag an lár stuaghanna cómhfhada den imlíne, agus (ii) gabhann stuaghanna cómhfhada uilleacha cothroma ag an lár.




(i) Hipotéis

'Sé O lár an ⊙ ABCD, agus tá $\hat{\alpha} = \hat{\alpha}_1$

Tátall

Tá an stuagh AB = an stuagh CD.

Cruthúnas

Má castar an plána timpeall O go gcuirtear $\hat{\alpha}$ ar a cómhuillinn $\hat{\alpha}_1$, gluaiseann A ar imlíne an ⊙ go dtí C, agus gluaiseann B go dtí D.

∴ leagtar an stuagh AB go cruinn ar CD.

∴ tá na stuaghanna cómhfhada. Q.E.D.

(ii) Hipotéis

'Sé O lár an ⊙ ABCD, agus tá an stuagh AB = an stuagh CD.

Tátall

Tá $\hat{\alpha} = \hat{\alpha}_1$.

Cruthúnas

Má cuirtear A ar C le casadh timpeall O, gluaiseann B go dtí pointe éigin eile ar an imlíne.

Ó tharla an stuagh AB = an stuagh CD, is ar D a thuiteas sé,

∴ leagtar an uille $\hat{\alpha}$ ar an uillinn $\hat{\alpha}_1$; ∴ $\hat{\alpha} = \hat{\alpha}_1$

Q.E.D.

Atora

Má's stuaghanna cómhfhada iad AB is CD, córdaí cómhfhada isea na córdaí AB is CD.

mar leagtar an córda AB anuas ar an gcórda CD.

[Ghheofar cruthúnas i gcaib. III gur fíor an coinbhéarsa].

Tosach leathanach 17 sa LSS.

## 2.13 Teoirim III

*I gciorcal ar bith, (i) gearann uilleacha cothroma ag an lár stuanna cómhfhada den imlíne, agus (ii) gabhann stuanna cómhfhada uilleacha cothroma ag an lár.*

Tá Fíoghair anseo sa LSS, leathanach 17.

(i)

*Hipotéis:*
'Sé $O$ lár an $\odot ABCD$, agus tá $\hat{\alpha} = \hat{\alpha}_1$.

*Tátall:*
Tá an stua $AB$ = an stua $CD$.

*Cruthúnas:*
Má castar an phlána timpeall $O$ go gcuirtear $\hat{\alpha}$ ar a cómhuillinn $\hat{\alpha}_1$, gluaiseann $A$ ar imlíne an $\odot$ go dtí $C$, agus gluaiseann $B$ go dtí $D$.
.i. leagtar an stua $AB$ go cruinn ar $CD$.
∴ tá na stauanna cómhfhada. □

(ii)

*Hipotéis:*
'Sé $O$ lár an $\odot ABCD$, agus tá an stua $AB$ = an stua $CD$.

*Tátall:*
Tá $\hat{\alpha} = \hat{\alpha}_1$.

*Cruthúnas:*
Má cuirtear $A$ ar $C$ le casadh timpeall $O$, gluaiseann $B$ dtí pointe éigin eile ar an imlíne.
Ó thárla an stua $AB$ = an stua $CD$, is ar $D$ a thuiteas sé .
∴ Leagtar an uille $\hat{\alpha}$ ar an uilinn $\hat{\alpha}_1$; .i. $\hat{\alpha} = \hat{\alpha}_1$.

**Theorem 3.** *In any circle (i) equal angles at the centre cut equal arcs on the perimeter, and (ii) equal arcs subtend equal angles at the centre.*

(i)

*Hypothesis:*
$O$ is the centre of the $\odot ABCD$, and $\hat{\alpha} = \hat{\alpha}_1$.

*Conclusion:*
The arc $AB$ = the arc $CD$.

*Proof:*

If the plane is rotated about $O$ until $\hat{\alpha}$ is placed on the equal angle $\hat{\alpha}_1$, $A$ on the perimeter of the ⊙ moves to $C$, and $B$ moves to $D$.

.i. the arc $AB$ is laid exactly on $CD$.

∴ the arcs have the same length. □

(ii)

*Hypothesis:*

$O$ is the centre of the $\odot ABCD$, and the arc $AB$ = the arc $CD$.

*Conclusion:*

$\hat{\alpha} = \hat{\alpha}_1$.

*Proof:*

If $A$ is laid on $C$ by a rotation about $O$, then $B$ moves to some other point on the perimeter.

Since the arc $AB$ = the arc $CD$, it is on $D$ that it lands.

∴ The angle $\hat{\alpha}$ is laid on the angle $\hat{\alpha}_1$; .i. $\hat{\alpha} = \hat{\alpha}_1$.

## Atora

*Má's stuanna cómhfhada iad $AB$ is $CD$, córdaí cómhfhada is ea na córdaí $AB$ is $CD$.*

Mar leagtar an córda $AB$ anuas ar an gcórda $CD$.

[Gheofar cruthúnas i gcaib. III gur fíor an coinvéarsa.]

**Corollary.** *If $AB$ and $CD$ are equal arcs, then the chords $AB$ and $CD$ have the same length.*

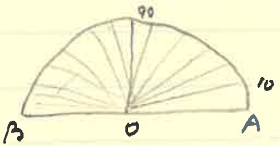
For the chord $AB$ is laid on the chord $CD$.

[We shall find a proof in Chapter III that the converse is also true.]

## Nóta 1

Baintear feidhm as teoirim III san uilleantomhas .i. áis chun uilleacha a thomhas. Leithchiorcal is ea é, agus an imlíne roinnte in 180 comhchoda aige. Tugtar grád (1° a scríobhtar) ar an uillinn a ghabhas stuagh ar bith de na 180 stuagha cothroma ag an lár. Tagann sin go bhfuil;

180° = dhá dhronuillinn.

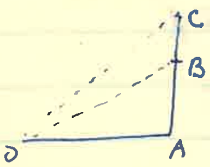



Chun uille a thomhas leagtar O ar rinn na h-uilleann agus cóirítear OA (nó OB) chun go luighfidh sé ar ghéig amháin. Áirítear an stuagh a ghearras an uille ar imlíne an leithchiorcail, agus uaidh sin cinntear méad na h-uilleann.

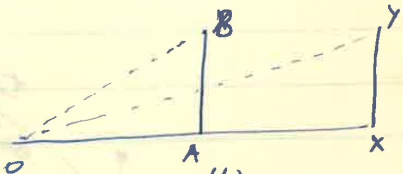
## Nóta 2

An té a bhreathnaíos ar dhá phointe spásúla P agus Q i ndiaidh a chéile, tig leis gairmheas a dhéanamh don uillinn PÔQ a ghabhas an dronlíne PQ ag a shúil féin O (tré casadh na súile a mheas, is dócha). Tá gléas ag seilbhéara chun an uille sin a thomhas go cruinn.

Is le h-uilleacha a ndéanann an gnáth-duine faid a mheas. I bhfioghair (a) is col dó gur faide AC ná AB de bhrí gur mó an uille AÔC a ghabhann AC ag an t-súil O.




(a)




(b)

Níor cheart dó a cheapadh áfach go bhfuil AB níos faide ná XY, cé gur mó an uille AÔB ná XÔY, mar is faide amach uaidh XY ná AB anois.

Mar an gcéanna nuair feictear dúinn nach mó an ghealach ná liathróid peile, is é is bun leis sin gurb ionann beagnach an uille a ghabhas an ghealach ag an t-súil agus an uille a ghabhas liathróid peile a bheirtear sa láimh uirthi.

Tosach leathanach 18 sa LSS.

## Nótá 1

Baintear feidhm as teoirim III san uilleanntomhas .i. áis chun uilleacha a thomhas. Leithchiorcal

Tá Fíoghair anseo sa LSS, leathanach 18.

is ea é , agus an imlíne roinnte in 180 cóchoda aige. Tugtar *grád* ($1^\circ$ a scríobhtar) ar an uillinn a ghabhas stua ar bith de na 180 stuanna cothroma ag an lár. Fágann sin go bhfuil

$$180^\circ = \textit{dhá dhronuillinn.}$$

Chun uille a thomhas leagtar $O$ ar rinn na huilleann agus cóirítear $OA$ (nó $OB$) chun go luíonn sé ar ghéig amháin. Áirítear an stua a ghearas an uille ar imlíne an leathchiorcail, agus uaig sin cinntear méad na huilleann.

## Nótá 2

An té a bhreathnaíos ar dha phointe spásúla $P$ agus $Q$ i ndiaidh a chéile, tig leis gairmheas a dhéanamh don uillinn $\widehat{POQ}$ a ghabhas an dronlíne $PQ$ ag a shúil féin (tré casadh na súile a mheas, is dócha). Tá gléas ag *seilbhéara* chun an uille sin a thomhas go cruinn.

Is le huilleacha a dhéanann an gnáth-duine faid a mheas. I bhfíogair (a) is eol

Tá Fíoghair anseo sa LSS, leathanach 18, bun.

dó gur faide $AC$ ná $AB$ de bhrí gur mó an uille $\widehat{AOC}$ a ghabhann $AC$ ag an t-súil $O$.

Níor cheart dó a cheapadh áfach go bhfuil $AB$ níos faide ná $XY$, cé gur mó an uille $\widehat{AOB}$ ná $\widehat{XOY}$, mar is faide amach uaidh $XY$ na $AB$ anois.

Mar an gcéanna nuair feichtear dúinn nach mó an ghealach ná liathróid peile, is é is bun leis sin gurb ionann beagnach an uille sin a ghabhas an ghealach ag an tsúil agus an uille a ghabhas liathróid peile a mbeirtear sa láimh uirthi.

## Note 1

Theorem III is applied in the protractor .i. a tool for measuring angles. It is a semicircle, with perimeter divided into 180 equal arcs. The angle subtended by any one of the 180 equal arcs at the centre is called *one degree* (written as $1^\circ$). Thus

$$180^\circ = \textit{two right angles.}$$

To measure an angle, you lay $O$ on the vertex of the angle and you arrange $OA$ (or $OB$) so that it lies along one of the arms. You inspect the arc that the angle cuts on the perimeter of the circle, and from that you read off and determine the size of the angle.

## Note 2

A person who considers the space between two points $P$ and $Q$ in space, one after the other, can make a rough estimate of the angle $\widehat{POQ}$ that the straight line $PQ$ subtends at his own eye (by estimating how much his eye moves, probably). Surveyors have an instrument for taking an accurate measurement of that angle.

An ordinary person usually judges distance by using angles. In Fig (a) he knows that $AC$ is longer than $AB$ because the angle $\widehat{AOC}$ subtended at his eye $O$ by $AC$ is the greater.

However he should not think that $AB$ is longer than $XY$, even though the angle $\widehat{AOB}$ is greater than $\widehat{XOY}$, because this time $XY$ is further away than $AB$.

In the same way, when it appears that the Moon is no larger than a football, that happens because the angle subtended by the Moon is more-or-less the same as that subtended by a football held in your hands.

## Tearmaí

Géaruille is ea uille gur lú í ná dronuille; tugtar maol-uille ar uillinn atá idir 90° agus 180°.

Uilleacha cóimhlíontacha is ea dhá uillinn gurb ionann is dronuille a suim, ach má's é 180° an tsuim tugtar uilleacha forlíontacha orthu.

Is uille aisfhillteach í uille ar bith atá níos mó ná 180°.

## Cleachtaithe

1) Cad í an uille a gearrann snáthad mhór an chloig tríthi i gcaitheamh (i) cheathrú uaire, (ii) deich nóiméad, (iii) leath-uaire, (iv) cúig nóiméad fichead.

2) Ó phointe O tarraingitear ceithre dronlínte OA, OB, OC, OD, ionnus go bhfuil $A\hat{O}B = 25°$, $B\hat{O}C = 130°$, $C\hat{O}D = 50°$, agus uilleacha deimhneacha is ea iad go léir.

Teaspáin go bhfuil OB is OD in aon dronlíne amháin ach nach bhfuil OA agus OC amhlaidh. Má síntear OA cruthuigh go gcóimhroinneann sí an uille $C\hat{O}D$.

3) Tá 30° san uillinn $A\hat{O}B$ agus castar an uille sin tré 90° ~~tá $A\hat{O}C = B\hat{O}D = 90$.~~ timpeall O go gcuirtear OA fan OC agus go gcuirtear OB fan OD. ~~Faigh (i) an uille $B\hat{O}C$, agus (ii)~~

Faigh (i) an uille $B\hat{O}C$, (ii) an uille idir OA agus cómhroinnteoir na h-uilleann $B\hat{O}C$.

4) Gearrann dhá dhronlíne a chéile ag O, agus cómhroinneann dronlíne áirithe eile uille amháin de na ceithre cinn ag O. Cruthuigh go gcómhroinneann sí uille eile ag O san am céanna. Teaspáin freisin go bhfuil sí ingearach le cómhroinnteoir an dá uillinn eile.

5) Tá na pointí A, B, C, D an fhad chéanna ó phointe O, agus tugtar dúinn go bhfuil $A\hat{O}B = C\hat{O}D$. Cruthuigh go bhfuil (i) $AB = CD$, agus (ii) $AC = BD$.

6) Castar dronuille $O\hat{A}X$ tré 60° timpeall O. Faigh amach le h-uilleanntomhas an líne go leagtar AX anuas uirthi.

Tosach leathanach 19 sa LSS.

## Tearmaí

*Géaruille* is ea uille gur lú í ná dronuille; tugtar *maoluille* ar uilleann atá idir 90° agus 180°.

Uilleacha *cóimhlíontacha* is ea dhá uilinn gurb ionann is dronuille a suim, ach más 180° a suim tugtar uilleacha *fóirlíontacha* orthu.

Is uille *aisfhillteach* uille ar bith atá níos mó ná 180°.

## Terms

An *acute angle* is an angle smaller than a right angle. An angle between 90° and 180° is called an *obtuse angle*.

*Complementary angles* are two angles whose sum is a right angle, but if the sum is 180° they are called *supplementary angles*.

A *reflex angle* is any angle greater than 180°[1].

## Cleachtaithe

1. Cad é an uille a gcasann snáthaid mór an chloig trithi a gcaitheamh (i) ceathrú uaire, (ii) deich nóiméad, (iii) leath-uaire, (iv) cúig nóiméad fichead.

2. Ó phointe $O$ tarraingítear ceithre dronlínte $OA, OB, OC, OD$, ionnus go bhfuil $\widehat{AOB} = 25°$, $\widehat{BOC} = 130°$, $\widehat{COD} = 50°$, agus uilleacha deimhneach is ea iad go léir.

   *Teaspáin* go bhfuil $OB$ is $OD$ in aon dronlíne amháin ach nach bhfuil $OA$ agus $OC$ amhlaidh. Má síntear $OA$ cruthuigh go gcómhroinnneann sí an uille $\widehat{COD}$.

3. Tá 30° san uilleann $\widehat{AOB}$ agus castar an uille sin trí 90° timpeall $O$ go gcuirtear $OA$ fan $OC$ agus go gcuirtear $OB$ fan $OD$.

   Faigh (i) an uille $\widehat{BOC}$, (ii) an uille idir $OA$ agus cómhroinnteoir na h-uileann $\widehat{BOC}$.

4. Gearrann dhá dhronlínte a chéile ag $O$, agus cómhroinneann dronlíne áirithe eile uille amháin de na cheithre cinn ag $O$. Cruthuigh go gcómhroinneann sí uille eile ag $O$ ag an am céanna. Teaspáin freisin go bhfuil sí ingearach le cómhroinnteoir an dá uillinn eile.

5. Tá na pointí $A, B, C, D$ an fhad céanna ó phointe $O$, agus tugtar dúinn go bhfuil $\widehat{AOB} = \widehat{COD}$. *Cruthuigh* go bhfuil (i) $AB = CD$, agus (ii) $AC = BD$.

6. Castar dronuille $\widehat{OAX}$ trí 60° timpeall $O$. Faigh amach le h-uilleanntomhas an líne go leagtar $AX$ anuas uirthi.

[1] and less than 360°

## Exercises

1. Through what angle does the big hand of the clock turn during (i) a quarter of an hour, (ii) ten minutes, (iii) half an hour, (iv) tewnty-five minutes.

2. Draw four straight lines $OA, OB, OC, OD$, from a point $O$, so that $\widehat{AOB} = 25°$, $\widehat{BOC} = 130°$, $\widehat{COD} = 50°$, and all these angles are positive.

   *Show* that $OB$ and $OD$ are in one straiht line, but that it is otherwise for $OA$ and $OC$. If $OA$ is extended, show that it bisects the angle $\widehat{COD}$.

3. There are 30° in the angle $\widehat{AOB}$, and that angle is rotated through 90° about $O$ so that $OA$ is placed along $OC$ and $OB$ along $OD$.

   Find (i) the angle $\widehat{BOC}$, (ii) the angle between $OA$ and the bisector of the angle $\widehat{BOC}$.

4. Two straight lines cut one another at $O$, and a certain other straight line one of the four angles at $O$. Prove that it also bisects another one of the angles at $O$. Show also that it is perpendicular to the bisector of the other two angles.

5. The points $A, B, C, D$ are all the same distance from the point $O$, and we are given that $\widehat{AOB} = \widehat{COD}$. *Prove* that (i) $AB = CD$, and (ii) $AC = BD$.

6. A right angle $\widehat{OAX}$ is turned through 60° around $O$. Find with a protractor the line to which $AX$ is moved.

# Caibidil III

## Scáthú an Phlána i nDronlíne

Dá n-iompaítí bun os cionn cárta a bhí leagtha ar an mbord, d'fheicfí dhúinn nach n-athródh sé sin fad líne ar bith (nó méad uilleann ar bith) sa gcárta. De bhrí gurb í an taobh a bhí thíos roimhe sin atá in uachtar anois, ní féidir an ghluaiseacht sin a chur i ngníomh le casadh ar bith den chineál ar thagramar dó i gcaib II.

Mar an gcéanna, is féidir leathanach leabhair a iompó thart ar chúis go mbí sé in aon phlána amháin leis an bplána ónar thógaigh sé, gan chuile phointe a chur ar ais arís mar a raibh sé i dtosach. Fanann pointí na cúise féin socair dá bharr, ach athraítear ionad gach aon phointe eile.

Páirt chuimseach de phlána atá i gceist sna triálacha sin thuas, ach léiríonn siad tréith áirithe den phlána gheoiméatrach.

Is féidir plána geoiméatrach a chasadh timpeall ar dhronlíne ar bith $l$ ann, chun go nglacfidh an plána i n-iomlán an t-ionad ina raibh sé i dtosach, agus i gcaoi go dtéann gach pointe den phlána (cé is moite de phointí na líne $l$) go dtí pointe eile ar an bplána.

<u>Scáthú</u> an phlána <u>sa dronlíne $l$</u> a tugtar ar an ngluaiseacht sin.

Má's ar $P_1$ a leagtar pointe $P$, is léir gur anuas ar $P$ a leagtar $P_1$, de bhrí go gcuirfear gach pointe ar ais sa sean-ionad de bharr dhá scáthú as a chéile in $l$. Tugtar <u>scátha</u> a chéile in $l$ ar phointí mar $P$ agus $P_1$.

Ní miste dúinn anois an prionnsabal seo a chur ar bun.

### Bun-Phrionnsabal II

Is cómhfhada na línte, agus is cómhméad na huilleacha go leagtar ceann acu ar an gceann eile de bharr an plána a scáthú i ndronlíne.

# Caibidil 3

# Scáthú an Phlána i nDromlíne

Tosach leathanach 20 sa LSS.

# Reflecting the Plane in a Straight Line

Dá n-iompaítí bun ós cionn cárta a bhí leagtha ar an mbord, d'fheicfí dhúinn nach n-athródh sé sin fad líne ar bith (nó méad uilleann ar bith) sa gcárta. De bhrí gurb í an taobh a bhí thíos roimhe sin atá in uachtar anois, ní féidir an ghluaiseacht sin a chur i ngníomh le casadh ar bith den chineál ar thagramar dó i gcaib. II.

Mar an gcéanna is féidir leathanach leabhair a iompó thart ar chiúis go mbí sí in aon phlána amháin leis an bplána ónar thosnaigh sé, gan chuile phointe a chur ar ais arís mar a raibh sé i dtosach. Fanann pointí na ciúise féin socair dábharr, ach athraítear ionad gach aon phointe eile.

Páirt chuimseach de phlána atá i gceist sna

triálacha sin thuas, ach léiríonn siad tréith áirithe den phlána geómétrach.

Is féidir plána geómétrach a chasadh timpeall ar dhronlíne ar bith $\ell$ ann, chun go nglacfaidh an plána i n-iomlán an t-ionad ina raibh sé i dtosach, agus i gcaoi go dtéann gach pointe den phlána (cé is moite de phointí na líne $\ell$) go dtí pointe eile ar an bplána.

*Scáthú an phlána sa dronlíne* $\ell$ a tugtar ar an bgluaiseacht sin.

Má's ar $P_1$ a leagtar pointe $P$, is léir gur anuas ar $P$ a leagtar $P_1$, de bhrí gp gcuirfear gach pointe ar ais sa sean-ionad de bharr dhá scáthúas a chéile in $\ell$. Tugtar *scátha* a chéile in $\ell$ ar phointí mar $P$ agus $P_1$.

Ní miste dúinn anois an prionnsabal seo a chur ar bun:

## 3.1 Bun-Phroinnsabal II

*Is cómhfhada na línte, agus is cómhméad na huilleacha go leagtar ceann acu ar an gceann eile de bharr an plána a scáthúi ndronlíne.*

If a card lying on the table is turned upside-down, we see that there is no change to the length of any line or the size of any angle on the card. Since the side that was down beforehand is now up, that change cannot be achieved by any rotation of the kind studied in Chapter 2.

In the same way, you can turn the page of a book on its spine so that it lies in the same plane as it occupied to start with and no point ends up where it started. The points along the spine stay put, but all other points move.

These experiments involve only a limited part of the plane, but they illustrate particular propeties of the geometric plane.

A geometric plane can be turned about any straight line $\ell$ in it, in such a way that the entire plane ends up where it started, and every point of the plane (except the points on the line $\ell$) moves to a different point of the plane.

This movement is called *reflecting the plane in the straight line* $\ell$.

If the point $P$ moves to $P_1$, then clearly $P_1$ moves to $P$, so two successive reflections in the same line $\ell$ bring every point back to its original position. Points like $P$ and $P_1$ are called *reflections of one another in* $\ell$.

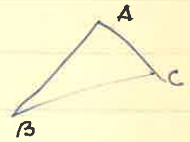
Now we have to lay down the following principle:

## 3.2 Axiom II

*When the plane is reflected in a straight line, lines are the same length as their reflections, and angles are the same size as their reflections.*

Téarmaí

Triantán: sin fíoghair iadta dúnta imeall trí dhronlínte, viz. na sleasa. Tugtar reanna an triantáin ar phointí teangmhála na slios. Uille den triantán is ea an uille (istigh) idir aon dá shlios.




Scríobhtar an nod Δ in áit triantáin go minic.

Triantán comhchosach.

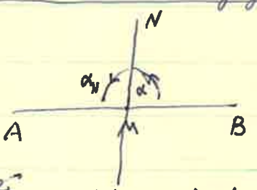
Sin triantán fá na dhá shlios chomhfhada. Is gnáthach bonn an Δ a thabhairt ar an slios eile.

Triantán comhshleasach: sin Δ fá na thrí sleasa comhfhada.

Ais Shuiméitreachta.

Nuair scáitear fíoghair fhlánach i ndronlíne l, má tharlaíonn go leagtar gach pointe P den fhíoghair sin ar phointe éigin eile den fhíoghair chéanna, ionnas nach n-athraítear ionad na fíoghaire iomláine dá bharr, deirtear go bhfuil an fhíoghair sin suiméitreach san dronlíne l, agus go bhfuil l ina ais shuiméitreachta aici.

e.g. (1) Tá ais shuiméitreachta ag gach dronlíne chuimseach AB.




Abair go bhfuil MN ingearach le AB, áit gurb é M lár AB. Ós dronuilleacha iad $\hat{\alpha}, \hat{\alpha}_1$, leagtar $\hat{\alpha}$ ar a comhuillinn $\hat{\alpha}_1$ de bharr scáthú in MN. 'Sé sin, cuirtear MB fan MA, agus ó tharla MB = MA is ar A a thuiteas B.

Mar an gcéanna cuirtear MA anuas go cruinn ar MB.

∴ ais shuiméitreachta is ea MN ag an dronlíne AB.

e.g. (2) Tá dhá ais shuiméitreachta ag dhá dhronlíne theangmhálacha.

Abair gurb é OX comhroinnteoir na h-uilleann deimhní idir líne 1 agus líne 2, ionnas go bhfuil $\hat{\alpha} = \hat{\alpha}_1$.

Tosach leathanach 21 sa LSS.

## Téarmaí

*Triantán:* sin fíogair iadta dárb imeall trí dronlínte, viz. na *sleasa*. Tugtar *reanna* an triantáin ar phointe teagmhála na slios. Uille den triantán is ea an uille (istigh) idir aon dá shlios.

Scríobhtar an nod $\Delta$ in áit triantáin go minic.

*Triantán cómhchosach:* Sin triantán fána dhá shlios chómhfhada. Is gnáthach *bonn* an $\Delta$ a thabhairt ar an slios eile.

*Triantán cómhshleasach:* sin $\Delta$ fána thrí sleasa cómhfhada.

*Ais Shuiméitreachta.* Nuair scáitear fíoghair phlánach i ndromlíne $\ell$, má thárlaíonn go leagtar gach pointe $P$ den fhíoghair ar phointe éigin eile den fhíoghair céanna, ionas nach n-athraítear ionad na fíoghaire iomláine dá bharr, diertear go bhfuil an fhíoghair *suiméitreach* san dronlíne $\ell$, agus go bhfuil $\ell$ ina *ais shuiméitreachta* aici.

## Definitions

A *triangle* is a closed figure with three straight lines a its perimeter, viz. the *sides*. The points where the sides meet are called *vertices*. An angle of the triangle refers to the (inside) angle between two sides.

We often write $\Delta$ instead of 'triangle'.

*Isosceles triangle:* That is a triangle having two sides of the same length. The other side is usually called the *base* of the $\Delta$.

*Equilateral triangle:* That's a $\Delta$ having all three sides of the same length.

*Axis of Symmetry:* If, when some figure is reflected in a straight line $\ell$, it happens that each point $P$ of the figure lands on some other point of the same figure, so that the overall position of the figure does not change, then we say that the figure is *symmetric* about the the straight line $\ell$, and that $\ell$ is an *axis of symmetry* for it.

e.g. (1) *Tá ais shuiméitreachta ag gach dronlíne chuimseach AB.*

Abair go bhfuil $MN$ ingearach le $AB$, áit gurb é $M$ lár $AB$. Ó's dronuilleacha iad $\hat{\alpha}, \hat{\alpha}_1$, leagtar $\hat{\alpha}$ ar a cómhuilinn $\hat{\alpha}_1$, de bharr scáthú in $MN$. 'Sé sin, cuirtear $MB$ fan $MA$, agus ó thárla $MB = MA$ is ar $A$ a thuiteas $B$.

Mar an gcéanna cuirtear $MA$ anuas go cruinn ar $MB$.

.i. ais shuiméitreachta is ea $MN$ ag an dronlíne $AB$.

e.g. (1) *Every bounded straight line AB has an axis of symmetry. chuimseach AB.*

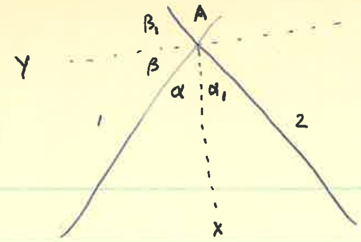
Suppose $M$ is the midpoint of $AB$, and $MN$ s perpendicular to $AB$. Since $\hat{\alpha}$ and $\hat{\alpha}_1$ are right angles, $\hat{\alpha}$ s laid on the equal angle $\hat{\alpha}_1$ when we reflect in $MN$. Thus $MB$ is laid along $MA$, and since it happens that $MB = MA$, $B$ lands on $A$.

In the same way $MA$ is laid exactly on $MB$.

.i. $MN$ is an axis of symmetry for the straight line $AB$.

e.g. (2) *Tá dhá ais shuiméitreachta ag dhá dhronlínte theagmhálacha.*

Abair gurb é $OX$ cómhroinnteoir na huilleann deimhní idir líne 1 agus líne 2, ionas go bhfuil $\hat{\alpha} = \hat{\alpha}_1$.




De bharr an phlána a scáthú in AX, fanann AX socair agus cuirtear $\hat{\alpha}$ ar a comhuillinn $\hat{\alpha}_1$. Cuirtear dá réir líne 1 fan na dronlíne 2, agus mar an gcéanna cuirtear líne 2 ar líne 1.

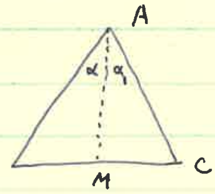
Is soiléir gur ais shuiméitreachta eile í AY, comhroinnteoir na h-uilleann diúltaí idir líne 1 is líne 2.

Nóta

Teaspáinfear ar ball cén chaoi a n-aimsítear ionaid na n-aisí suiméitreachta sin.

## Teoirim IV

Tá triantán comhchosach suiméitreach i gcomhroinnteoir na stuacuilleann.




**Hipotéis** Tugtar AB = AC, agus gurb é AM comhroinnteoir $\hat{A}$ i.e. $\hat{\alpha} = \hat{\alpha}_1$.

**Tátal** Tá an $\triangle ABC$ suiméitreach in AM.

**Cruthúnas**

De bharr scáthú in AM, cuileann AB ar AC mar tá $\hat{\alpha} = \hat{\alpha}_1$.

Ó thárla AB = AC, is anuas ar C a thuiteas B, agus mar an gcéanna cuirtear C ar B.

De bhrí go bhfanann M socair, cuirtear MB ar MC (agus cuirtear MC ar MB)

Q.E.D.

De réir Bhun-Phrionsabail II fágann sin:-

(i) $MB = MC$ ; (ii) $A\hat{M}B = A\hat{M}C = 90°$ ; (iii) $\hat{B} = \hat{C}$.

Atora 1 I dtriantán chomhchosach 'sí comhroinnteóir na stuacuilleann ais shuiméitreachta an bhoinn.

Atora 2 Triantáin chomhchosacha dhifriúla atá ar aon bhonn amháin, is fioghair fána ais shuiméitreachta a ghearras siad.

mar ais shuiméitreachta ag chuile thriantán acu is ea ais shuiméitreachta an bhoinn.

Tosach leathanach 22 sa LSS.

Tá Fíoghair anseo sa LSS, leathanach 22.

De bharr an plána a scáthú in $AX$, fanann $AX$ socair, agus cuirtear $\hat{\alpha}$ ar a comhuilleann $\hat{\alpha}_1$. Cuirtear dá réir líne 1 fan na dronlíne 2, agus mar an gcéanna cuirtear líne 2 ar líne 1.

Is soiléir gur ais shuiméitreachta eile í $AY$, cómhroinnteoir na huilleann diúltaíidir líne 1 agus líne 2.

## Nóta

Teaspáinfear ar ball cé'n chaoi a n-aimsítear ionaid na n-aisí suiméitreachta sin.

e.g. (2) *Two straight lines that meet have two axes of symmetry.*

Let $OX$ be a bisector of the positive angle betweeen line 1 and line 2, so that $\hat{\alpha} = \hat{\alpha}_1$. When the plane is reflected in $AX$, the line $AX$ remains fixed, and $\hat{\alpha}$ is placed on the equal angle $\hat{\alpha}_1$. Hence line 1 s placed along the straight line 2, and in the same way line 2 is placed on line 1.

Clearly, another axis of symmetry is $AY$, the bisector of the negative angle between line 1 and line 2.

## Note

It will be shown later how to find the position of those axes of symmetry.

## 3.3 Teoirim IV

*Tá triantán cómhchosach suiméitreach i gcómhroinnteoir na stuacuilleann.*

Tá Fíoghair anseo sa LSS, leathanach 22.

*Hipotéis:*

Tugtar $AB = AC$, agus gurb é $AM$ cómhroinnteoir $\hat{A}$, i.e. $\hat{\alpha} = \hat{\alpha}_1$.

*Tátall:* Tá an triantán suiméitreach in $AM$.

*Cruthúnas:*

De bharr scáthú in $AM$, titeann $AB$ ar $AC$, mar tá $\hat{\alpha} = \hat{\alpha}_1$.

Ó thála $AB = AC$, is anuas ar $C$ a thiteas $B$, agus mar an gcéanna cuirtear $C$ ar $B$.

De bhrí go bhfanann $M$ socair, cuirtear $MB$ ar $MC$ (agus cuirtear $MC$ ar $MB$). □

De réir Bhun-Phrionsabail II fágann sin:-

(i) $MB = MC$; (ii) $\widehat{AMB} = \widehat{AMC} = 90°$; (iii) $\hat{B} = \hat{C}$.

**Theorem 4.** *An isosceles triangle is symmetrical in the bisector of the apex angle.*

*Hypothesis:*
We are given that $AB = AC$, and that $AM$ is the bisector of $\hat{A}$, i.e. $\hat{\alpha} = \hat{\alpha}_1$.

*Conclusion:*The triangle is symmetrical in $AM$.

*Proof:*
When we reflect in $AM$, $AB$ lands on $AC$, because $\hat{\alpha} = \hat{\alpha}_1$.
Simce $AB = AC$, $B$ lands on $C$, and in the same way $C$ lands on $B$.
Since $M$ stays fixed, $MB$ is placed on $MC$ (and $MC$ is put on $MB$). □
By Axiom II, it follows that:
(i) $MB = MC$; (ii) $\widehat{AMB} = \widehat{AMC} = 90°$; (iii) $\hat{B} = \hat{C}$.

## Atora 1

*I dtriantán cómhchosach 'sí cómhroinnteoir na stuacuilleann ais shuiméitreachta an bhoinn.*

## Atora 2

*Triantáin chómhchosacha dhifriúla atá ar aon bhonn amháin, is fíoghair fana hais shuiméitreachta a ghineas siad.*

Mar is ais shuiméitreachta ag chuile thriantán acu is ea ais shuiméitreachta an bhoinn.

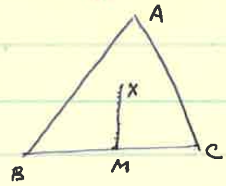
**Corollary 1.** *In an isosceles triangle the bisector of the apex angle is the axis of symmetry of the base.*

**Corollary 2.** *Different isosceles triangles that share a common base combine to make a figure with an axis of symmetry.*

For the axis of symmetry of the base is an axis of symmetry for each of the triangles.

Teoirim V

Triantán cómhcosach isea triantán ar bith ina bhfuil dhá uillinn ar cómhméad.




Hipotéis  Tuglar $A\hat{B}C = A\hat{C}B$

Tátall  Tá $AB = AC$.

Cruthúnas

Abair gurb é MX ais shuiméitreachta an bhoinn.

De bharr scáthu in MX cuirlear MB, géag den uillinn $\hat{B}$, ar MC gur géag í den uillinn chothruim $\hat{C}$.

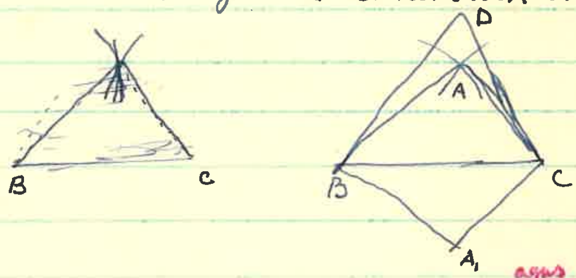
∴ Leagtar BA fan CA, agus mar an gcéanna cuirlear CA fan BA. Fágann sin go dtéann A, pointe teagmhála BA is CA, go dtí pointe teagmhála CA is BA: ∴ fanann A socair.

∴ Pointe ar MX isea A, agus tá an Δ suiméitreach in MXA.

De thairbhe Phrionnsabail II mar sin tá AB = AC.

Nóta 1. Is iomdha ceist geoméitreach a réitítear le Teoirim IV atora 2.

Is mar seo a leanas a tógtar Δ cómhchosach ar bhonn BC.




Tarraing dhá chioreal chothroma gur láir dóibh B agus C, agus abair gur pointe teagmhála é A den dá chioreal sin (má's ann dó). Is leor stuaganna beaga den dá ⊙ a líniú chun ionad A a chinntiú.

Ó's gatha cioreal cothrom iad BA agus BC, táid comhfhada agus A cómhcosach isea ABC. Má athraítear gatha na gcioreal gcothrom is triantáin chómhchosacha dhifriúla a gheibhtear, agus luigheann A, D, $A_1$ etc. ar ais shuiméitreachta an bhoinn BC.

Nóta 2 Scríobhfar a.s. in ionad "ais shuiméitreachta" go minic.

Tosach leathanach 23 sa LSS.

## 3.4 Teoirim V

*Triantán cómhcosach is ea triantán ar bith ina bhfuil dhá uillinn ar cómhéad.*

Tá Fíoghair anseo sa LSS, leathanach 23.

*Hipotéis:*
Tugtar $\widehat{ABC} = \widehat{ACB}$.

*Tátall:*
Tá $AB = AC$.

*Cruthúnas:*
Abair gurb é $MX$ ais suiméitreachta an bhoinn.

De bharr scáthú in $MX$ cuirtear $MB$, géag den uilinn $\hat{B}$, ar $MC$ gur géag í den uilinn chothrom $\hat{C}$.

∴ Leagtar $BA$ fan $CA$, agus mar an gcéanna cuirtear $CA$ fan $BA$. Fágann sin go dtéann $A$, pointe teagmhála $BA$ is $CA$, go dtí pointe teagmhála $CA$ is $BA$: .i. fanann $A$ socair.

∴ Pointe ar $MX$ is ea $A$, agus tá an $\Delta$ suiméitreach in $MXA$.

De thairbhe Phrionnsabail II mar sin tá $AB = AC$. □

**Theorem 5.** *Any triangle having two angles of equal size is isosceles.*

*Hypothesis:*
We are given $\widehat{ABC} = \widehat{ACB}$.

*Conclusion:*
$AB = AC$.

*Proof:*
Let $MX$ be the axis of symmetry of the base.

If we reflect in $MX$, then $MB$, which is one arm of the angle $\hat{B}$, is placed on $MC$, which is an arm of the equal angle $\hat{C}$.

∴ $BA$ is laid along $CA$, and in the same way $CA$ is laid along $BA$. Thus $A$, the point where $BA$ meets $CA$, goes to the common point of $CA$ and $BA$: .i. $A$ stays fixed.

∴ $A$ is a point on $MX$, and the $\Delta$ is symmetric in $MXA$.

Hence, by Axiom II, we have $AB = AC$. □

### Nóta 1

Is iomaí ceist geóméitreach a réitítear le Teoirim IV, Atora 2.

Is mar seo a leanas a tógtar $\Delta$ cómhchosach ar bhonn $BC$.

Tá Fíoghair anseo sa LSS, leathanach 23.

Tarraing dhá chiorcal chothoma gur láir dóibh $B$ agus $C$, agus abair gur pointe teagmhála é $A$ den dá chiorcal sin (má's ann dó ). Is leor stuanna beaga den dá $\odot$ a líniú chun ionad $A$ a chinntiú .

Ó's gatha ciorcal cothrom iad $BA$ agus $BC$, táid comhfhada agus $\Delta$ comhcosach is ea $ABC$. Má h-aithrítear gatha na gciorcal gcothrom is triantáin chóchosacha dhifriúla a gintear, agus luíonn $A, D, A_1$, etc ar ais shuiméitreachta an bhoinn $BC$.

## Nota 2

Scríobhfar a.s. in ionad "ais shuiméitreachta" go minic.

## Note 1

Many geometrical questions can be solved by using Theorem 4, Corollary 2.

Here is a way to construct an isosceles triangle on the base $BC$:

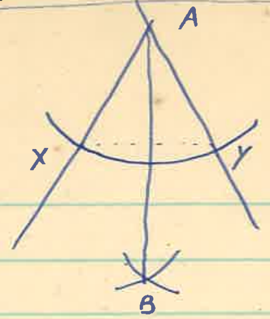
Draw two equal circles having centres $B$ and $C$, and suppose the point of intersection of those two circles (if it exists) is $A$. It is enough to draw little short arcs of the two circles in order to determine the posiition of $A$.

Since $BA$ and $BC$ are radii of equal circles, they have the same length, and so $ABC$ is an isosceles $\Delta$. If you change the radius of the circles you generate a different equilateral triangle, and $A, D, A_1$, etc all lie on the axis of symmetry of the base $BC$.

## Note 2

We shall aften write a.s. instead of "axis of symmetry".

## Ceist 1   Uille a chómhroinnt le Compás agus Riail.



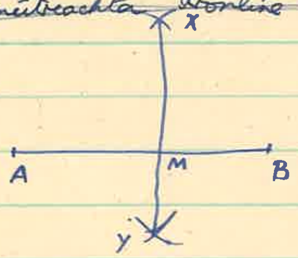

**Réiteach.** Tarraing ⊙ ar bith gurb é A a lár, a ghearras géaga na h-uilleann in X agus Y. Triantán cómhchosach ar an mbonn XY isea AXY. Ar XY déan A cómhcosach ar bith eile BXY agus ceangail AB. 'Sé AB cómhroinnteoir na h-uilleann Â.

**Cruthúnas**

'Sé AB a.s. na fioghaire AXBY de réir Teoirme IV, atora 2.

∴ Cómhroinneann sé an uille Â.

## Ceist 2   Dronlíne chuimseach a chómhroinnt;
~~nó, ais shuimeitreachta dronlíne cuimsigh a aimsiú.~~

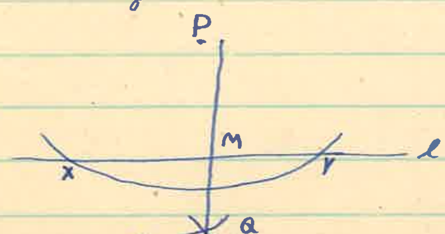



**Réiteach** Abair gurb í AB an dronlíne. Ar AB déan dhá thriantán chómhchosacha ar bith XAB agus YAB, agus ceangail XY d'á chéile. Cómhroinneann XY an dronlíne AB.

**Cruthúnas**

'Sí XY a.s. na dronlíne AB (Teoirim IV) ∴ AM = MB.

## Ceist 3   An t-ingear a tharraint ar dhronlíne ó phointe ~~áirithe~~ áirithe taobh amuigh




**Réiteach** Abair gurb í l an dronlíne agus gurb é P an pointe amuigh. Le P mar lár tarraing ⊙ ar bith a (Tóg x eile) ghearras l in X agus Y, abair. Triantán cómhchosach isea PXY. Déan A cómhcosach eile QXY ar XY. 'Sé PQ, líne cheangail P is Q, an t-ingear.

Tosach leathanach 24 sa LSS.

## Ceist 1

Uille a chómhroinnt le compás agus riail.

Tá Fíoghair anseo sa LSS, leathanach 24.

*Réiteach:*

Tarraing ⊙ ar bith gurb é $A$ a lár, a ghearas géaga na huilleann in $X$ agus $Y$. Triantán cómhchosach an an mbonn $XY$ is ea $AXY$. Ar $XY$ déan $\Delta$ comhchosach ar bith eile $BXY$ agus ceangail $AB$. 'Sé $AB$ cómhroinnteoir na huilleann $\hat{A}$.

*Cruthúnas:*

'Sé $AB$ a.s. na fíoghaire $AXBY$ de réir Teoirma IV, Atora 2.
∴ Cómhroinneann sé an uille $\hat{A}$.

## Question 1

Bisect an angle with ruler and compass.

*Solution:*

Draw any circle with centre $A$, cutting the arms of the angle at $X$ and $Y$. Then $AXY$ is an isosceles triangle on the base $XY$. On $XY$ draw any other isosceles $\Delta$ $BXY$ aand join ceangail $AB$. Then $AB$ is the bisector of the angle $\hat{A}$.

*Proof:*

$AB$ is the a.s. of the figure $AXBY$, by Theorem 4, Corollary 2.
∴ It bisects the angle $\hat{A}$.

## Ceist 2

Dronlíne choimseach a chómhroinnt.

Tá Fíoghair anseo sa LSS, leathanach 24.

*Réiteach:*

Abair gurb í $AB$ an dronlíne. Ar $AB$ déan dhá thriantán chómhchosacha ar bith $XAB$ agus $YAB$, agus ceangail $XY$ d'á chéile. Cómhroinneann $XY$ and dronlíne $AB$.

*Cruthúnas:*

'Sé $XY$ a.s. na dronlíne $AB$ (Teoirim IV). ∴ $AM = MB$.

## Question 2

To bisect a bounded straight line.

*Solution:*

Suppose $AB$ is the straight line. On $AB$ construct any two isosceles triangles $XAB$ and $YAB$, and join $XY$ together. Then $XY$ bisects the straight line $AB$.

*Proof:*

$XY$ is the a.s. of the straight line $AB$ (Theorem 4). $\therefore AM = MB$.

## Ceist 3

An t-ingear a tharraingt ar dhronlíne ó phointe áirithe taobh amuigh.

Tá Fíoghair anseo sa LSS, leathanach 24.

*Réiteach:*

Abair gurb í $\ell$ an líne agus gurb é $P$ an pointe amuigh.
Le $P$ mar lár tarraing $\odot$ ar bith a ghearas $\ell$ in $X$ agus $Y$, abair.
Triantán cómhchosach is ea $PXY$. Déan $\Delta$ cómhcosach eile $QXY$ ar $XY$.

'Sé $PQ$, líne cheangail $P$ is $Q$, an t-ingear.

Cruthúnas.

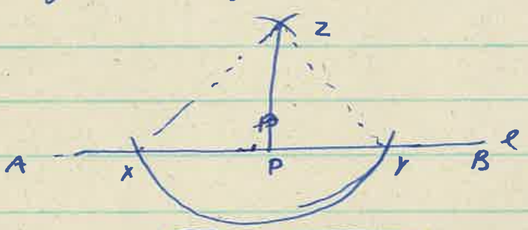
'Sé PQ ais shiméitreachta na fioghaire PXQY; $\therefore P\hat{M}X = P\hat{M}Y = 90°$.

Nóta. An t-ingear ó P ar l a deirtear, agus teaspáinfear anois nach bhfuil ann ach ceann amháin.

De bharr an pléna a scáthú in l is soiléir go dtéigeann ingear ar bith ó P trí an bpointe $P_1$, scáth an phointe P in l. Ach ní fhéadfadh dhá dhronlíne dhifriúla a dhul trí P is $P_1$.

$\therefore$ Níl ach ingear amháin ó P ar l.

Ceist 4 An t-ingear a thógáil ar dhronlíne ag pointe áirithe inti.




[Cás speisialta é seo de cheist 1 nuair is mar rinn na huilleann dírí APB a samhlaítear P]

Réiteach Abair gurb é l an dronlíne is gurb é P an pointe. Le P mar lár tóg ciorcal ar bith a ghearras l in X agus Y. Ar XY déan Δ comhchosach ar bith ZXY. 'Sé ZP an t-ingear.

Cruthúnas

Sa Δ comhchosach ZXY, 'sé P lár an bhoinn, agus de bharr gurb ionann a.s. an bhoinn agus a.s. an triantáin, 'sé PZ an a.s. Δin.

$\therefore Z\hat{P}X = Z\hat{P}Y = 90°$.

Cleachtaithe

1. Trí M, lár na dronlíne AB tarraingítear an dronlíne ⊥ le AB. Más pointe ar bith den ingear é P, cruthuigh PA = PB.

2. I dtriantán ABC tá AB = AC agus is iad L, M, láir na sleas AB agus AC. Cruthuigh, de bharr an Δ a scáthú san a.s., go bhfuil BM = CL. Má theagmhaíonn BM agus CL in O, cruthuigh OL = OM.

3. Is pointí iad D, E i mbonn an Δ chomhchosaigh ABC ionnas go bhfuil BD = CE. Cruthuigh AD = AE, agus $B\hat{A}D = E\hat{A}C$.

Tosach leathanach 25 sa LSS.

*Cruthúnas:*
'Sé $PQ$ ais shuiméitreachta na fíoghaire $PXQY$; $\therefore \widehat{PMX} = \widehat{PMY} = 90°$.

## Question 3

To draw the perpendicular line to a straight line from some point outside it.

*Solution:*
Suppose $\ell$ is the line and $P$ the point outside it.
With $P$ as centre draw any ⊙, cutting $\ell$ at $X$ and $Y$, say.
$PXY$ is an isosceles triangle. Make another isosceles $\Delta\, QXY$ on $XY$.
Then $PQ$, the line joining $P$ and $Q$, is the perpendicular.

*Proof:*
$PQ$ is the axis of symmetry of the figure $PXQY$; $\therefore \widehat{PMX} = \widehat{PMY} = 90°$.

## Nóta

*An t-ingear* ó $P$ *ar* $\ell$ a deirtear, agus teaspáinfear anois nach bhfuil ann ach ceann amháin.

De bharr an plána a scáthú in $\ell$ is soiléir go dtéighann ingear ar bith ó $P$ tré'n bpointe $P_1$, scáth an phointe $P$ in $\ell$. Ach ní fhéadfadh dhá dhronlíne dhifriúla a dhul trí $P$ is $P_1$.

$\therefore$ Níl ach ingear amháin ó $P$ ar $\ell$.

## Note

One says *the perpendicular from $P$ on $\ell$*, and we shall now show that only one exists.

On reflecting the plane in $\ell$ it is clear that any perpendicular from $P$ passes through $P_1$, the reflection of $P$ in $\ell$. But it cannot happen that two different straight lines pass through $P$ and $P_1$.

$\therefore$ There is only one perpendicular from $P$ on $\ell$.

## Ceist 4

An t-ingear a thógáil ar dhronlíne ag pointe áirithe innti.

Tá Fíoghair anseo sa LSS, leathanach 25.

[Cás spesialta de cheist 1 é seo nuair mar rinn na huilleann dirí $APB$ a samhlaítear $P$.]

*Réiteach:*
Abair gurb í $\ell$ an dronlíne agus gurb é $P$ an pointe.
Le $P$ mar lár tóg ciorcal ar bith a ghearas $\ell$ in $X$ agus $Y$. Ar $XY$ déan $\Delta$ cóchosach ar bith $ZXY$. 'Sé $ZP$ an t-ingear.

*Cruthúnas:*

Sa $\Delta$ comhchosach $ZXY$ 'sé $P$ lár an bhoinn, agus de bhrí gurb ionann a.s. an bhoinn agus a.s. an triantáin, 'sé $PZ$ an a.s. sin.

$\therefore \widehat{ZPX} = \widehat{ZPY} = 90°$.

## Question 4

To construct the perpendicular to a straight line at some particular point on it.

[This is a special case of Question 1, if you think of $P$ as the vertex of the angle $APB$.]

*Solution:*

Suppose $\ell$ is the straight line and $P$ the point.

With $P$ as centre draw any circle, cutting $\ell$ at $X$ and $Y$. On $XY$ make any isosceles $\Delta$ $ZXY$. Then $ZP$ is the perpendicular.

*Proof:*

In the isosceles $\Delta$ $ZXY$, $P$ is the centre of the base, and since the a.s. of the base is the same as the a.s. of the traingle, that a.s. is $PZ$.

$\therefore \widehat{ZPX} = \widehat{ZPY} = 90°$.

4) Rinn uillinn díríthe ísea A agus le A mar lár tarraingítear dhá chiorcal. Gearrann an chéad chiorcal géaga na h-uillinne in X agus Y, ach is in Z agus W a ghearras an dara ceann iad. De bharr an plána a scáthú san a.s. cruthuigh (i) go gcuirtear an dronlíne XW ar an dronlíne YZ; (ii) go bhfuil pointe teagmhála XW agus YZ ar an a.s. Dá chionn sin teaspáin modh eile chun uille a chómhroinnt.

5) Is pointí iad P, Q ar thaobh amháin de dhronlíne l agus

$P_1, Q_1$, siad $P_1, Q_1$, scátha P is Q in l. Gearrann PQ an dronlíne l in X. Cruthuigh (i) $P_1Q_1 = PQ$; (ii) $PQ_1 = P_1Q$, (iii) go ngabhann an dronlíne $P_1Q_1$ tré X.

6) Fíoghair phlánach fána ceithre sleasa cothroma ísea ABCD, agus siad AC is BD na treasnáin. Teaspáin gur aisí suiméitreachta ag an bhfíoghair iomlán iad AC, BD. Cruthuigh (i) go gcómhroinneann na treasnáin na h-uilleacha a ngabhann siad triothu, (ii) go bhfuil na treasnáin ingearach le chéile.

7) 'Sé M bun an ingir ó P ar dhronlíne l, agus pointe eile den dronlíne sin ísea X. Má's iontuigthe go bhfuil dhá shlios triantáin le chéile níos faide ná an tríú slios, teaspáin go bhfágann sin go bhfuil PX > PM.

[Leide: an fhíoghair a scáthú in l]

8. Má tá pointe O ar chómhroinnteoir ar bith den dhá chómhroinnteóirí na h-uilleann idir na línte l agus m, cruthuigh gur cómhfhada na h-ingir ón bpointe O ar l agus m.

9. Triantán cómhchosach é ABC ina bhfuil AB = AC. Gearrann cómhroinnteoir na h-uilleann B an slios AC in X, agus gearrann cómhroinnteoir na h-uilleann C an slios AB in Y. Cruthuigh BX = CY.

10 Dhá thriantán chómhchosacha isea ABC, ABD ar an mbonn chéanna AB. 'Siad X, Y láir CA is CB, agus 'siad Z, W láir AD agus DB. Cruthuigh YZ = XW.

## Cleachtaithe

1. Tré $M$, lár an dronlíne $AB$ tarraingítear an dronlíne $\perp$ le $AB$. Má's pointe ar bith den ingear é $P$, cruthuigh $PA = PB$.

2. I dtriantán $ABC$ tá $AB = AC$ agus siad $L, M$ láir na slios $AB$ agus $AC$. Cruthaigh, de bharr an $\Delta$ a scáthúsan a.s., gp bhfuil $BM = CL$. Má teagmhaíonn $BM$ agus $CL$ in $O$, cruthuigh $OL = OM$.

3. Is pointí iad $D, E$ i mbonn an $\Delta$ chómhchosaigh $ABC$ ionas go bhfuil $BD = CE$. Cruthaigh $AD = AE$, agus $\widehat{BAD} = \widehat{EAC}$.

   Tosach leathanach 26 sa LSS.

4. Rinn uileann áirithe is ea $A$ agus le $A$ mar lár tarraingítear dhá chiorcail. Gearrann an chéad chiorcal géaga na huilleann in $X$ agus $Y$, ach is in $Z$ agus $W$ a ghearas an dara ceann iad. De bharr an plána a scáthú san a.s. cruthuigh (1) go gcuirtear an dronlíne $XW$ ar an dronlíne $YZ$; (ii) go bhfuil pointe teagmhála $XW$ agus $YZ$ are an a.s. Dá chionn sin ceap módh eile chun uille a chómhroinnt.

5. Is pointí iad $P, Q$ ar thaobh amháin de dhronlíne $\ell$ agus siad $P_1, Q_1$ scátha $P$ is $Q$ in $\ell$. Gearann $PQ$ an dronlíne $\ell$ in $X$. Cruthaigh (i) $P_1Q_1 = PQ$; (ii) $PQ_1 = P_1Q$; (iii) go ngabhann an dronlíne $P_1Q_1$ tré $X$.

6. Fioghair phlánach fána ceithre sleasa cothroma is ea $ABCD$, agus siad $AC$ is $BD$ na treasnáin. Teaspáin gur aisí suiméitreachta ag an bhfioghair iomláin iad $AC$, $BD$. Cruthaigh (i) go gcómhroinneann na treasnáin na huilleacha a ngabhann siad triothu, (ii) go bhfuil na treasnáin ingearach le chéile.

7. 'Sé $M$ bun an ingir ó $P$ ar dhronlíne $\ell$, agus pointe eile den dronlíne sin is ea $X$. Má's iontuigthe go bhfuil dhá shlios triantáin le chéile níos faide ná an tríú slios, teaspáin go bhfágann sin go bhfuil $PX > PM$.

   [Leide: an fhioghair a scáthú in $\ell$.]

8. Má tá pointe $O$ ar chómhroinnteoir ar bith de dhá chómhroinnteoirí na huilleann idir na línte $\ell$ agus $m$, cruthuigh gur cómhfhada na hingear ón bpointe $O$ ar $\ell$ agus $m$.

9. Triantán cómhchosach é $ABC$ ina bhfuil $AB = AC$. Gearann cómhroinnteoir na huilleann $B$ an slios $AC$ in $X$, agus gearrann cómhroinnteoir na huilleann $C$ an slios $AB$ in $Y$. Cruthaigh $BX = BY$.

10. Dháthriantáin chómhchosacha is ea $ABC$, $ABD$ ar an mbonn chéanna $AB$. 'Siad $X, Y$ láir $CA$ is $CB$, agus 'siad $L, W$ láir $AD$ agus $DB$. Cruthaigh $YZ = XW$.

## Exercises

1. Through $M$, the centre of the straight line $AB$, draw a straight line perpendicular to $AB$.

   If $P$ is any point of that perpendicular, prove that $PA = PB$.

2. In the triangle $ABC$ we have $AB = AC$ and $L, M$ are the centres of the sides $AB$ and $AC$. Prove, by reflecting the $\Delta$ in the a.s., that $BM = CL$.

   If $BM$ and $CL$ meet at $O$, prove $OL = OM$.

3. $D$ and $E$ are points on the base of the isosceles $\Delta$ $ABC$ such that $BD = CE$. Prove that $AD = AE$, and $\widehat{BAD} = \widehat{EAC}$.

4. $A$ is the vertex of a certain angle, and two circles are drawn with centre $A$. The first circle cuts the arms of the angle at $X$ and $Y$, but the second cuts them at $Z$ and $W$. By reflecting the plane in the a.s. prove
   (1) that the straight line $XW$ is laid on the straight line $YZ$; (ii) that the common point of $XW$ and $YZ$ lies on the a.s. On that basis, invent another way to bisect an angle.

5. $P, Q$ are points on one side of a straight line $\ell$ and $P_1, Q_1$ are the reflections of $P$ and $Q$ in $\ell$. $PQ$ cuts the straight line $\ell$ at $X$. Prove (i) $P_1Q_1 = PQ$; (ii) $PQ_1 = P_1Q$; (iii) that the straight line $P_1Q_1$ passes through $X$.

6. $ABCD$ is a plane figure having four equal sides, and $AC$ and $BD$ are the diagonals. Show that $AC$ and $BD$ are axes of symmetry for the whole figure. Prove (i) that the diagonals bisect the angles they pass through, (ii) that the diagonals are perpendicular to one another.

7. $M$ is the bottom (foot) of the perpendicular from $P$ on a straight line $\ell$, and $X$ is another point of that straight line. If it is taken for granted that two sides of any triangle are together greater than the third side, show that it follows that $PX > PM$.

   [Hint: reflect the figure in $\ell$.]

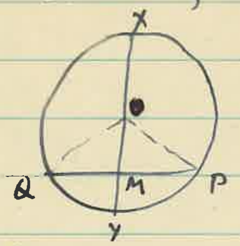
8. If a point $O$ is on either of the two bisectors of the angles between the lines $\ell$ agus $m$, prove that the perpenticulars from the point $O$ on $\ell$ and $m$ have the same length.

9. $ABC$ is an isosceles triangle in which $AB = AC$. The bisector of the angle $B$ cuts the side $AC$ at $X$, and the bisector of the angle $C$ cuts the side $AB$ at $Y$. Prove $BX = BY$.

10. $ABC$, $ABD$ are two isosceles triangles on the same base $AB$. $X, Y$ are the centres of $CA$ and $CB$, and $L, W$ are the centres of $AD$ agus $DB$. Prove that $YZ = XW$.

Teoirim VI

I gciorcal ar bith, ais shuimeitreachta is ea gach láirlíne.




Hipotéis — Láirlíne ar bith is ea XY i gciorcal gurb é O a lár.

Tógáil — Tré phointe ar bith P ar an imlíne tarraing an córda PQ atá ⊥ le XY. Ceangail OP, OQ.

Cruthúnas

Is gatha an ⊙ iad OP, OQ ionnas gur Δ comhchosach é OPQ.

'Sé OM an t-ingear ón stuaic ar an mbonn PQ

∴ Sé OM a.s. an Δ OPQ, agus fágann sin gur scátha a chéile in XY iad na pointí P agus Q.

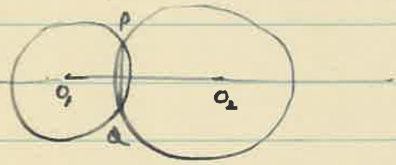
Mar an gcéanna, cuirfear gach pointe den imlíne ar phointe eile den imlíne de bharr scáthú in XY.

Q.E.D

Atoradh 1 — Sé an láirlíne ingearach a.s. chórda chiorcail ar bith.

Atoradh 2 — Tá líne cheangail lár córda le lár an ⊙ féin, ingearach leis an gcórda.

Atoradh 3 — Má ghearrann dhá chiorcal a chéile i ndhá phointe, sí líne cheangail na lár a.s. an chomhchórda.




Ó's córda san dá ⊙ é PQ caithfidh a.s. PQ dul tré $O_1$ agus tré $O_2$. ∴ Sí an líne $O_1O_2$ í.

Tosach leathanach 27 sa LSS.

## 3.5 Teoirim VI

*I gciorcal ar bith, ais shuiméitreachta is ea gach lálíne.*

Tá Fíoghair anseo sa LSS, leathanach 27.

*Hipotéis:*

Lárlíne ar bith is ea $XY$ i gciorcal gurb é $O$ a lár.

*Tógáil:*

Tré phointe ar bith $P$ ar an imlíne tarraing ancórda $PQ$ atá $\perp$ le $XY$. Ceangail $OP$, $OQ$.

*Cruthúnas:*

Is gatha an $\odot$ iad $OP, OQ$ ionas gur $\Delta$ cómhchosach é $OPQ$.
'Sé $OM$ an t-ingear ó'n stuac ar an mbonn $PQ$.
$\therefore$ Sé $OM$ a.s. an $\Delta$ $OPQ$, agus fágann sin gur scátha a chéile in $XY$ iad na pointí $P$ agus $Q$.
Mar an gcéanna, cuirfear gach pointe den imlíne ar phointe eile den imlíne de bharr scáthú in $XY$. □

**Theorem 6.** *In any circle, each diameter is an axis of symmetry.*

*Hypothesis:*

$XY$ is some diameter of a circle with centre $O$.

*Construction:*

Through any point $P$ on the perimeter draw a chord $PQ \perp$ to $XY$. Join $OP$, $OQ$.

*Proof:*

$OP, OQ$ are radii of the $\odot$, so that $OPQ$ is an isosceles $\Delta$.
$OM$ is the perpendicular from the apex on the base $PQ$.
$\therefore$ $OM$ is the a.s. of $\Delta$ $OPQ$, and it follows that the points $P$ and $Q$ are the reflections of one another in $XY$.
Similarly, each point of the perimeter is placed on some other point of the perimeter when reflected in $XY$. □

### Atora 1

*Sí an lárlíne ingearach a.s. chórda chiorcail ar bith.*

### Atora 2

*Tá líne cheangail lár córda le lár an $\odot$ féin, ingearach leis an gcórda.*

### Atora 3

*Má ghearrann dhá chiorcal a chéile i ndhá phointe, sí líne cheangail na lár a.s. an chómhchórda.*

Tá Fíoghair anseo sa LSS, leathanach 27.

Ó's córda san dá ⊙ é $PQ$ caithfidh a.s. $PQ$ dul tré $O_1$, agus tré $O_2$. dbs Sí an líne $O_1O_2$ í.

**Corollary 1.** *The perpendicular diameter is the a.s. of any chord.*

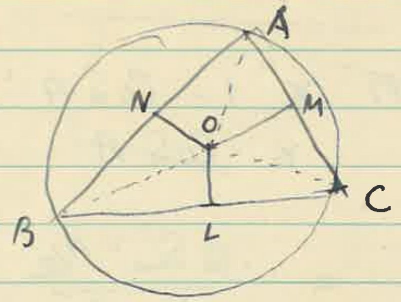
**Corollary 2.** *The line joining the centre of a chord to the centre of the ⊙ is perpendicular to the chord.*

**Corollary 3.** *If two circles cut one another in two points, then the line joining the centres is the a.s. of their common chord.*

Since $PQ$ is a chord in both circles, the a.s. of $PQ$ has to go through $O_1$, and through $O_2$. ∴ it is the line $O_1O_2$.

Teoirim VII

I dtriantán ar bith tagann aisí suiméitreachta na slios le chéile in aon phointe amháin.




Tógáil  Tarraing MO agus NO aisí suiméitreachta na slios AC is AB.
Tá le cruthú go ngabhann a.s. an tsleasa BC tré O.

Cruthúnas

Ós pointe é O in a.s. AB, scátha a chéile in NO sea A agus B.
∴ OA = OB.
Mar an gcéanna tá O ar a.s. AC, ionnus go bhfuil OA = OC.
Fágann san OA = OB = OC.
Ach de thairbhe OB = OC gabhann a.s. BC tré O (Teoirim IV)
Q.E.D.

Atora 1  Gabhann an ⊙ gurb é O a lár agus ar ga dó OA (=OB=OC) tré reanna an △ ABC.
Iomchiorcal an △ a tugtar ar an gciorcal úd; 'sé O iomlár an triantáin.

Atora 2  Ní ghabhann ach ciorcal amháin tré trí phointí ar bith nach bhfuil cóimhlíneach.
Mar, cuintear lár (agus ga) singil amháin de réir tógála na teoirime thuas.
Is ionann an atora seo is a rá nach bhfuil thar dhá phointe teagmhála ag dhá chiorcal dhifriúla.

Atora 3  Ní féidir trí dronlínte cómhfhada a tharraingt ó phointe P go dtí imlíne chiorcail, marab é P féin lár an chiorcail.
Mar, de thairbhe PA = PB = PC, tá P ar a.s. na dtrí slios; ∴ 'sé lár an ⊙ABC é.

Tosach leathanach 28 sa LSS.

## Teoirim VII

*I dtriantán ar bith tagann aisí suiméitreachta na slios le chéile in aon phointe amháin.*

Tá Fíoghair anseo sa LSS, leathanach 28.

*Tógáil:*
Tarraing $MO$ agus $NO$ aisí suiméitreachta na slios $AC$ is $AB$.
Tá le cruthú go ngabhann a.s. an tsleasa $BC$ tré $O$.

*Cruthúnas:*
Ó's pointe é $O$ in a.s. $AB$, scátha a chéile in $NO$ is ea $A$ agus $B$.

$$\therefore OA = OB.$$

Mar an gcéanna tá $O$ ar a.s. $AC$, ionnas go bhfuil $OA = OC$.
Ach de thairbhe $OB = OC$ gabhann a.s. $BC$ tré $O$ (Teoirim IV). □

**Theorem 7.** *In any triangle the axes of symmetry of the three sides meet in a single point.*

*Construction:*
Draw $MO$ and $NO$, the axes of symmetry of the sides $AC$ and $AB$.
We have to prove that the a.s. of the side $BC$ passes through $O$.

*Proof:*
Since $O$ is a point in the a.s. of $AB$, $A$ and $B$i are reflections of one another in $NO$.

$$\therefore OA = OB.$$

Similarly, $O$ is on the a.s. of $AC$, so that $OA = OC$.
But since $OB = OC$ the a.s. of $BC$ passes through $O$ (Theorem 4). □

## Atora 1

*Gabhann an ⊙ gurb é $O$ a lár agus ar ga dó $OA$ $(= OB = OC)$ tré reanna an $\Delta$ $ABC$.*

*Iomchiorcal* an $\Delta$ a tugtar ar an gciorcal úd; 'Sé $O$ *iomlár* an triantáin.

## Atora 2

*Ní ghabhann ach ciorcal amháin tré trí phointí ar bith nach bhfuil cóimhlíneach.*

Mar , cinntear lár (agus ga) singil amháin de réir tógála na teorime thuas.
Is ionann an atora seo is a rá nach bhfuil thar dhá phointe teagmhála ag dhá chiorcail dhifriúla.

## Atora 3

*Ní féidir trí dronlínte cómhfhada a tharraingt ó phointe $P$ go dtí imlíne chiorcail, morab é $P$ féin lár an chiorcail.*

Mar , de thairbhe $PA = PB = PC$, tá $P$ ar a.s. na dtrí slios; .i. 'sé lár an $\odot ABC$ é.

**Corollary 1.** *The $\odot$ with centre $O$ and radius $OA$ $(= OB = OC)$ goes through the vertices of the $\Delta$ $ABC$.*

The *circumcircle of the* $\Delta$ is the name given to that circle; $O$ is the *circumcentre* of the triangle.

**Corollary 2.** *There is just one circle that passes through three given non-collinear points.*

For the construction in the above theorem determines a single centre (and radius).

Another way to state this corollary is to say that two different circles can have no more than two points in common.

**Corollary 3.** *It is impossible to draw three equally-long straight lines from a point $P$ to the perimeter of a circle, unless $P$ itself is the centre of the circle.*

For , since $PA = PB = PC$, the point $P$ is on the a.s. of the three sidea, .i. it is the centre of the $\odot ABC$ é.

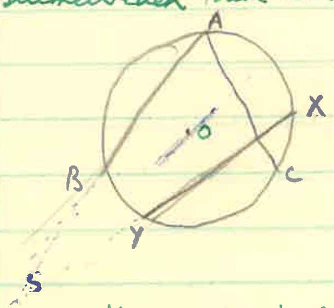
Nótaí (1) Baineann dhá stuagh le córda ciorcail ar bith AB; tugtar an mion-stuagh AB ar an gceann is giorra orthu.

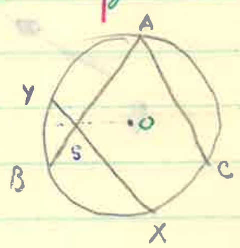
(2) Nuair leagtar stuagh ciorcail AB ar stuagh chómhfhada CD de bharr scáthú i láirlíne, is soiléir gur stuaghanna iad AB agus CD atá i dtreoanna contrárdha ar an imlíne.

Má's le casadh timpeall an láir áfach, a cuirtear stuagh AB ar cheann eile XY, tá na stuaghanna AB agus XY in aon treo amháin ar imlíne an chiorcail.

## Teoirim VIII

Má tá dhá chórda ciorcail cómhfhada le chéile (agus ∥) táid siméitreach san láirlíne tré na bpointe teagmhála.




Má's ar an imlíne a ghearras na córdaí cómhfhada AB, AC a chéile, níl aon chórda eile tré A atá cómhfhada leo (teoirim VII), agus tá an teoirim soiléir de réir teoirme VI.

a tora 3

Hipotéis

Abair anois go bhfuil AB = XY, agus ainmnigh na córdaí sin i gcaoi go bhfuil na mion-stuaghanna AB, XY contrárdha maidir le treo.

Tátall Scátha a chéile i láirlíne isea na córdaí AB agus XY.

Cruthúnas

De bharr an plána a scáthú in a. s. AX, leagtar an córda XY ar chórda éigin den dá chórda AB, AC tré A atá cómhfhada leis.

Ní féidir gur ar AC a thuiteas sé, de bhrí go bhfuil na mion-stuaghanna XY agus AC in aon treo amháin.

Fágann sin go gcuirtear an córda (agus an stuagh) XY anuas ar AB.

Q. E. D.


under Rot BOC BA → CD : angle between BA and CD = $\hat{BOC}$

Bisector of this angle ∥ AC and so equal ∠ BAC

∴ ∠ BOC = 2 ∠ BAC

Tosach leathanach 29 sa LSS.

### Nótaí

(1) Baineann dhá stua le córda ciorcail ar bith $AB$; tugtar an *mion-stua* $AB$ are an gceann is giorra orthu.
(2) Nuair leagtar stua ciorcail $AB$ ar stua chómhfhada $CD$ de bharr scáthú i lárlíne, is soiléir gur stuanna iad $AB$ agus $CD$ atá i dtreonna *contrárdha* ar an imlíne.

Má's le casadh timpeall an láir áfach, a cuirtear stua $AB$ or cheann eile $XY$, tá na stuanna $AB$ agus $XY$ in aon treo amháin ar imlíne an chiorcail.

### Notes

(1) There are two arcs associated to any given chord $AB$ of a circle; the shorter of them is called the *minor arc* $AB$.
(2) When a circle arc $AB$ is laid on an equally-long arc $CD$ as a result of reflection in some diameter, it is clear that the arcs $AB$ and $CD$ run in *opposite* directions on the perimeter.

However, if it is a rotation about the centre that lays the arc $AB$ on another one $XY$, then the arcs $AB$ and $XY$ go in the same direction on the perimeter of the circle.

## 3.6 Teoirim VIII

*Má tá dhá chórda ciorcail cómhfhada le chéile, táid suiméitreach san lárlíne tré na bpointe teagmhála.*

Tá Fíoghair anseo sa LSS, leathanach 29.

Má's ar an imlíne a ghearas na córdaí cómhfhada $AB$,$AC$ a chéile, níl aon chórda eile tré $A$ atá cómhfhada leo (Teoirim VII) agus tá an teoirim soiléir de réir teoirme VI.

*Hipotéis:*
Abair anois go bhfuil $AB = XY$, agus ainmnigh na córdaí sin ionnas go bhfuil na mion-stuanna $AB$, $YXY$ contrárdha maidir le treo.

*Tátall:*
Scátha a chéile i lárlíne is ea na córdaí $AB$ agus $XY$.

*Cruthúnas:*
De bharr an plána a scáthú in a.s. $AX$, leagtar an córda $XY$ ar córda éigin den dá chórda $AB$, $AC$ tré $A$ atá cófhada leis.

Ní féidir gur ar $AC$ a thiteas sé , de bhrí go bhfuil na mion-stuanna $XY$ agus $AC$ is aon treo amháin.

Fágann sin go gcuirtear an córda (agus an stua) $XY$ anuas ar $AB$.

Tá Fíoghair anseo sa LSS, leathanach 29, ag bun ar chlé.

**Theorem 8.** *If two chords of a circle have the same length, they are symmetrical in the diameter through the point where they meet.*

If equally-long chords $AB$,$AC$ cut one another on the perimeter, then there is no other chord through $A$ that has the same length as them (Theorem 7) and (in that case) the theorem is clear from Theorem 6.

*Hypothesis:*

Suppose now that $AB = XY$, and name those chords so that the minor arcs $AB$, $XY$ have opposite directions.

*Conclusion:*

The chords $AB$ and $XY$ are reflections of one another in a diameter.

*Cruthúnas:*

When the plane is reflected a.s. $AX$, the chord $XY$ is laid on one of the two chords $AB$, $AC$ through $A$ that have the same length as it.

It cannot land on $AC$, because the minor arcs $XY$ and $AC$ have the same direction.

Thus the chord (and the arc) $XY$ lands on $AB$.

Tá Fíoghair anseo sa LSS, leathanach 29, ag bun ar chlé.

Stray text in English in MSS (page 29, bottom, next to the figure):
under rot BOC $BA\ toCD$: angle between $BA$ and $CD = \widehat{BOC}$.

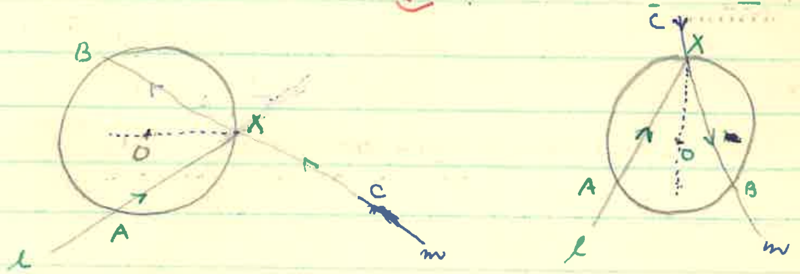
Bisector of this angle $\parallel AC$ and so equal $\hat{}BAC$.

$$\therefore \hat{}BOC = 2\hat{}BAC.$$

Atóra 1  Is cómhfhada na stuaghanna a ghearras córdaí cómhfhada ar imlíne chiorcail.

Atóra 2  Sin dhá chaoi chun córda ciorcail a leagan anuas ar chórda cóthrom; (a) le scáthú, a chuireas XY ar AB, (b) le casadh, a chuireas YX ar AB.

Atóra 3  Má leagtar dronlíne $l$ ar dhronlíne $m$ de bharr an plána a chasadh timpeall ar phointe O, luíonn O ar chómhroinnteoir de dhá chómhroinnteoir na h-uilleann idir $l$ is $m$.




Nuair tarraingtear an ⊙ tré X gur lár dó O, má's in A, B a ghearras sé $l$ agus $m$, feicfear go gcuirtear A ar X agus go gcuirtear X ar B.

∴ ~~córdaí cómhfhada isea AX agus~~ BX

∴ Leagtar AX ar an gcórda cómhfhada XB, agus táid suimétreach san láirlíne XO de réir na teoirme.

## Cleachtaithe

1) Dhá chórda ciorcail isea AB, CD atá ⊥ le láirlíne áirithe. Cruthuigh (i) AC = BD, (ii) AD = BC, (iii) go dtagann AD is BC le chéile ar láirlíne.

2) Téigheann trí ciorcail dhifriúla tré dhá phointe A, B. Teaspáin go bhfuil láir na gciorcal sin in aon líne amháin.

3) Nuair tugtar dhá phointe A, B, agus dronlíne $l$, tarraing ⊙ tré A is B go bhfuil a lár ar $l$.

4) 'Sé O lár an bhoinn BC sa △ ABC, agus 'sé an cineál triantáin é go bhfuil OA = OB = OC. Cruthuigh (1) go dtagann aist suimétreachta AB agus AC le chéile in O; agus (2) go bhfuil $\hat{A} = \hat{B} + \hat{C}$.

5) Aimsigh pointe O a bheas an fhad chéanna ó thrí phointe áirithe a tugtar.

6) Teaspáin cén chaoi a gcómhroinntear stua ciorcail.

7) Má tá na córdaí ciorcail AB agus XY cómhfhada, cruthuigh go bhfuil na h-ingir ón lár orthu cómhfhada. Cruthuigh freisin gur fíor a choinvéarsa sin.

8) Má's cómhfhada na h-ingir ó P ar dhá dhronlíne theagmhálacha cruthuigh go bhfuil P ar chómhroinnteoir d'uillinn idir an dá dhronlíne.

9. Gearrann dronlíne dhá chiorcal chómhláracha sna pointí A, B, C, D i n-ord a chéile ar an dronlíne. Cruthuigh AB = CD.

10. Tré phointe P taobh istigh de chiorcal áirithe, tarraing an córda atá cómhroinnte ag P. Tabhair cruthúnas ~~teid~~ led thógáil.

lth30

## Atora 1

*Is cómhfhada na stuanna a ghearas córdaí cómhfhada ar imlíne chiorcail.*

## Atora 2

*Sin dhá chaoi chun córda ciorcail a leagan anuas ar chórda chothrom: (a) le scáthú , a cuireas $XY$ ar $AB$, (b) le casadh, a chuireas $YX$ ar $AB$.*

## Atora 3

*Má leagtar dronlíne $\ell$ ar dhronlíne $m$ de bharr an plána a chasadh timpeall ar phointe $O$, luíonn $O$ ar chómhroinnteoir de dháchómhroinnteoir na huilleann idir $\ell$ is $m$.*

Tá Fíoghair anseo sa LSS, leathanach 30.

Nuair tarraingítear an ⊙ tré $X$ gur lár dó $O$, má's in $AB$ a ghearas sé $\ell$ agus $m$, feicfear go gcuirtear $A$ ar $X$ agus go gcuirtear $X$ ar $B$.

.i. Leagtar $AX$ are an gcórda cómhfhada $XB$, agus táid suiméitreach san lárlíne $XO$ de réir na teoirme.

**Corollary 1.** *Chords of equal length cut arcs of equal length on the permimeter of a circle.*

**Corollary 2.** *Here are two ways to lay a chord of a circle on an equal chord: (a) by a reflection that lays $XY$ on $AB$, (b) by a rotation that lays $YX$ on $AB$.*

**Corollary 3.** *If the straight line $\ell$ is laid on the straight line $m$ as a result of rotating the plane about a point $O$, then $O$ ilies on one of the two bisectors of the angles between $\ell$ and $m$.*

When the ⊙ through $X$ with centre $O$ is drawn, if it cuts $\ell$ and $m$ in $AB$, it will be seen that $A$ is placed on $X$ and $X$ is placed on $B$.

.i. $AX$ is laid on the equally-long chord $XB$, and they are symmetrical in the line $XO$ according to the theorem.

## Cleachtaithe

1. Dhá chórda ciorcail is ea $AB$,$CD$ atá ⊥ le láirlíne áirithe. Cruthaigh (i) $AC = BD$, (ii) $AD = BC$, (iii) go dtagann $AD$ is $BC$ le chéile ar (an) láirlíne.

2. Téigheann trí ciorcail dhifriúla tré dhá phointe $A, B$. Teaspáin go bhfuil láir na gciorcail sin nin aon dronlíne amháin.

3. Nuair tugtar dhá phointe $A, B$ agus dronlíne $\ell$, tarraing ⊙ tré $A$ is $B$ a bhfuil a lár ar $\ell$.

4. 'Sé $O$ lár an bhoinn $BC$ sa $\Delta\ ABC$, agus 'sé an cineál triantáin é go bhfuil $OA = OB = OC$. Cruthaigh (1) go dtagann aisí suiméitreachta $AB$ agus $AC$ le chéile in $O$; agus (2) go bhfuil $\hat{A} = \hat{B} + \hat{C}$.

5. Aimsigh pointe $O$ a bhéas an fhad chéanna ó thrí phointí áirithe a tugtar.

6. Teaspáin cé'n chaoi a gcómhroinntear stua ciorcail.

7. Má tá na córdaí ciorcail $AB$ agus $XY$ cómhfhada, cruthuigh go bhfuil na hingir ó'n lár orthu cómhfhada. Cruthaigh freisin gur fíor a choinvéarsa sin.

8. Má's cómhfhada na hingir ó $P$ are dhá dhronlíne teagmhálacha cruthuigh go bhfuil $P$ ar chómhroinnteoir d'uillinn idir an dá dhronlíne.

Tosach leathanach 31 sa LSS.

9. Gearann dronlíne dhá chiorcal chómhláracha sna pointí $A, B, C, D$ in ord a chéile ar an dronlíne. Cruthaigh $AB = CD$.

10. Tré phointe $P$ taobh istigh de chiorcail áirithe, tarraing an córda atá cómhroinnte ag $P$. Tabhair cruthúnas led thógaáil.

## Exercises

1. $AB$,$CD$ are two chords of a circle that are $\perp$ to a certain diameter. Prove (i) $AC = BD$, (ii) $AD = BC$, (iii) that $AD$ and $BC$ meet on that diameter.

2. Three different circles pass through two points $A, B$. Show that the centres of those circles lie on a single straight line.

3. Given two points $A, B$ and a straight line $\ell$, draw a $\odot$ through $A$ and $B$ having its centre on $\ell$.

4. $O$ is the centre of the base $BC$ in $\Delta\ ABC$, and it is the kind of triangle in which $OA = OB = OC$. Prove (1) that the axes of symmetry of $AB$ and $AC$ meet at $O$; and (2) that $\hat{A} = \hat{B} + \hat{C}$.

5. Find a point $O$ that will be the same distance from three given points.

6. Show how to bisect an arc of a circle.

7. If the bisectors of the circle chords $AB$ and $XY$ are the same length, prove that the perpendiculars to them from the centre are the same length. Also, prove that the converse is valid.

8. If the perpendiculars from $P$ on two iintersecting straight lineare are equally long, prove that $P$ is on the bisector of an angle between the two straight lines.

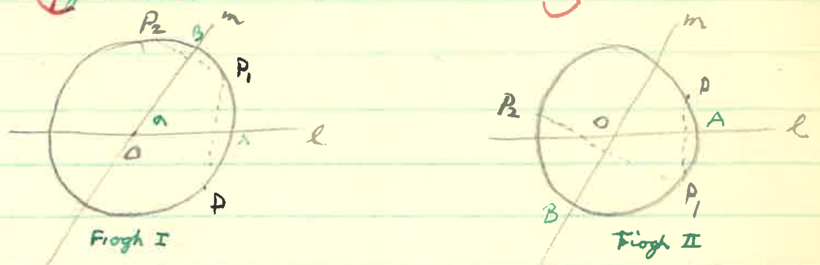
9. A straight line cuts two concentric circles at the points $A, B, C, D$ in that order on the straight line. Prove that $AB = CD$.

10. Through a given point $P$ inside a certain circle, draw the chord that is bisected by $P$. Prove that your construction works.

# Teoirmí Breise

## Teoirim A

Is ionann dhá scáthú an phlána as a chéile sna dronlínte teagmhálacha $l$ is $m$, agus an plána a chasadh timpeall a bpointe teagmhála trí dhá oiread na huilleann idir $l$ is $m$.




Is léir go bhfanann O socair de bharr an dá scáthú.

## Tógáil

Tóg pointe ar bith P sa bplána agus línigh an ⊙ trí P gurb é O a lár. Aimsigh $P_1$ scáth an phointe P in $l$, agus $P_2$ scáth an phointe $P_1$ in $m$, gur pointí iad ar imlíne an ⊙ de réir teoirme VI.

Binneann na trí pointí $P\,P_1\,P_2$ in ord a chéile treo áirithe ar an imlíne, agus is do stuanna san treo sin a thagras an cruthúnas.

Scríobhfar $\widehat{AB}$ chun an stua AB a chomharthú.

## Cruthúnas

Comhroinneann $l$ an stuagh $PP_1$ (teoirim VI).

∴ tá $\widehat{PP_1} = 2 \times \widehat{AP_1}$.

Mar an gcéanna, tá $\widehat{P_1P_2} = 2 \times \widehat{P_1B}$, agus le suimiú faightear

$$\widehat{PP_2} = 2 \times \widehat{AB}.$$

∴ Buailtear P ar $P_2$ de bharr chasta timpeall O, gurb ionann é agus dhá oiread an chasta a chuireas $l$ ar $m$.

Bé nach mar a chéile na huilleacha $A\hat{O}B$ san dá léaráid (uilleacha den chineál $\hat{\alpha}$ agus $-(180°-\hat{\alpha})$ is ea iad ó chast ~~is ionann le chéile céanna a fhreagraíos don dá uillinn sin,~~ is cuma cé acu a cuirtear i gceist sa teoirim, de bhrí gurb é an casadh céanna a fhreagraíos don dá uillinn $2\hat{\alpha}$ agus $-360° + 2\alpha$.

**Nóta** Má's scáthú in $m$ a déantar i dtosach agus ná scáthlear an plána san líne $l$ ina dhiaidh sin, is ionann é sin agus

Tosach leathanach 32 sa LSS.

## 3.7 Teoirmí Breise

## 3.8 Teoirim A

*Is ionann dhá scáthú an phlána as a chéile sna dronlínte teagmhálacha $\ell$ is $m$, agus an plána a chasadh timpeall a bpointe teagmhála tré dhá oiread na huilleann idir $\ell$ is $m$.*

Tá Fíoghair anseo sa LSS, leathanach 32.

Is léir go bhfanann $O$ socair de bharr an dá scáthú.

*Tógáil:*

Tóg pointe ar bith $P$ sa phlána agus línigh an ⊙ tré $P$ gurb é $O$ a lár. Aimsigh $P_1$ scáth an phoinnte $P$ in $\ell$, agus $P_2$ scáth an phointe $P_1$ in $m$, gur pointí iad ar imlíne an ⊙ de réir teoirme VI.

Cinneann na trí pointí $PP_1P_2$ in ord a chéile treo áirithe ar an imlíne, agus is do stuanna *san treo sin* a thagras an cruthúnas.

Scríbhfar $\widehat{AB}$ chun an stua $AB$ a chomharchú.

*Cruthúnas:*

Cómhroinnrann $\ell$ an stua $PP_1$ (teoirim VI).

$\therefore$ tá $\widehat{PP_1} = 2 \times \widehat{AP_1}$.

Mar an gcéanna, tá $\widehat{P_1P_2} = 2 \times \widehat{P_1B}$, agus le suimiú fáighter

$$\widehat{PP_2} = 2 \times \widehat{AB}.$$

.i. Cuirtear $P$ ar $P_2$ de bharr chasta timpeall $O$, gurb ionann é agus dhá oiread an chasta a chuireas $\ell$ ar $m$.

Cé nach mar a chéile ne huilleacha $\widehat{AOB}$ san dá léaráid (uilleacha den chineál $\hat{\alpha}$ agus $-(180° - \hat{\alpha})$ is ea iad ó ???) is cuma cé acu a cuirtear i gceist sa teoirim, de bhrí gurb é an casadh céanna a fhreagraíos don dá uillinn $2\hat{\alpha}$ agus $-360° + 2\hat{\alpha}$.

## 3.9 Extra Theorems

**Theorem A.** *The same effect is produced by ireflecting the plane in succession in two intersecting straight lines $\ell$ and $m$, and by rotating the plane about the point of intersection through twice the angle between $\ell$ and $m$.*

It is clear that $O$ remains fixed under the two reflections.

*Construction:*

Take any point $P$ in the plane and draw the ⊙ through $P$ with centre $O$. Find $P_1$, the reflection of the point $P$ in $\ell$, and $P_2$, the reflection of the point $P_1$ in $m$, both of which will be points on the perimeter of the ⊙ by Theorem 6.

The three points $PP_1P_2$ in that order determine a particular direction on the perimeter, and the proof will refer to arcs *in that direction*.

We shall denote the arc $AB$ by $\widehat{AB}$.

*Proof:*

$\ell$ bisects the arc $PP_1$ (Theorem 6).

$\therefore\ \widehat{PP_1} = 2 \times \widehat{AP_1}$.

Similarly, $\widehat{P_1P_2} = 2 \times \widehat{P_1B}$, and adding gives

$$\widehat{PP_2} = 2 \times \widehat{AB}.$$

.i. $P$ is placed on $P_2$ by a rotation about $O$ equal to twice the rotation that places $\ell$ on $m$.

Even though the angles $\widehat{AOB}$ in the two illustrations are different (they are angles of the form $\hat{\alpha}$ and $-(180^\circ - \hat{\alpha})$ from <illegible>[1]) it doesn't matter which of them is referred to in the theorem, because the same rotation corresponds to the two angles $2\hat{\alpha}$ ands $-360^\circ + 2\hat{\alpha}$.

---

[1] possibly Chapter something

casadh trí dhá oiread an chosta a leagas $m$ ar $l$.

Sin casadh atá contrárdha don chasadh a thagrann an teoirim dó.

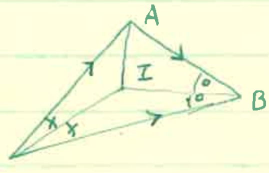
Téarma

Tugtar dronlínte comhreathacha ar dhronlínte a thagas le chéile in aon phointe amháin.

e.g. Aist suiméitreachta na dtrí slios i dtriantán ar bith.

Teoirim B

I dtriantán ar bith, dronlínte comhreathacha isea comhroinnteóirí na n-uilleann istigh.




Hipotéis

Comhroinnteóirí na n-uilleann $\hat{B}$ is $\hat{C}$ isea $BI$ agus $CI$.

Tátall

Tá le cruthú gurb é $AI$ comhroinnteoir na h-uilleann $\hat{A}$.

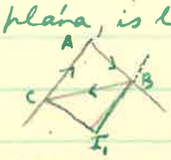
Cruthúnas.

De bhárr dhá scáthú in $BI$ agus $CI$ as éadan, cuirtear $AB$ fan $CB$ i dtosach, agus leagtar $CB$ fan $CA$ ansin.

∴ Cuirtear $AB$ fan na líne $ACA$ de bharr an dá scáthú, gurb ionann iad agus casadh áirithe timpeall $I$.

∴ Comhroinneann $AI$ an uille $C\hat{A}B$ [Teoirim VIII, Atora 3].

Nóta 1. Má's in $BI_1$, $CI_1$ (comhroinnteóirí na n-uilleann $B$ agus $C$ amuigh) a scáithtear an plána, is léir go gcuirfear $AB$ fan $CA$ arís, ionnas




go bhfuil $I_1$ ar chomhroinnteoir na h-uilleann $A$ freisin.

∴ Tá comhroinnteóirí na n-uilleann $\hat{B}$ agus $\hat{C}$ amuigh, comhreathach le comhroinnteoir na h-uilleann $\hat{A}$ istigh.

### Nóta

Má's scáthú in $m$ a déantar i dtosach agus má scáithtear an plána san líne $\ell$ ina dhiaidh sin, is ionann é sin agus

Tosach leathanach 33 sa LSS.

casadh tré dhá oiread an chasta a leagas $m$ ar $\ell$.
Sin casadh atá contrárdha don cheann a thagrann an teoirim dó.

### Téarma

Tugtar dronlínte *cómhreathacha* ar dhronlínte a thagas le chéile in aon phointe amháin.
e.g. Aisí suiméitreachta na dtrí slios i dtriantán ar bith.

### Note

If you start by reflecting in $m$ and then reflect the plane in the line $\ell$ after that, the result is the same as
a rotation that is twice the rotation that places $m$ on $\ell$.
That is an opposite rotation to the one that is involved in the theorem.

### Definition

(Several) lines are said to be *concurrent* if they all meet in a single point.
e.g. The axes of symmetry of the three sides of an arbitrary triangle.

## 3.10 Teoirim B

*I dtriantán ar bith, dronlínte cómhreathacha is ea cómh-roinnteoirí na n-uilleann istigh.*

Tá Fíoghair anseo sa LSS, leathanach 33.

*Hipotéis:*
Cómhroinnteoirí na n-uilleann $\hat{B}$ is $\hat{C}$ is ea $BI$ agus $CI$.

*Tátall:*
Tá le cruthú gurb é $AI$ cómhroinnteoir na huilleann $\hat{A}$.

*Cruthúnas:*
De bhár dhá scáthú in $BI$ agus $CI$ as éadan, cuirtear $AB$ fan $CB$ i dtosach, agus leagtar $CB$ fan $CA$ ansin.
.i. Cuirtear $AB$ fan na líne $CA$ de bharr an dá scáthú , gurn ionann iad agus casadh áirithe timpeall $I$.
∴ cómhroinneann $AI$ an uille $\widehat{CAB}$ [Teoirm VIII, Atora 3].

**Theorem B.** *In any triangle the bisectors of the inside angles are concurrent.*

*Hypothesis:*

$BI$ and $CI$. are the bisectors of the angles $\hat{B}$ and $\hat{C}$.

*Conclusion:*

We have to prove that $AI$ bisects the angle $\hat{A}$.

*Proof:*If we reflect in $BI$ and $CI$, one after the other, then first of all $AB$ is laid along $CB$, and then $CB$ along $CA$.
.i. $AB$ is laid along the line $CA$ as a result of the two reflections, which together produce the same effect as a certain rotation about $I$.

$\therefore$ có$AI$ bisects the angle $\widehat{CAB}$ [Theorem 8, Corollary 3].

casadh trí dhá oiread an chosta a leagas m ar l.

Sin casadh atá contrárdha don chasadh a thagrann an teoirim dó.

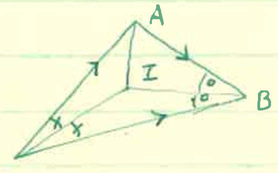
## Téarma

Tugtar dronlínte cómhreathacha ar dhronlínte a thagas le chéile in aon phointe amháin.

e.g. Áist suiméitreachta na dtrí slios i dtriantán ar bith.

## Teoirim B

I dtriantán ar bith, dronlínte cómhreathacha is ea cómh-roinnteóirí na n-uilleann istigh.




**Hipotéis**

Cómhroinnteóirí na n-uilleann $\hat{B}$ is $\hat{C}$ is ea BI agus CI.

**Tátall**

Tá le cruthú gurb é AI cómhroinnteóir na h-uilleann $\hat{A}$.

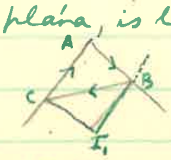
**Cruthúnas**

De bharr dhá scáthú in BI agus CI as éadan, cuirtear AB fan CB i dtosach, agus leagtar CB fan CA ansin.

∴ cuirtear AB fan na líne ACA de bharr an dá scáthú, gurb ionann iad agus casadh áirithe timpeall I.

∴ cómhroinneann AI an uille $C\hat{A}B$ [Teoirim VIII, Atora 3].

**Nóta 1.** Má's in $BI_1$, $CI_1$ (cómhroinnteóirí na n-uilleann B agus C amuigh) a scáithtear an plána, is léir go gcuirfear AB fan CA arís, ionnas




go bhfuil $I_1$ ar chómhroinnteóir na h-uilleann A freisin.

∴ Tá cómhroinnteóirí na n-uilleann $\hat{B}$ agus $\hat{C}$ amuigh, cómhreathach le cómhroinnteóir na h-uilleann $\hat{A}$ istigh.

## Nóta 1

Má's in $BI_1$, $CI_1$ (cómhroinnteoirí na n-uilleann $B$ agus $C$ amuigh) a scáithtear an plána, is léir go gcuirfear $AB$ fan $CA$ arís, ionas

Tá Fíoghair anseo sa LSS, leathanach 33.

go bhfuil $AI_1$ mar chómhroinnteoir na huilleann $A$ freisin.
.i. Tá cómhroinnteoirí na n-uilleann $\hat{B}$ agus $\hat{C}$ amuigh, cómhreathach le cómhroinnteoir na h-uilleann $\hat{A}$ istigh.

Tosach leathanach 34 sa LSS.

Tugtar *inlár* an triantáin ar $I$; 'sé $I_1$ an t-eislár ós cóir $A$. Tá eisláir eile ós cóir $B$ agus $C$.

## Nóta 2

Is cómhfhada na hingear ó $I$ (agus ó $I_1$) ar shleasa an triantáin.
Mar scátha a chéile sna cómhroinnteoirí is ea iad.

## Note 1

If (instead) we reflected the plane in $BI_1$, $CI_1$ (the bisectors of the external angles $B$ and $C$), it is clear that $AB$ would again be placed along $CA$, so that $AI_1$ is also the bisector of the angle $A$.
.i. The bisectors of the external angles $\hat{B}$ and$\hat{C}$ are concurrent with the bisector of the internal angle $\hat{A}$.

$I$ is called the *incentre* of the triangle; $I_1$ is called the exocentre opposite $A$. There are other exocentres opposite $B$ and $C$.

## Note 2

The perpendiculars from $I$ on the sides of the triangle are all the same length (as are those from $I_1$).

For they are reflections of one another in the bisectors.

# Caibidil IV

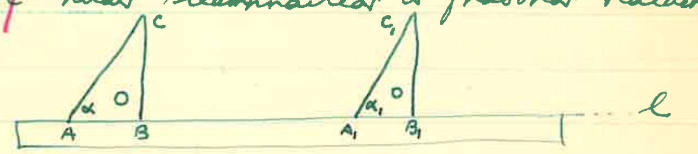
## Aistriú an Phlána, Paralléileacht.

Nuair castar plána timpeall ar phointe dhe, stuaigh ciorcail isea lorg gach pointe den phlána (cé's moite de lár an chasta). Sa gcaibidil seo is mian linn trácht ar chaoi eile chun plána a shleamhnú air féin, ionnas gur ar dhronlíne a ghluaiseas pointe ar bith.

Aistrigh leabhar ar chlár an bhúird i gcaoi go sleamhnaíonn ciumhais den leabhar fan dronlíne a marcáiltear ar an mbord roimh ré. Feicfear nach mbaineann casadh nó iompó don leabhar san imtheacht dó.

Línigh fíoghair phlánach agus dronlíne éigin $l$ ar do pháipéar, agus rianuigh arís iad ar pháipéar thrí-shoillseach a leagtar anuas orthu. Sleamhnuigh an páipéar uachtarach ar an gceann íochtrach i gcaoi go bhfanann rian na dronlíne $l$ ar $l$ féin ar feadh na gluaiseachta. Léiríonn sé sin ceard a bhaineas d'ionad na fíoghaire de bharr an aistriuithe.

Tugtar aistriú an phlána fan na dronlíne $l$ ar an gcineál sin gluaiseachta. Is aistriú a cuirtear i ngníomh ar dhronbhacart nuair sleamhnaítear ar fhaobhar rialach é.




Tá dhá threo chontrárdha ar $l$ chun aistriú a dhéanamh ionnte, viz. (i) nuair is ó $A$ go dtí $A_1$ a théigheas $A$, agus (ii) an t-aistriú ina ngluaiseann $A_1$ go dtí $A$, a chuireas an chéad aistriú ar neamhní.

Má's é $B_1$ an t-ionad a ghabhas $B$ san aistriú ó $A$ go dtí $A_1$, tuiteann $AB$ anuas go cruinn ar $A_1B_1$. Mar an gcéanna cuirfear géaga na h-uilleann $\hat{\alpha}$ ar ghéaga na h-uilleann $\hat{\alpha}_1$. Léiríonn sé sin réasún a rá go bhfuil $A_1B_1 = AB$, agus $\hat{\alpha} = \hat{\alpha}_1$, agus ní miste an prionsabal seo a leanas a chur ar bun:

# Caibidil 4

# Aistriú an Phlána. Paralléileacht

Tosach leathanach 35 sa LSS.

## Translating the Plane. Parallelism

Nuair caster plána timpeall ar phointe dhe, stua ciorcail is ea lorg gach pointe den phlána (cé's moite de láran chasta). Sa gcaibidil seo is mian linn trácht ar chaoi eile chun plána a shleamhnú air féin, ionas gur ar dhronlíne a ghluaiseas pointe ar bith.

Aistrigh leabhar ar dhromchla an bhoird i gcaoi go shleamhnaíonn ciúis an leabhar fan dronlíne a marcáiltear ar an mbord roimhré. Feicfear nach mbaineann casadh nó iompó don leabhar san imeacht dó. Línigh fíoghair phlánach agus dronlíne éigin $\ell$ ar an pháipéar, agus rianaigh arís iad ar pháipéar trí-shoillseach a leagtar anuas orthu. Sleamhnaigh an páipéar uachtarach ar an gceann íochtarach i gcaoi go bhanann rian na dronlíne $\ell$ ar $\ell$ féin ar feadh na gluaiseachta. Léiríonn sé sin ceard a bhaineas d'ionad na fíoghaire de bharr an aistrithe.

Tugtar *aistri*ú an phlána fan na dronlíne $\ell$ ar an gcineál sin gluaiseachta. Is aistriú a cuirtear i ngníomh ar dhronbhacart nuair shleamhnaítear ar fhaobhar rialach é.

Tá Fíoghair anseo sa LSS, leathanach 35.

Tá dhá treo chontrádha ar $\ell$ chun aistriú a dheanamh ionntu, viz. (i) nuair is ó $A$ go dtí $A_1$ a théigheas $A$, agus (ii) an t-aistriú ina ngluaiseann $A_1$ go dtí $A$, a chuireas an chéad aistriú ar meamhní.

Má'sé $B_1$ an t-ionad a ghabhas $B$ san aistriú ó $A$ go dtí $A_1$, tuiteann $AB$ anuas go cruinn ar $A_1B_1$. Mar an gcéanna cuirfear géaga na huilleann $\hat{\alpha}$ ar ghéaga na huilleann $\hat{\alpha}_1$. Luíonn sé le réasúngo bhfuil $A_1B_1 = AB$, agus $\hat{\alpha} = \hat{\alpha}_1$, agus ní miste an prionnsabal seo a leanas a chur ar bun:

When the plane is rotated around one of its points, the locus of each point of the plane (apart from the centre of the rotation) is a circle. In this chapter we want to discuss another way to slide the plane on itself, so that each point moves in a straight line.

Move a book on the surface of the table in such a way that the spine of the book slides along a straight line marked on the table in advance. You will observe that the book is not turned or rotated as it moves. Draw a planar figure and some straight line $\ell$ on the paper, and draw them again on transparent paper laid on top. Slide the upper paper on the lower in such a way that the copy of the straight line $\ell$ stays on top of $\ell$ throughout the movement. That shows what happens to the position of the figure as a result of the movement.

That kind of movement is called a *translation* of the plane along the straight line $\ell$. You are translating a set square when you slide it along the edge of a ruler.

There are two opposite ways along $\ell$ to make translations, viz. (i) when $A$ moves from $A$ to $A_1$, and (ii) the translation in which $A_1$ moves to go $A$, which cancels out the first translation.

If $B_1$ is the position to which $B$ moves with the translation from $A$ to $A_1$, then $AB$ lands exactly on $A_1B_1$. In the same way, the arms of the angle $\hat{\alpha}$ are placed on the arms of the angle $\hat{\alpha}_1$. It stands to reason that $A_1B_1 = AB$, and $\hat{\alpha} = \hat{\alpha}_1$, and we need to lay down the following principle:

## Bun-Phrionnsabal III

Is cómhfhada na línte agus is cómhmhéad na h-uilleacha go luíonn ceann acu go cruinn ar an gceann eile de bharr an plána a aistriú.

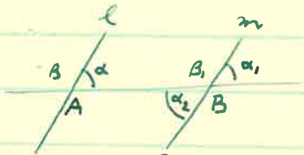
Cinneann aon dá phointe ar bith A is $A_1$ aistriú áirithe, viz. an t-aistriú fan $AA_1$ ina gcuirtear A anuas ar $A_1$. 'Sé an t-aistriú ó $A_1$ go dtí A an t-aistriú contrárdha.

Ó thárla $AB = A_1B_1$, tá $AA_1 = BB_1$, agus d'á bhrí sin cuireann na pointí difriúla ar $l$ an fhad chéanna díobh. Fágann sin gurb é an t-aistriú céanna é A go dtí $A_1$ nó B go dtí $B_1$.

## Téarmaí

Treasnaí a tugtar ar dhronlíne a ghearras dhá dhronlíne eile.

Dronlínte parallélacha isea dhá dhronlíne gur féidir líne acu a leagan ar an líne eile le h-aistriú.




Má's ar an bparallél $m$ a cuirtear $l$ de bharr áistrithe fan AB, ó A go dtí B, is soiléir go leagtar an uille $\hat{\alpha}$ anuas ar $\hat{\alpha}_1$. Tugtar uilleacha freagarthacha orthu siúd, agus uilleacha freagarthacha isea $\hat{\beta}$ agus $\hat{\beta}_2$ freisin. Uilleacha alteárnacha a tugtar ar $\hat{\alpha}$ agus $\hat{\alpha}_2$.

## Teoirim IX

Má ghearrann treasnaí dhá dhronlíne eile i gcaoi go bhfuil,

(i) na h-uilleacha freagarthacha ar cómhméad,

nó (ii) na h-uilleacha alteárnacha ar cómhméad,

nó (iii) suim an dá uillinn istigh atá ar aon taobh amháin den treasnaí cothrom le dhá dhronuillinn,

ansin dronlínte parallélacha isea an dá dhronlíne.

Tosach leathanach 36 sa LSS.

### Bun-Phrionnsabal III

*Is cófhada na línte agus is cómhméad na huilleacha go luíonn ceann acu go cruinn ar an gceann eile de bharr an plána a aistriú.*

Cinneann aon dá phointe ar bith $A$ is $A_1$, aistriú áirithe, viz. an t-aistriú fan $AA_1$ ina gcuirtear $A$ anuas ar $A_1$. 'Sé an t-aistriú ó $A_1$ go dtí $A$ an t-aistriú contrárdha.

Ó thárla $AB = A_1B_1$, tá $AA_1 = BB_1$, agus dá bhrí sin cuireann na pointí difriúla ar $\ell$ an fhad chéanna díobh. Fágann sin gurb é an t-aistriú céanna é $A$ go dtí $A_1$, nó $B$ go dtí $B_1$.

### Axiom III

*When the plane is translated each line is moved to a line of equal length and each angle is moved to an angle of equal size.*

Any two points $A$ and $A_1$ determine a particular translation, viz. the translation along $AA_1$ in which $A$ moves to $A_1$. The translation from $A_1$ to $A$ is the opposite translation.

Since $AB = A_1B_1$, we have $AA_1 = BB_1$, and hence all the different points on $\ell$ travel the same distance. Thus the translation from $A$ to $A_1$ is the same as $B$ to $B_1$.

### Téarmaí

*treasnaí* a tugtar ar dhronlíne a ghearas dhá dhronlíne eile.
*Dronlínte parallélacha* is ea dhá dhronlíne gur féidir líne amhaáin acu a leagan ar an líne eile le haistriú.

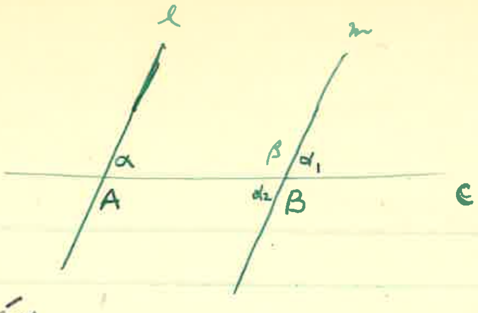
Tá Fíoghair anseo sa LSS, leathanach 36.

Má's ar an bparalléil $m$ a cuirtear $\ell$ de bharr aistrithe fan $AB$, ó $A$ go dtí $B$, is soiléir go leagtar an uille $\hat{\alpha}$ anuas ar $\hat{\alpha}_1$. Tugtar *uilleacha freagarthacha* orthu siúd, agus uilleacha freagarthacha is ea $\hat{\beta}$ agus $\hat{\beta}_1$ freisin. *Uilleacha altéarnacha* a tugtar ar $\hat{\alpha}$ agus $\hat{\alpha}_2$.

### Definitions

A line that cuts two other lines is called a *transversal*.
*Parallel lines* are lines, one of which can be moved onto the other by a translation.

If the parallel $m$ is where a $\ell$ moves as a result of the translation along $AB$, from $A$ to $B$, it is obvious that the angle $\hat{\alpha}$ moves onto $\hat{\alpha}_1$. Those angles are called *corresponding angles* and $\hat{\beta}$ and $\hat{\beta}_1$ are also corresponding angles. *Alternate angles* is the term used for $\hat{\alpha}$ and $\hat{\alpha}_2$.




(I) Hipotéis

Trasnaí isea AB a ghearras $l, m$ ionnus go bhfuil $\hat{\alpha} = \hat{\alpha}_1$

Tátall

Línte paralléalacha isea $l, m$.

Cruthúnas

De bharr an plána a aistriú fan AB ó A go dtí B, sleamhnaíonn an dronlíne AB uirthi féin.

∴ buntear AB fan BC.

Ach, ó tharla $\hat{\alpha} = \hat{\alpha}_1$, is ar m a leagtar an géag l.

∴ Tá l paralléalach le m de réir an t-sonnruithe.

(II) Hipotéis. Tá na h-uilleacha altéarnacha $\hat{\alpha}, \hat{\alpha}_2$ ar cómhméad.

Tátall Línte paralléalacha isea $l, m$.

Cruthúnas

Tugtar $\hat{\alpha} = \hat{\alpha}_2$. Ach tá $\hat{\alpha}_2 = \hat{\alpha}_1$ (teoirim II)

∴ Tá $\hat{\alpha} = \hat{\alpha}_1$, agus línte paralléalacha isea $l, m$, de réir (I).

(III) Hipotéis Is ionann agus dhá dhronuillinn an tsuim $\hat{\alpha} + \hat{\beta}$.

Tátall Línte paralléalacha isea $l, m$.

Cruthúnas

Tugtar $\hat{\alpha} + \hat{\beta} = 180°$. Ach tá $\hat{\alpha}_1 + \hat{\beta} = 180°$ (teoirim I).

Fágann sin $\hat{\alpha} = \hat{\alpha}_1$, ionnus gur línte paralléalacha iad de réir (I).

Q.E.D

## 4.1 Teoirim IX

*Má ghearrann treasnaí dhá dhronlíne eile i gcaoi go bhfuil,*
*(i) na huilleacha freagarthacha ar cómhméad,*
*nó (ii) na huilleacha altéarnacha ar cómhméad,*
*nó (iii) suiman dá uillinn istigh atá ar aon taobh amháin den treasnaí cothrom le dhá dhronuillinn,*
*ansin dronlínte parallélacha is ea an dá dhronlíne.*

Tosach leathanach 37 sa LSS.

Tá Fíoghair anseo sa LSS, leathanach 37.

(i)

*Hipotéis:*
Treasnaí is ea $AB$ a ghearas $\ell, m$ ionas go bhfuil $\hat{\alpha} = \hat{\alpha}_1$.

*Tátall:*
Línte parallélacha is ea $\ell, m$.

*Cruthúnas:*
De bharr an plána a aistriú fan $AB$ ó $A$ go dtí $B$, sleamhnaíonn an dronlíne $AB$ uirthi féin.
$\therefore$ Cuirtear $AB$ fan $BC$.
Ach, ó thárla $\hat{\alpha} = \hat{\alpha}_1$, is ar $m$ a leagtar an géag $\ell$.
$\therefore$ Tá $\ell$ parallélach le $m$ de réir an t-sonnruithe.

(ii)

*Hipotéis:*
Tá na huilleacha altéarnacha $\hat{\alpha}, \hat{\alpha}_2$ ar cómhméad.

*Tátall:*
Línte parallélacha is ea $\ell, m$.

*Cruthúnas:*
Tugtar $\hat{\alpha} = \hat{\alpha}_2$. Ach tá $\hat{\alpha}_2 = \hat{\alpha}_1$ (Teoirim II).
$\therefore$ Tá $\hat{\alpha} = \hat{\alpha}_1$, agus línte paralélacha is ea $\ell, m$, de réir (i).
(iii)

*Hipotéis:*
Is ionann agus dhá dhronuillinn an tsuim $\hat{\alpha} + \hat{\beta}$.

*Tátall:*
Línte parallélacha is ea $\ell, m$.

*Cruthúnas:*
Tugtar $\hat{\alpha} + \hat{\beta} = 180°$. Ach tá $\hat{\alpha}_1 + \hat{\beta} = 180°$ (Teoirim I). Fágann sin $\hat{\alpha} = \hat{\alpha}_1$, ionas gur línte parallélacha iad de réir (ii). □

**Theorem 9.** *If a transversal cuts two other straight lines in such a way that*
*(i) the corresponding angles are of equal size,*
*or (ii) the alternate angles are of equal size,*
*or (iii) the sum of two inside angles on the same side of the transversal is equal to two right angles,*
*then the two straight lines are parallel.*

noindent (i)

*Hypothesis:*
$AB$ is a transversal cutting $\ell, m$ so that $\hat{\alpha} = \hat{\alpha}_1$.

*Conclusion:*
The lines $\ell, m$ are parallel.

*Proof:*
When the plane is slid along $AB$ from $A$ to $B$, the straight line $AB$ slides on itself.
$\therefore$ $AB$ is moved to $BC$.
But, since $\hat{\alpha} = \hat{\alpha}_1$, the line $\ell$ is moved onto $m$.
$\therefore$ $\ell$ is parallel to $m$ according to the definition.

(ii)

*Hypothesis:*
The alternate angles $\hat{\alpha}, \hat{\alpha}_2$ are the same size.

*Conclusion:*
The lines $\ell, m$ are parallel.

*Proof:*
We are given that $\hat{\alpha} = \hat{\alpha}_2$. But $\hat{\alpha}_2 = \hat{\alpha}_1$ (Theorem 2).
$\therefore$ $\hat{\alpha} = \hat{\alpha}_1$, and $\ell, m$ are paralle lines, according to (i).
(iii)

*Hypothesis:*
The sum $\hat{\alpha} + \hat{\beta}$ is equal to two right angles.

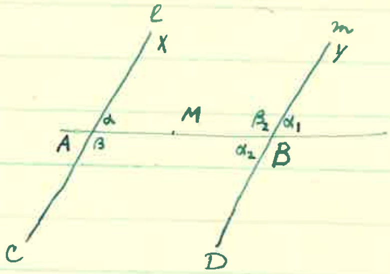
*Conclusion:*
The lines $\ell, m$ are parallel.

*Proof:*
We are given that $\hat{\alpha} + \hat{\beta} = 180°$. But $\hat{\alpha}_1 + \hat{\beta} = 180°$ (Theorem 1). It follows that $\hat{\alpha} = \hat{\alpha}_1$, so that the lines are parallel according to (ii). □

## Teoirim XV

(a) Is féidir dronlíne a leagan anuas ar dhronlíne pharallélaí de bharr an plána a chasadh trí 180°.

(b) Ní fhéadfadh dhá dhronlíne pharallélacha a theagmháil le chéile.




(a) Hipotéis

Tá aistriú éigin ann (ó A go dtí B, abair) a leagas $l$ ar $m$

Tógáil

Faigh M lár na dronlíne AB.

Tátall

Leagtar $l$ ar $m$ (agus cuirtear $m$ ar $l$) de bharr chasta trí 180° timpeall M.

Cruthúnas

De bhrí go gcuirtear $l$ ar $m$ de bharr an aistruithe ó A go dtí B, tá $\hat{\alpha} = \hat{\alpha}_1$. Ach tá $\hat{\alpha}_1 = \hat{\alpha}_2$ (teoirim II).

Fágann sin $\hat{\alpha} = \hat{\alpha}_2$, agus mar an gcéanna tá $\hat{\beta} = \hat{\beta}_2$.

Má castar an plána trí 180° timpeall M, cuirtear MA fan na líne MB, agus ó tharla MB = MA is anuas ar B a thuiteas A.

Ach de thairbhe $\hat{\alpha} = \hat{\alpha}_2$ is fan na líne BD a leagfar AX.

∴ Cuirtear $l$ fan na dronlíne $m$, agus mar an gcéanna cuirtear $m$ fan $l$.

Q.E.D.

(b) Hipotéis. Dronlínte parallélacha isea $l$, $m$.

Tátall. ní féidir leo teagmháil le chéile, pé treo ina síntear iad.

Cruthúnas

Dá ngearradh AX is BY i bpointe Z thuas, de bharr an chasta trí 180° timpeall M cuirfí Z ar phointe teagmhála BD agus AC thíos, ionnus go mbeadh dhá phointe theagmhála ag na dronlínte $l$ is $m$.

Ach ní fhéadfadh sé sin a bheith amhlaidh.

∴ Ní ghearrann $l$ is $m$ a chéile thuas ná thíos.

Q.E.D.

Ar chruthaigh é seo a chruthú féin, de bharr aistrithe amháin? Nuair a tugtar an t-aistriú $\vec{AB}$ do $l$, aistreofar Z (atá ar $m$) i líne dhíreach go pointe eile Z, de $m$, agus ní fan $m$ a aistrítear an t-aistriú.

Tosach leathanach 38 sa LSS.

## 4.2 Teoirim X

*(a) Is féidir dronlíne a leagan anuas ar dhronlíne pharallélach de bharr an phlána a chasadh tré* $180^\circ$.
*(b) Ní fhéadfadh dhá dhronlíne pharallélacha theagmháil le chéile.*

Tá Fíoghair anseo sa LSS, leathanach 38.

(a)

*Hipotéis:*
Tá aistriú éigin ann (ó $A$ go dtí $B$, abair) a leagas $\ell$ ar $m$.

*Tógáil:*
Faigh $M$ lár na dronlíne $AB$.

*Tátall:*
Leagtar $\ell$ ar $m$ (agus cuirtear $m$ ar $\ell$) de bharr chasta tré $180^\circ$ timpeall $M$.

*Cruthúnas:*
De bhrí go gcuirtear $\ell$ ar $m$ de bharr an aistrithe ó $A$ go dtí $B$, tá $\hat{\alpha} = \hat{\alpha}_1$. Ach tá $\hat{\alpha}_1 = \hat{\alpha}_2$ (Teoirim II).
Fágann sin $\hat{\alpha} = \hat{\alpha}_2$, agus mar an gcéanna tá $\hat{\beta} = \hat{\beta}_2$.
Má castar an plána tré $180^\circ$ timpeall $M$, cuirtear $MA$ fan na líne $MB$, agus ó thárla $MB = MA$ is anuas ar $B$ a thiteann $A$.
Ach de thairbhe $\hat{\alpha} = \hat{\alpha}_2$ is fan na líne $BD$ a leagfar $AX$.
.i. Cuirtear $\ell$ fan na dronlíne $m$, agus mar an gcéanna cuirtear $m$ fan $\ell$. □

(b)

*Hipotéis:*
Dronlínte parallélacha is ea $\ell, m$.

*Tátall:*
Ní féidir leo teagmháil le chéile, pé treo ina síntear iad.

*Cruthúnas:*
Dá ngearradh $AX$ is $BY$ i bpointe $Z$ thuas, de bharr an chasta tré $180^\circ$ timpeall $M$ cuirfí $Z$ ar phointe teagmhála $BD$ agus $AC$ thíos, ionas ionas go mbeadh dhá phointe teagmhála ag na dronlínte $\ell$ is $m$.
Ach ní féidir sin a bheith amhlaidh.
$\therefore$ Ní ghearann $\ell$ is $m$ a chéile thuas ná thíos. □
[1]

**Theorem 10.** *(a) A straight line can be moved onto a parallel straight line by rotating the plane through* $180^\circ$.
*(b) Two parallel straight lines cannot meet.*

[1] Tá ábhar scríofa san imeall agus líne tríd.

(a)

*Hypothesis:*

There is some translation (from $A$ to $B$, say) that lays $\ell$ on $m$.

*Construction:*

Find $M$, the centre of the straight line $AB$.

*Conclusion:*

$\ell$ is laid on $m$ (and $m$ s put on $\ell$) by a rotation through 180° about $M$.

*Proof:*

Since $\ell$ is put on $m$ by the translation from $A$ to $B$, we have $\hat{\alpha} = \hat{\alpha}_1$. But $\hat{\alpha}_1 = \hat{\alpha}_2$ (Theorem 2).

It follows that $\hat{\alpha} = \hat{\alpha}_2$, and in the same way $\hat{\beta} = \hat{\beta}_2$.

If the plane is rotated through 180° around $M$, then $MA$ is placed along the line $MB$, and since $MB = MA$ it is on $B$ that $A$ lands.

But since $\hat{\alpha} = \hat{\alpha}_2$, $AX$ is laid along $BD$.

.i. $\ell$ is put along the straight line $m$, and similarly $m$ s laid along $\ell$. □

(b)

*Hypothesis:*

$\ell$ and $m$ are parallel straight lines.

*Conclusion:*

They will never meet, no matter how far they are extended.

*Proof:*

If $AX$ were to meet $BY$ at the point $Z$ above, the result of a rotation through 180° about $M$ would be to place $Z$ on the point where $BD$ and $AC$ below, so that there would be two common points on the lines $\ell$ and $m$.

But that cannot happen.

∴ $\ell$ and $m$ do not cut one another, above or below. □

Atora. Ní féidir aistriú a chur i ngníomh le casadh singil timpeall ar phointe ar bith.

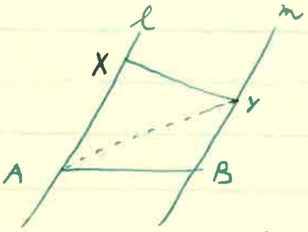
Mar, dá mba phointe é Z nár chorraigh, cuirfí ~~ZX ar ZY~~ ZA ar ZB agus ba phointe leargmhála dhá dhronlíne pharallélacha é Z.

Nóta. Sin dhá chaoi chum l a leagan anuas ar an bparallél m, agus an pointe A a thuitfim ar B, viz. (i) an plána a aistriú fan AB, agus (ii) an plána a chasadh tré 180° timpeall M.

Is contrárdha dá chéile áfach na treóanna go leagtar l iontu san dá chás.

Acsióm Playfair

De réir an tsonnruithe nuair tugtar dhá dhronlíne a bheith parallélach, is ionann é is a rá go bhfuil aistriú amháin ann, ar a laghad, a leagas l ar m; e.g. ó A go dtí B fan na dronlíne AB.




Má's treasnaí eile é AY, is léir go gcuirfear l ar m arís de bharr an dá aistriú ó A go dtí B agus ó B go dtí Y as a chéile, ach is ar an bpointe Y a thuitfeas A anois.

Dá n-aistrítí an plána ó A go dtí Y fan na dronlíne AY, cuirfí A ar Y agus leagfaí l ar pharallél éigin tré Y. Ní léir dúinn anois ~~áfach~~ áfach (agus ní féidir a chruthú le bun-phrionnsabal III) gur anuas go cruinn ar m a leagfar í.

Tosach leathanach 39 sa LSS.

### Atora

*Ní féidir aistriú a chur i ngníomh le casadh singil timpeall ar phointe ar bith.*

Mar, dá mba phointe é $Z$ nár chorruigh, cuirfí $ZA$ ar $ZB$ agus ba phointe teagmhála dhá dhronlíne pharallélacha é $Z$.

### Nóta

Sin dhá chaoi chun $\ell$ a leagan anuas ar an bparalléil $m$, agus an pointe $A$ a thitim ar $B$, viz. (i) an plána a aistriú fan $AB$ agus (ii) a plána aa chasadh tré 180° timpeall $M$.

Is contrádha dá chéile áfach na treoanna go leagtar $\ell$ ionta san dá chás.

**Corollary.** *A translation cannot be achieved by a single rotation around any point at all.*

For if $Z$ were the point that does not move, then $ZA$ would move to $ZB$ and those two parallel lines would meet at $Z$.

### Note

There are two ways to move $\ell$ onto the parallel $m$, so that the point $A$ falls on $B$, viz. (i) to translate the plane along $AB$ and (ii) to rotate the plane through 180° about $M$.

However, in the two cases $\ell$ is laid in contrary directions.

Gheofar amach le triálacha áfach gur mar sin atá, ionnas gur mar a chéile an dá aistriú A go dtí B agus B go dtí Y as éadan, agus an t-aistriú singil A go dtí Y.

Mar an gcéanna má's pointe ar bith eile é X ar $l$, is ionann an t-aistriú X go dtí Y agus an dá aistriú X go dtí A agus A go dtí Y as a chéile, rud a leagas $l$ ar $m$ arís, ach go dtuiteann an pointe X ar an bpointe Y dá bharr.

'Séard a léiríos na triálacha sin gur cuma cén pointe de $l$ a cuirtear ar Y dá bharr aistriuithe, is fan na líne $m$ a cuirtear $l$ i gcomhnaí, ionnas gurb é $m$ an t-aon dronlíne amháin trí Y atá paralléileach le $l$.

## Bun-Phrionnsabal IV (Acsióm Playfair)

Trí phointe ar bith ar phlána, ní théigheann ach dronlíne amháin atá paralléileach le dronlíne áirithe.

Is furasta a theaspáint gurb ionann acsióm Playfair agus teoirim ar bith den dá theoirim seo a leanas.

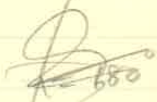
## Teoirim XII (Coinvéarsa Teoirme IX)

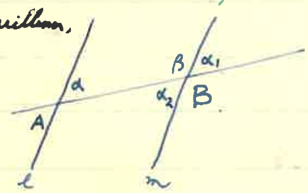
Má ghearrann treasnaí ar bith dhá dhronlíne pharalléileacha, tá,

(i) na h-uilleacha freagarthacha ar cómhméad,

(ii) na h-uilleacha ailtéarnacha ar cómhméad,

(iii) sum an dá uillinn istigh atá ar aon taobh amháin cothrom le dhá dhronuillinn.






**Hipotéis** Treasnaí ar bith ida AB a ghearras na dronlínte paralléileacha $l$ agus $m$.

**Tátall** Tá (i) $\hat{\alpha} = \hat{\alpha}_1$ ; (ii) $\hat{\alpha} = \hat{\alpha}_2$ ; (iii) $\alpha + \beta = 180°$.

**Cruthúnas**

Nuair a h-aistrítear an plána fan AB go gcuirtear A ar B, leagtar an dronlíne $l$ anuas ar $m$. (Acsióm Playfair).

## 4.3 Acsióm Playfair

De réir an tsonnruithe nuair tugtar dhá dhronlíne a bheith parallélach, is ionann é is a rá go bhfuil aistriú amháin ann, ar a laghad, a leagas $\ell$ ar $m$; e.g. ó $A$ go dtí $B$ fan na dronlíne $AB$.

Tá Fíoghair anseo sa LSS, leathanach 39.

Má's treasnaí eile é $AY$, is léir go gcuirfear $\ell$ ar $m$ arís de bharr an dá aistriú ó $A$ go dtí $B$ agus ó $B$ go dtí $Y$ as a chéile, ach is ar an bpointe $Y$ a thitfeas $A$ anois.

Dá n-aistrítí an plána ó $A$ go dtí $Y$ fan na dronlíne $AY$, cuirfí $A$ ar $Y$ agus leagfaí $\ell$ ar pharalléil éigin tré $Y$. Ní léir dúinn anois áfach (agus ní féidir a chruthú le bun-phrionnsabal III) go anuas go cruinn ar $m$ a leagfar í .

Tosach leathanach 40 sa LSS.

Gheofar amach le triálacha áfach gur mar sin atá , ionas gur mar a chéile an dá aistriú $A$ go dtí $B$ agus $B$ go dtí $Y$ as éadan, agus an t-aistriú singil $A$ go dtí $Y$.

Mar an gcéanna má's pointe ar bith eile í $X$ ar $\ell$, is ionann an t-aistriú $X$ go dtí $Y$ agus an dá aistriú $X$ go dtí $A$ agus $A$ go dtí $Y$ as a chéile, rud a leagas $\ell$ ar $m$ arís, ach go dtiteann an pointe $X$ ar an bpointe $Y$ dá bharr.

'Séard a léiríos na triálacha sin gur cuma cén pointe de $\ell$ a cuirtear ar $Y$ de bharr aistrithe, is fan na líne $m$ a cuirtear $l$ i gcomhnaí , ionas gurb é $m$ an t-aon dronlíne amháin tré $Y$ atá parallélach le $\ell$.

### Bun-Phrionnsabal IV (Acsióm Playfair)

*Tréphointe ar bith ar phlána, ní théigheann ach dronlíne amháin atá parallélach le dronlíne áirithe.*

Is furasta a theaspáint gurb ionann acsíom Playfair agus teoirim ar bith den dá theoirim seo a leanas.

## 4.4 Playfair's Axiom

According to the definition, when we are given that two straight lines are parallel, that is the same as saying that there is one translation, at least, that moves $\ell$ onto $m$; e.g. from $A$ to $B$ along the straight line $AB$.

If $AY$ is another transversal, it is clear that $\ell$ is laid on $m$ again as a result of the translations from $A$ to $B$ and from $B$ to $Y$ together, but now the point $Y$ is where $A$ ends up.

If the plane were translated from $A$ to $Y$ along the straight line $AY$, then $A$ would be placed on $Y$ and $\ell$ would be laid on some parallel through $Y$. However, it is not obvious (and it is impossible to prove using Axiom III) that $\ell$ ends up exactly on $m$.

Experiment, however, will show that this is the case, so that the two translations are equivalent: from $A$ to $B$ and from $B$ to $Y$ combined, and the single translation from $A$ to $Y$.

Similarly, if $X$ is any other point on $\ell$, then the translation from $X$ to $Y$ and the two translations $X$ to $A$ and $A$ to $Y$ in sequence produce the same effect, which is to lay $\ell$ on $m$ again, except that now the point $X$ ends up at the point $Y$ as a result.

What these experiments show is that it makes no difference which point of $\ell$ is placed at $Y$ by the translation, the line $\ell$ is always laid along the line $m$, so that $m$ is the only straight line through $Y$ that is parallel to $\ell$.

## Axiom IV (Playfair's Axiom)

*There is only one straight line through any give point of the plane, parallel to a given straight line.*

It is easy to show that Playfair's axiom is equivalent to either of the two following theorems.

(i) ∴ Leagtar $\hat{\alpha}$ anuas ar an uillinn fhreagarthaigh $\hat{\alpha}_1$ ∴ $\hat{\alpha} = \hat{\alpha}_1$.

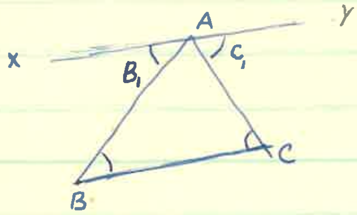
(ii) Ó tharla $\hat{\alpha}_1 = \hat{\alpha}_2$ (teoirim II), fágann sin $\hat{\alpha} = \hat{\alpha}_2$.

(iii) Tá $\hat{\alpha} + \hat{\beta} = \hat{\alpha}_1 + \hat{\beta} = 180°$ (teoirim I)

Q.E.D.

Teoirim XIII

I dtriantán ar bith is ionann suim na dtrí n-uilleann istigh agus dhá dhronuillinn.




Hipotéis Triantán ar bith ABC.

Tógáil Abair gurb í XAY an t-aon pharailéil amháin le BC a ghabhas tré A.

Tátall Tá $\hat{A} + \hat{B} + \hat{C} = 180°$.

Cruthúnas

Ó's trasnaí é AB a ghearras na línte parailéalacha BC, XY, tá na h-uilleacha ailtéarnacha $\hat{B}$ agus $\hat{B}_1$ ar cómhméad.

Mar an gcéanna trasnaí eile isea AC, ionnus go bhfuil $\hat{C} = \hat{C}_1$.

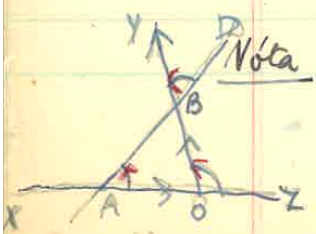
Fágann sin $\hat{B} + \hat{A} + \hat{C} = \hat{B}_1 + \hat{A} + \hat{C}_1 = 180°$ (teoirim I).

Q.E.D.

Atoradh

Má sínter slios triantáin is ionann an uille amuigh agus suim an dá uillinn neamhchómhgaracha istigh.

[illegible]? Mar is í fóirlíon na h-uilleann comhgaraí í, agus sin suim an dá uillinn eile de réir na teoirme.




Nóta Má's bothair dhíreacha iad XZ is OB, ~~tá~~ casadh tré'n uillinn $Z\hat{O}B$ le déanamh ag O, ag gluaisteánaí a thagas ó X agus gur mian leis iompó i dtreo Y. Sin casadh an-ghéar. Má tá bothar eile AB trasna áfach, beg leis dhá iarracht a thabhairt faoi i.e. $O\hat{A}B$ ag O agus $D\hat{B}Y$ ag B. Feách gurb ionann ~~suim~~ an dá chasadh bheag agus an casadh iomlán ag O.

## 4.5 Teoirim XI (Coinvéarsa Theoirme IX)

*Má ghearrann treasnaí* ar bith *dhá dhronlíe pharallélacha, tá,*
*(i) na nuilleacha freagarthacha ar cóméad,*
*(ii) na huilleacha altéarnacha ar cóméad,*
*(iii) suim an dá uilinn istigh atá ar aon taobh amháin chothrom le dhá dhronuillinn.*

Tá Fíoghair anseo sa LSS, leathanach 40.

*Hipotéis:*
Treasnaí ar bith is ea $AB$ a ghearas na dronlínte parallélacha $\ell$ agus $m$.

*Tátall:*
Tá (i) $\hat{\alpha} = \hat{\alpha}_1$; (ii) $\hat{\alpha} = \hat{\alpha}_2$; (iii) $alpha + \beta = 180°$.

*Cruthúnas:*
Nuair a haistrítear an plána fan $RAB$ go gcuirtear $A$ ar $B$, leagtar an dronlíne $\ell$ anuas ar $m$ (Acsíom Playfair).

Tosach leathanach 41 sa LSS.

(i) ∴ Leagtar $\hat{\alpha}$ anuas ar an uillinn fhreagarthaigh $\hat{\alpha}_1$, .i. $\hat{\alpha} = \hat{\alpha}_1$.

(ii) Ó thárla $\hat{\alpha}_1 = \hat{\alpha}_2$ (Teoirim II), fágann sin $\hat{\alpha} = \hat{\alpha}_2$.

Tá $\hat{\alpha} + \hat{\beta} = \hat{\alpha}_1 + \hat{\beta} = 180°$ (Teoirim I). □

**Theorem 11** (Converse of Theorem 9)**.** *If* any transversal at all *cuts two parallel straight lines, then*
*(i) the corresponding angles are the same size,*
*(ii) the alternate angles are the same size,*
*(iii) the sum of the two inside angles on one side add to two right angles.*

*Hypothesis:*
$AB$ is any translation that cuts the parallel straight lines $\ell$ and $m$.

*Conclusion:*
(i) $\hat{\alpha} = \hat{\alpha}_1$; (ii) $\hat{\alpha} = \hat{\alpha}_2$; (iii) $alpha + \beta = 180°$.

*Proof:*
When the plane is translated along $RAB$ to put $A$ on $B$, the straight line $\ell$ is placed on $m$ (Playfair's Axiom).

(i) ∴ $\hat{\alpha}$ lands on the corresponding angle $\hat{\alpha}_1$, .i. $\hat{\alpha} = \hat{\alpha}_1$.

(ii) Ó thárla $\hat{\alpha}_1 = \hat{\alpha}_2$ (Theorem 2), and hence $\hat{\alpha} = \hat{\alpha}_2$.

$\hat{\alpha} + \hat{\beta} = \hat{\alpha}_1 + \hat{\beta} = 180°$ (Theorem 1). □

## 4.6 Teoirim XII

*I dtriantán ar bith is ionann suim na dtrí n-uilleann istigh agus dhá dhronuillinn.*

Tá Fíoghair anseo sa LSS, leathanach 41.

*Hipotéis:*
Triantán is ea $ABC$.

*Tógáil:*
Abair gurb í $XAY$ an t-aon pharalléil ahmáin le $BC$ a ghabhas tré $A$.

*Tátall:*
Tá $\hat{A}+\hat{B}+\hat{C}=180°$.

*Cruthúnas:*
Ó's treasnaí é $AB$ a ghearas na línte parallélacha $BC, XY$, tá na huilleacha altéarnacha $\hat{B}$ agus $\hat{B}_1$ ar cómhméad.
Mar an gcéanna treasnaí eile is ea $AC$, ionas go bhfuil $\hat{C}=\hat{C}_1$.
Fágann sin $\hat{B}+\hat{A}+\hat{C}=\hat{B}_1+\hat{A}+\hat{C}_1=180°$. □

**Theorem 12.** *In any triangle at all the sum of the three inside angles is equal to two right angles.*

*Hypothesis:*
$ABC$ is a triangle.

*Construction:*
Let $XAY$ be the sole parallel to $BC$ that passes through $A$.

*Conclusion:*
$\hat{A}+\hat{B}+\hat{C}=180°$.

*Proof:*
Since $AB$ is a transversal that cuts the parallel lines $BC, XY$, the alternate angles $\hat{B}$ and $\hat{B}_1$ are the same size.
Similarly, $AC$ is another transversal, so that $\hat{C}=\hat{C}_1$.
Thus $\hat{B}+\hat{A}+\hat{C}=\hat{B}_1+\hat{A}+\hat{C}_1=180°$. □

### Atora

*Má síntear slios triantáin is ionann an uille amuigh agus suim an dá uillinn meamhchógaracha istigh.*

Tá Fíoghair anseo sa LSS, leathanach 41, ceann beag, ar dheis.

Mar 'sí fóirlíon na huilleann comhgoraí í , agus sin suim an dá uilinn eile de réir na teoirme.

Tá Fíoghair anseo sa LSS, leathanach 41, imeall clé .

## Nóta

Tá Fíoghair anseo sa LSS, leathanach 41, imeall clé .

Má's bothair dhireacha iad $XY$ is $OB$ tá casadh tré'n uillinn $\widehat{ZOB}$ le déanamh ag $O$, ag gluaisteánaí a thagas ó $X$ agus gur mian leis iompó i dtreo $Y$. Sin casadh an-ghéar. Má tá bothar eile $AB$ treasna áfach, tig leis dhá iarracht a thabhairt faoi i.e. $\widehat{OAB}$ ag $A$[2] agus $\widehat{DBY}$ ag $B$. Féach gurb ionann suim an dá chasadh bheag leis an casadh iomlán ag $O$.

Tá Fíoghair anseo sa LSS, leathanach 41, san imeall.

**Corollary.** *When one side of a triangle is extended, the external angle is equal to the sum of the two non-adjacent angles inside.*

Because it is the complement of the adjacent angle, and that is the sum of the other two angles, according to the theorem.

## Note

If $XY$ and $OB$ are straight roads A driver would have to make a turn through an angle $\widehat{ZOB}$ at $O$, if he were coming from $X$ and wanted to turn in the direction of $Y$. That is a very sharp turn. If, however, there is another cross-road $AB$, he can make two efforts of it, i.e. $\widehat{OAB}$ at $A$ and $\widehat{DBY}$ at $B$. Observe that the sum of the two small rotations is the same as the whole rotation at $O$.

[2] botún sa LSS: $O$ in áit $A$

**Téarmaí** Scríobhtar an nod // in áit "parallélach".

**Parallélogram** isea ceathairshleasán ar bith ina bhfuil gach slios // leis an slios atá ós a chóir. Scríobhtar ▱ in áit "parallélograim" amanta.

## Cleachtaithe

1) Má tá $\hat{\alpha} = 60°$ sa bhfioghair i dteoirim X, faigh $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\beta}$.

2) Línigh ar do pháipéar dhá dhronlíne // tré dronbhacart a sleamhnú ar fhaobhar rialach. Tarraing trí treasnaithe ar bith agus faigh le h-uilleanntomhas na h-uilleacha ailtéarnacha.

3) Trí dronlínte isea $l, m, n$. Tá $l$ // le $n$, agus tá $m$ // le $n$. Tarraing treasnaí ar bith agus luaidh aistriú (I) a chuireas $l$ ar $m$; (II) a chuireas $m$ ar $n$; (III) a chuireas $l$ ar $n$.

Má tá dhá dhronlíne // le dronlíne eile, cruthuigh go bhfuil siad féin // le chéile.

4) Tá dhá dhronlíne ⊥ le dronlíne áirithe; cruthuigh go bhfuil siad //.

5) Cordaí comhfhada i gciorcal isea AB, CD conas go bhfuil na mion-stuanna AB is CD sa treó céanna ar an imlíne. Cruthuigh de thairbhe cheiste 4 go bhfuil AD // le BC.

6) Teasgmhaíonn dhá dhronlíne $l, m$ le chéile i bpointe A, agus táid // leith ar leith le dhá dhronlíne eile $l_1$ agus $m_1$ a ghearras a chéile in B. Cruthuigh gurb ionann an uille dheimhneach idir $l$ is $m$ agus an uille dheimhneach idir $l_1$ is $m_1$.

[Lide: an t-aistriú ó A go dtí B a bhreithniú]

7) De thairbhe (6) cruthuigh go bhfuil gach uille i bparallélogram cothrom leis an uillinn atá ós a cóir.

8) I bparallélogram ar bith ceithre dronuilleacha isea suim na gceithre n-uilleann istigh.

9) Tarraing triantán ar bith OAB agus aimsigh ionad nua $OA_1B_1$ an triantáin sin ara chasadh tré 180° timpeall O.

Cruthuigh go bhfuil $B_1A_1$ // le AB.

10) Pointí isea X, Y ar shleasa an triantáin chomhchosaigh ABC conas go bhfuil XY // leis an mbonn BC. Cruthuigh gur Δ comhchosach eile é AXY.

11) I dtriantán comhshleasach ar bith, teaspáin go bhfuil 60° in gach uillinn.

12) I dtriantán comhchosach áirithe is mó faoi dhó gach bonn-uille ná an stuacuille. Áirmhigh toisí na dtrí n-uilleann.

13) Pointe isea P taobh istigh den Δ ABC. Cruthuigh

$B\hat{P}C = B\hat{A}C + A\hat{B}P + A\hat{C}P$.

[Lide: sín an líne AP agus breithnigh uilleacha na Δ BAP agus CAP]

Tosach leathanach 42 sa LSS.

## Téarmaí

Scríobhtar an nod ∥ in áit “parallélach".
*Parallélogram* is ea ceatharshleasán ar bith ina bhfuil gach slios ∥ leis an sios atá ós a chóir. Scríobhtar ▱ in áit “paralélogram" amannta.

## Definitions

We write the symbol ∥ instead of “parallel".
A *Parallelogram* is any quadilateral n which each side is ∥ to the side opposite it. We sometimes write ▱ instead of “parallelogram".

## Cleachtaithe

1. Má tá $\hat{\alpha} = 60°$ sa bhfíoghair i dteoirim X, faigh $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}$.

2. Línigh ar do pháipéar dhá dhronlíne ∥ tré dronbhacart a sleamhnú ar faobhar rialach. Tarraing trí treasnaithe ar bith agus agus faigh le huilleanntomhas na huilleacha altéarnacha.

3. Trí dronlínte is ea $\ell, m, n$. Tá $\ell \parallel$ le $m$ agus tá $m \parallel$ le $n$. Tarraing treasnaí ar bith agus luaidh aistriú (i) a chuireas $\ell$ ar $m$; (ii) a chuireas $m$ ar $n$; (iii) a chuireas $\ell$ ar $n$.

   Má tá dhá dhronlíne ∥ le dronlíne eile, cruthuigh go bhfuil siad féin ∥ le chéile.

4. Tá dhá dhronlíne ⊥ le dronlíne áirithe; Cruthaigh go bhfuil siad ∥.

5. Cordaí cómhfhada i gciorcal is ea $AB, CD$ ionas go bhfuil na mion-stuanna $AB$ is $CD$ sa treó céanna ar an imlíne. Cruthuigh de thairbhe Ceist 4 go bhfuil $AD \parallel$ le $BC$.

6. Teagmhaíonn dhá dhronlíne $\ell, m$ le chéile i bpointe $A$, agus táid ∥ leith ar leith le dhá dhronlíne eile $\ell_1$ agus $m_1$, a ghearras a chéile in $B$. Cruthaigh gurb ionann an uile deimhneach idir $\ell$ is $m$ agus an uille deimhneach idir $\ell_1$ is $m_1$.
   [Lide: an t-aistriú ó $A$ go $B$ a bhreithniú.]

7. De thairbhe (6) cruthuigh go bhfuil gach uille i bparallélogram cothrom leis an uillinn atá ós a cóir.

8. I bparallélogram ar bith ceithre dronuilleacha is ea suim na gceithre n-uilleann istigh.

9. Tarraing triantán ar bith $OAB$ agus aimsigh ionad nua $OA_1B_1$ an triantáin sin arna chasadh tré $180°$ timpeall $O$.

   Cruthaigh go bhfuil $B_1A_1 \parallel$ le $AB$.

10. Pointí is ea $X, Y$ ar shleasa an triantáin cómhchosach $ABC$ ionas go bhfuil $XY \parallel$ leis an mbonn $BC$. Cruthuigh gur $\Delta$ cómhchosach eile é $AXY$.

11. I dtriantán chómhshleasach ar bith, teaspáin go bhfuil $60°$ i ngach uillinn.

12. I dtriantán chómhchosach áirithe is mó faoi dhó gach bonn-uille ná an stuacuille. Aimsigh toisí na dtrí n-uilleann.

13. Pointe is ea $P$ taobhistigh den $\Delta$ $ABC$. Cruthuigh

$$\widehat{BPC} = \widehat{BAC} + \widehat{ABP} + \widehat{ACP}.$$

   [Lide: ín an líne $AP$ agus breithnigh uilleacha na $\Delta$ $BAP$ agus $CAP$.]

Tá Fíoghair anseo sa LSS, leathanach 42, ag bun, san imeall.

## Exercises

1. If $\hat{\alpha} = 60°$ in the figure in Theorem 10 find $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}$.

2. Draw on your paper two parallel lines by sliding a set square along a ruler. Draw any three transversals and measure the alternate angles with a protractor.

3. $\ell, m, n$ are three straight lines. $\ell \parallel$ to $m$ and $m \parallel$ to $n$. Draw any transversal at all and look for a translation (i) that puts $\ell$ on $m$; (ii) that puts $m$ on $n$; (iii) that puts $\ell$ on $n$.

   If two straight lines are parallel to another straight line, prove that they themselves are $\parallel$ with one another.

4. Two straight lines are $\perp$ to a particular straight line ; Prove that they are $\parallel$.

5. $AB, CD$ are chords of equal length in a circle and the minor arcs $AB$ and $CD$ have the same direction on the perimeter. Prove as a result of question 4 that $AD \parallel$ to $BC$.

6. Two straight lines $\ell, m$ meet at the point $A$, and they are $\parallel$ respectively with two other straight lines $\ell_1$ and $m_1$, which cut one another at $B$. Prove that the positive angle between $\ell$ and $m$ is equal to the positive angle between $\ell_1$ and $m_1$.
   [Hint: consider the translation from $A$ to $B$.]

7. By using (6) prove that each angle in a parallelogram is equal its opposite angle.

8. In amy parallelogram the sum of the four interior angles is four right angles.

9. Draw any triangle $OAB$ and find the new position $OA_1B_1$ of that triangle after rotating it through 180° about $O$.

   Prove that $B_1A_1 \parallel$ to $AB$.

10. $X, Y$ are points on the sides of the isosceles triangle $ABC$ such that $XY \parallel$ to the base $BC$. Prove that $AXY$ is another isosceles $\Delta$.

11. In any equilateral triangle, prove that each angle has 60°.

12. In a given isosceles triangle each base angle is twice as great as the apex angle. Find the measures of the three angles.

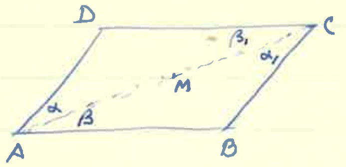
13. $P$ is a point inside the $\Delta$ $ABC$. Prove

$$\widehat{BPC} = \widehat{BAC} + \widehat{ABP} + \widehat{ACP}.$$

   [Hint: Draw the line $AP$ and examine the angles of the $\Delta$s $BAP$ and $CAP$.]

## Teoirim XIV

Nuair a h-aistrítear plána, is comhfhada, paralléalach na dronlínte go léir a cheanglaíos ionad tosaigh agus ionad deiridh gach pointe.




**Hipotéis** Cuirtear A ar B, agus cuirtear D ar C de bharr aistriúithe áirithe, ionnas go leagtar AD anuas ar ~~BC~~ an bparalléil BC.

$\therefore$ Tugtar AD = agus // le BC.

**Tátall** Tá AB = agus // le DC.

**Tógáil** Faigh M, lár na dronlíne AC.

**Cruthúnas**

Má castar an plána tré $180^\circ$ timpeall M, cuirtear A ar C agus leagtar AD fan na dronlíne CB (teoirim XI).

Ó tharla AD = BC, is anuas ar B a thuiteas D, agus mar an gceanna tuiteann B ar D.

Fágann sin go gcuirtear AB ar CD, ionnas go bhfuil siad comhfhada.

De bhrí go leagtar $\hat{\beta}$ ar an uillinn ailtéarnaigh $\hat{\beta}_1$, tá AB // le DC (teoirim XII)

$\therefore$ Tá AB = agus // le DC.

Q.E.D.

**Atora 1** Paralléalogram isea ceathairshleasán ar bith gur comhfhada, paralléalach dhá shlios de atá ar aghaidh a chéile ann.

**Atora 2**

Má tá dhá dhronlíne (mar AB is DC thuas) comhfhada, paralléalach, 'sé an t-aistriú céanna a bhreacas siad.

Tosach leathanach 43 sa LSS.

## 4.7 Teoirim XIII

*Nuair a h-aistrítear plána, is cómhfhada parallélach na dronlínte go léir a cheanglaíos ionad tosaigh agus ionad deiridh gach pointe.*

Tá Fíoghair anseo sa LSS, leathanach 43.

*Hipotéis:*

Cuirtear $A$ ar $B$, agus cuirtear $C$ ar $D$ de bharr aistrithe áirithe, ionas go leagtar $AD$ anuas ar bparalléil $BC$.

.i. Tugtar $AD =$ agus $\parallel$ le $BC$.

*Tátall:*

Tá $AB =$ agus $\parallel$ le $DC$.

*Tógáil:*

Faigh $M$, lár na dronlíne $AC$.

*Cruthúnas:*

Má castar an plána tré 180° timpeall $M$, cuirtear $A$ ar $C$ agus leagtar $AD$ fan na dronlíne $CB$ (Teoirim XI).

Ó thárla $AD = BC$, is anuas ar $B$ a thuiteas $D$, agus mar an scéanna tuiteann $B$ ar $D$.

Fágann sin go gcuirtear $AB$ ar $CD$, ionas go bhfuil siad cómhfhada.

De bhrí go leagtar $\hat{\beta}$ ar an uillinn altéarnaigh $\hat{\beta}_1$, tá $AB \parallel$ le $DC$ (Teoirim XII).

.i. Tá $AB =$ agus $\parallel$ le $DC$. □

**Theorem 13.** *When a plane is translated, the straight lines that join the initial and final positions of each point are all parallel.*

*Hypothesis:*

$A$ is placed on $B$, and $C$ on $D$ by a certain translation, so that $AD$ is laid down on a parallel $BC$.

.i. We are given that $AD =$ and $\parallel$ to $BC$.

*Conclusion:*

$AB =$ and $\parallel$ to $DC$.

*Construction:*

Find $M$, the centre of the straight line $AC$.

*Proof:*

If the plane is rotated through 180° around $M$, then $A$ is placed on $C$ and $AD$ is laid along the straight line $CB$ (Theorem 11).

Since $AD = BC$, $B$ is where $D$ goes, and similarly $B$ goes to $D$.

Thus $AB$ is placed on $CD$, so that they are the same length.

Since $\hat{\beta}$ is laid on the alternate angle $\hat{\beta}_1$, we have $AB \parallel$ to $DC$ (Theorem 12).

.i. $AB =$ and $\parallel$ to $DC$. □

## Atora 1

*Parallélogram is ea ceathairshleasán ar bith gur cómhfhada, parallélach dhá shlios de atá ar aghaidh a chéile ann.*

## Atora 2

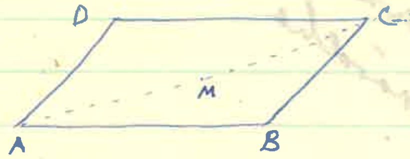
*Má tá dhá dhronlíne (mar AB is DC thuas) cómhfhada, parallélach, 'sé an t-aistriú céanna a bhreacas siad.*

**Corollary 1.** *Any quadrilateral having two opposite sides of equal length and parallel is a parallelogram.*

**Corollary 2.** *If two straight lines (like AB and DC above) are the same length and parallel, then they determine the same translation.*

Teoirim XIV (Suiméitreacht an Pharailléilograim)

Is air féin ar ais a leagtar parailléilogram de bharr an plána a chasadh tré 180° timpeall phointe áirithe




Hipotéis Parailléilogram isea ABCD .i. tá AB// le DC, agus tá AD// le BC.

Tógáil Faigh M, lár an treasnáin AC.

Tátall Is air féin ar ais a leagtar ABCD de bharr chasta tré 180° timpeall M.

Cruthúnas

Treasnaí ag an dá dhronlíne // AB, DC isea AC, agus de réir teoirme XI cuirtear A ar C agus leagtar AB fan CD, de bharr chasta tré 180° timpeall M.

Treasnaí ag an dá dhronlíne // AD, BC isea AC freisin, agus mar an gcéanna cuirtear CB fan AD de bharr an chasta sin.

Fágann sin go gcuirtear B, pointe teagmhála AB is CB, ar phointe theagmhála CD is AD .i. ar D.

∴ Leagtar gach rinn ar an rinn ós a chóir, agus leagtar gach slios ar an slios ar a aghaidh.

Q.E.D.

Atoradh Síolraíonn na tréithre seo go léir as suiméitreacht an pharailléilograim:—

(i) Tá AB = DC, agus tá AD = BC.

(ii) Tá $D\hat{A}B = D\hat{C}B$, agus tá $A\hat{B}C = A\hat{D}C$.

(iii) 'Sé M lár an treasnáin BD freisin, mar leagtar B ar D.

(iv) Má ghearrann dronlíne ar bith tré M dhá shlios, atá ar aghaidh a chéile, sna pointí X is Y, tá MX = MY.

[mar cuirtear MX ar MY de bharr an chasta tré 180°]

(v) Triantáin congrúacha isea ABC agus CDA.

Tosach leathanach 44 sa LSS.

## 4.8 Teoirim XIV (Suiméitreacht an Pharallelogram)

*Is air féin ar ais a leagtar parallélogram de bharr an plána a chasadh tré* $180^\circ$ *timpeall phointe áirithe.*

Tá Fíoghair anseo sa LSS, leathanach 44.

*Hipotéis:*
Parallélogram is ea $ABCD$ .i. tá $AB\|$ le $DC$, agus tá $AD\|$ le $BC$.

*Tógáil:*
Faigh $M$, lár an treasnáin $AC$.

*Tátall:*
Is air féin ar ais a leagtar $ABCD$ de bharr chasta tré $180^\circ$ timpeall $M$.

*Cruthúnas:*
Treasnaí ag an dá dronlíne $\|$ $AD, DC$ is ea $AC$, agus de réir Teoirme XI cuirtear $A$ ar $C$ agus leagtar $AB$ fan $CD$, de bharr chasta tré $180^\circ$ timpeall $M$.

Treasnaí ag an dá dhronlíne $\|$ $AD, BC$ is ea $AC$ freisin, agus mar an gcéanna cuirtear $CB$ fan $AD$ de bharr an chasta sin.

Fágann sin go gcuirtear $B$, pointe teagmhála $AB$ is $CB$, ar phointe teagmhála $CD$ is $AD$, .i. ar $D$.

∴ Leagter gach rinn ar an rinn ós a chóir, agus leagtar gach slios ar an slios ar a aghaidh. □

**Theorem 14** (The Symmetry of the Parallelogram)**.** *A parallelogram is laid on itself by a rotation of the plane about a certain point.*

*Hypothesis:*
$ABCD$ is a parallelogram .i. $AB\|$ to $DC$, and $AD\|$ le $BC$.

*Construction:*
Find $M$, the centre of the diagonal $AC$.

*Conclusion:*
$ABCD$ is laid on itself again by a rotation through $180^\circ$ about $M$.

*Proof:*
$AC$ is a transversal of the two $\|$ straight lines $AD, DC$, and by Theorem 11 $A$ is placed on $C$ and $AB$ along $CD$, by a rotation through $180^\circ$ about $M$.

$AC$ is also a transversal of the $\|$ straight line $AD, BC$, and in the same way $CB$ is placed along $AD$ by that rotation.

It follows that $B$, the point where $AB$ meets $CB$, is placed on the point where $CD$ meets $AD$, .i. on $D$.

∴ Each vertex is laid on the opposite vertex to it, and each side is laid on its opposite side. □

## Atora

*Síolraíonn na tréithre seo go léir as suiméitreacht an pharallélograim:—*
*(i) Tá $AB = DC$, agus $AD = BC$.*
*(ii) Tá $\widehat{DAB} = \widehat{DCB}$, agus tá $\widehat{ABC} = \widehat{ADC}$.*
*(iii) 'Sé $M$ lár an treasnáin $BD$ freisin, mar leagtar $B$ ar $D$.*
*(iv) Má ghearrann dronlíne ar bith tré $M$ dhá shlios, atá ar aghaidh a chéile, sna pointí $X$ is $Y$, tá $MX = MY$.*
*(v) Triantáin congrúacha is ea $ABC$ agus $CDA$.*

**Corollary.** *All the following properties follow from the symmetry of the parallelogram:—*
*(i) $AB = DC$, and $AD = BC$.*
*(ii) $\widehat{DAB} = \widehat{DCB}$, and $\widehat{ABC} = \widehat{ADC}$.*
*(iii) $M$ is also the centre of the diagonal $BD$ as well, because $B$ is laid on $D$.*
*(iv) If any straight line at all through $M$ cuts two opposite sides in the points $X$ and $Y$, then $MX = MY$.*
*(v) The triangles $ABC$ and $CDA$ are conguent.*

taispeántar

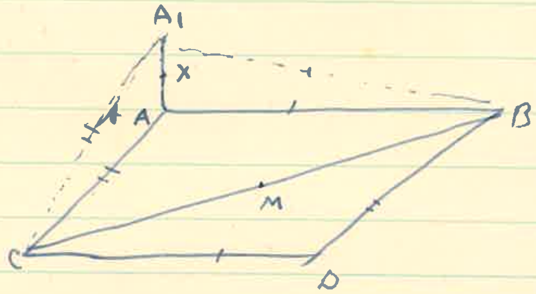
**Teorma** Chun fírinne teoirime a chruthú, is furasta amanta a thaspáint nach féidir í a shéanadh. 'Sé sin, teaspáintear go bhfuil contrárdha na teoirime bréagach agus fágann sin go gcaithfidh an teoirim féin a bheith fíor.

Cruthúnas neadíreach a tugtar ar a leithéid sin de chruthúnas

os cionn (...) in áit eile

## Teoirim XV

Má's comhfhada i gceathairshleasán na sleasa atá ar aghaidh a chéile, is parallélogram an ceathairshleasán




**Hipotéis** Sa 4-sl. ABCD tá AB = CD ~~DA = CB~~, agus tá AC = BD

**Tátal** Parallélogram isea ABCD

**Cruthúnas** Cas an Δ CBD trí 180° timpeall lár CB.

[Taispeántar] Teaspáintear go dtuiteann D ar A ~~toir~~ toisc nach féidir leis dul in áit ar bith eile.

Má thuiteann D ar $A_1$, beidh CD = $BA_1$ agus DB = $A_1C$

Ach tugtar AB = CD agus AC = DB.

∴ Tá AB = $BA_1$, agus AC = $A_1C$ ionnus gur dhá thriantán chomhchosacha iad $CAA_1$ agus $BAA_1$ ar an mbonn $AA_1$

Má's pointí difriúla iad A agus $A_1$, ~~agus má 'sé X lár $AA_1$~~, fágann sin go gcomhroinneann BC an líne $AA_1$ go h-ingearach (Teoirim IV)

Ach ní féidir é sin mar tá $A_1A$ ar an taobh chéanna de BC.

∴ Ní féidir gur ~~Fágann sin nach~~ pointí difriúla iad A agus $A_1$, ionnas go gcuirtear an Δ CDB anuas ar Δ BAC de bharr a chasta trí 180° timpeall M.

∴ parallélogram isea ABCD (teoirim XIV)

Tosach leathanach 45 sa LSS.

### Téarma

Chun fírinne teoirme a chruthú , is furasta amanta a theaspáint nach féidir í a shéanadh. 'Sé sin, *teaspáintear go bhfuil contrárdha na teoirme bréagach* agus fágann sin go gcaithfidh an teoirim féin a beith fíor.

*Crutúnas neadíreach* a tugtar ar a leitéid sin de chruthúnas.

### Definition

To prove that a theorem is true, it is sometimes easier to prove that it cannot be denied. That is, you *prove that the contrary of the theorem is false* and that implies that the theorem has to be true.

*Reductio ad absurdum* or *proof by contradiction* is the term used for that kind of proof.

## 4.9 Teoirim XV

[3] *Má's cómhfhada i gceathairshleasán na sleasa atá ar aghaidh a chéile, is parallélogram an ceathairshleasán.*

Tá Fíoghair anseo sa LSS, leathanach 45.

*Hipotéis:*

Sa 4-sh $ABCD$ tá $AB = CD$ agus tá $AC = BD$.

*Tátall:*

Parallélogram is ea $ABCD$.

*Cruthúnas:*

Cas an $\Delta CBD$ tré 180° timpeall lár $CB$.

Teaspáinfear go dtuiteann $D$ ar $A$ toisc nach féidir leis dul in áit ar bith eile.

Má thuiteann $D$ ar $A_1$, beidh $CD = BA_1$ agus $DB = A_1C$. Ach tugtar $AB = CD$ agus $AC = DB$.

∴ Tá $AB = BA_1$ agus $AC = A_1C$ ionas gur dhá thriantáin chómhchosacha iad $CAA_1$ agus $BAA_1$ ar an mbonn $AA_1$.

Má's pointí difriúla iad $A$ agus $A_1$, fágann sin go gcomhroinneann $BC$ an líne $AA_1$, go h-ingearach (Teoirim IV).

Ach ní féidir é sinmar tá $A, A_1$ ar an taobh céanna de $BC$.

.i. Ní féidir gur pointí difriúla iad $A$ agus $A_1$, ionas go gcuirtear an $\Delta CDB$ anuas ar $\Delta BAC$ de bharr a chasta tré 180° timpeall $M$.

∴ Parallélogram is ea $ABCD$ (Teoirim XIV).

[3] Ós cóir ??? in áteacha eile

**Theorem 15.** *If the opposite sides of a quadrilateral are the same length, then the quadrilateral is a parallelogram.*

*Hypothesis:*
In the quadrilateral $ABCD$ we have $AB = CD$ and $AC = BD$.

*Conclusion:*
$ABCD$ is a $\varparallelogram$.

*Proof:*
Rotate the $\Delta CBD$ through 180° about the centre of $CB$.
We will show that $D$ lands on $A$ because it could not go anywhere else.
If $D$ were to land on $A_1$, then $CD = BA_1$ and $DB = A_1C$. But we are given that $AB = CD$ and $AC = DB$.
$\therefore$ $AB = BA_1$ and $AC = A_1C$ so that $CAA_1$ and $BAA_1$ are two isosceles triangles on the base $AA_1$.
If $A$ and $A_1$ are different points, it follows that $BC$ bisects the line $AA_1$, at right angles (Theorem 4).
But that cannot occur, because $A, A_1$ are on the same side of $BC$.
.i. It cannt happen that $A$ and $A_1$ are different points, and so the $\Delta CDB$ is placed on $\Delta BAC$ by the rotation through180° about $M$.
$\therefore$ $ABCD$ is a parallelogram (Theorem 14).

## Cleachtaithe

1) Má tá uille pharallélograim ina dronuillinn, cruthuigh gur dronuilleacha iad na h-uilleacha uilig.

Tugtar dronuilleóg ar an gcineál parallélograim sin.

2) Má tá na sleasa go léir i bparallélogram cómhfhada le chéile, deimhnigh gur aisí suiméitreachta iad an dá threasnáin, agus go bhfuil na treasnáin ingearach le chéile.
[Rhombus isea an ▱ sa gcás sin. Má's dronuilleóg agus rhombus é an ▱, tugtar cearnóg air]

3) Ó dhá phointe P, Q ar dhronlíne *l* tarraingtear ingir PX agus QY ar dhronlíne pharallélaigh *m*. Cruthuigh (1) go bhfuil PX = QY; (11) má's pointí ar bith iad L, M ar *l* agus *m*, go bhfuil LM níos faide ná PX, mara bhfuil LM ⊥ le *m*.

4) Sa gceathairshleasán ABCD cómhroinneann na treasnáin a chéile. Cruthuigh gur ▱ é ABCD.

stet

5) I gceathairshleasán ABCD is ~~cómhfhada na sleasa~~ cómhméid dona h-uilleacha atá ar aghaidh a chéile. Cruthuigh gur ▱ é ABCD.
[Lide: ~~línigh $AD_1$ slath an △ ADC in aois suiméitreachta an treasnán AC. Cruthuigh ∠ACD = ∠$CAD_1$ = ∠CAB~~ Cuimhnighe gur ionann suim na gceithre n-uilleann agus ceithre dronuilleacha.]

6) Is cómhfhada na h-ingir ó phointe P ar dhá dhronlíne // *l* agus *m*. Cruthuigh go gcómhroinneann P treasnaí ar bith tré P.

7) Sa ▱ ABCD siad AX, CY na h-ingir ar an treasnán BD ó A agus C. Cruthuigh AX = CY.

8) (1) I dtriantán dronuilleach teasbáin gur uilleacha cómhlíontacha iad an dá uillinn eile,

(11) I dtriantán chómhchosach áirithe is ionann leath na stuac-uilleann agus trian de gach bhonnuillinn. Áireamh na h-uilleacha go léir, agus línigh triantán den chineál sin le compás agus riail.

9) I ngach rhombus cómhroinneann na treasnáin na h-uilleacha a ngabhann siad tríothu.

Tosach leathanach 46 sa LSS.

## Cleachtaithe

1. Má tá uille pharallélograim ina dronuillinn, cruthuigh gur dronuilleacha iad na h-uilleacha uilig.

   Tugtar *dronuille*óg ar an gcineál parallélograim sin.

2. Má tá na sleasa go léir i bparallélogram cómhfhada le chéile deimhnigh gur aisí suiméitreachta iad an dá threasnáin agus go bhfuil na treasnáin ingearach le chéile.

   [ *Rhombus* is ea an ▱ sa gcás sin. Má's dronuilleog agus rhombus an ▱, tugtar *cearn*óg air. ]

3. Ó dhá phointe $P, Q$ ar dhronlíne $\ell$ tarraingítear ingir $PX$ agus $QY$ ar dronlíne pharallélaigh $m$. Cruthaigh (i) go bhfuil $PX = QY$, (ii) má's pointí ar bith iad $L, M$ ar $\ell$ agus $m$, go bhfuil $LM$ níos faide ná $PX$, mura bhfuil $LM \perp$ le $m$.

4. Sa gceathairshleasán $ABCD$ cómhroinneann na treasnáin a chéile. Cruthaigh gur ▱ é $ABCD$.

5. I gceathairshleasán $ABCD$ is cómhéid do na h-uilleacha atá ar aghaidh a chéile. Cruthaigh gur ▱ é $ABCD$.

   [Lide: Cuimhnigh gurb ionann suim na gceithre n-uilleann agus ceithre dronuilleacha.]

6. Is cómhfhada na h-ingir ó phointe $P$ ar dhá dhronlíne $\parallel$ le $\ell$ agus $m$. Cruthaigh go gcómhroinneann $P$ treasnaí ar bith tré $P$.

7. Sa ▱ $ABCD$ siad $AX, CY$ na h-ingir ar an treasnán $BD$ ó $A$ agus $C$. Cruthaigh $AX = CY$.

8. (i) I dtriantán dronuilleach teaspáin gur uilleacha cómhlíontacha iad an dá uillinn eile.
   (ii) I dtriantácómhchosach áirithe is ionann leath na stuac-uilleann agus trian de gach bhonnuillinn. Áireamh na h-uilleacha go léir, agus línigh triantán den chineál sin le compás agus riail.

9. I ngach rhombus cómhroinneann na treasnáin na h-uilleacha a ngabhann siad triothu.

10) I gciorcail chothroma (a) gearrann stillreacha comhionanna ag na láir stuaghanna comhfhada ar na h-imlínte, agus a choinbhéarsa sin, (b) gabhann stuaghanna comhfhada de na h-imlínte uilleacha cothroma ag na láir.

[Leide: an t-aistriú a leagas lár ciorcail acu ar lár an chiorcail chothruim eile a bhreithniú, agus teoirim III a úsáid]

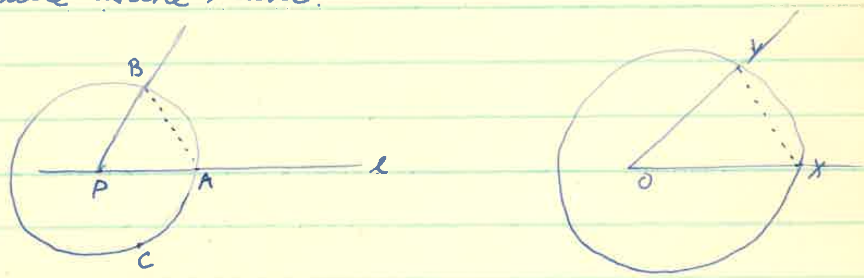
11. Más comhfhada trasnáin pharallelograim, taispeáin gur aistí suimeitreachta ann líne ceangail lár na slios atá ar aghaidh a chéile; agus dá bhrí sin gur dronuilleog í an ▭.

# — #

Baintear leas as cheist 10 thuas chun na ceisteanna seo a réiteach le compás agus riail.

Ceist I

Ag pointe P ar dhronlíne áirithe l, tarraing an dronlíne a ghníos uille áirithe léithi.




Abair gurb í $\angle XOY$ an uille áirithe a tugtar.

Réiteach

Tarraing ⊙ ar bith gurb é O a lár a ghearras géaga na h-uilleann in X agus Y, abair.

Tarraing ⊙ cothrom ar lár dó P agus tabhair A ar phointe den dá phointe ina ngearrann sé l. Tarraing ⊙ eile ar lár dó A gurb é fad an chórda XY a ga, a ghearras an ⊙ eile in B agus C.

'Sí PB (nó PC) an dronlíne a fheileas.

Cruthúnas

De réir na tógála, córdaí comhfhada i gciorcail chothroma iséa XY agus AB

$\therefore$ Tá $X\hat{O}Y = A\hat{P}B$.

[Mar an gcéanna tá $A\hat{P}C = X\hat{O}Y$ freisin]

Tosach leathanach 47 sa LSS.

10. I gciorcail chothroma (a) gearrann uilleacha cómhionanna ag na láir stuanna cómhfhada ar na h-imlínte, agus a choinvéarsa sin, (b) gabhann stuanna cómhfhada de na h-imlínte uilleacha cothoma ag an lár.

    [Lide: an t-aistriú a leagas lár ciorcail acu ar lár an chiorcail chothroim eile a bhreithniú , agus Teoirim III a úsáid.]

11. Má's cómhfhada treasnáin pharallélograim, teaspáin gur aisí suiméitreachta annlínte ceangail lár na slios atá ar aghaidh a chéile; agus d'á bhrí sin gur dronuilleog í an ▱ .

## Exercises

1. If one angle in a parallelogram is a right angle, prove that all the angles are right angles.

   That kind of parallelogram is called a *rectangle*.

2. If all the sides of a parallelogram have the same length prove that the diagonals are axes of symmetry and that the diagonals are perpendicular to one another.

   [The ▱ is a *rhombus* in that case. If a ▱ is a rectangle and a rhombus, it is called a *square*. ]

3. From two points $P, Q$ on a straight line $\ell$ drop perpendiculars $PX$ and $QY$ on a parallel straight line $m$. Prove (i) that $PX = QY$, (ii) if $L, M$ are any points at all on $\ell$ and $m$, then $LM$ is longer than $PX$, unless $LM \perp$ le $m$.

4. In the quadrilateral $ABCD$ the diagonals bisect one another. Prove that $ABCD$ is a ▱ .

5. In a quadrilateral $ABCD$ the opposite angles are equal. Prove that $ABCD$ is a ▱ .

   [Hint: Recall that the sum of the four angles equals four right angles. ]

6. The perpendiculars from a point $P$ on two $\parallel$ le straight lines $\ell$ and $m$ are the same length. Prove that $P$ bisects each transversal through $P$.

7. In the ▱ $ABCD$, the lines $AX and CY$ are the perpendiculars on the diagonal$BD$ from $A$ and $C$. Prove that $AX = CY$.

8. (i) In a right angle triangle show that the other two angles are complementary.
   (ii) In a particular isosceles triangle half the apex angle is equal to a third of each base angle. Calculate all the angles, and draw such a triangle using ruler and compass.

9. In each rhombus the diagonals bisect the angles they pass through.

10. In equal circles (a) equal angles at the centres cut arcs of equal length on the perimeters, and conversely to that, (b) arcs of equal length on the perimeters subtend equal angles at the centres.

    [Hint: consider the translation that moves the centre of one circle to the centre of the other, and use Theorem 3.]

11. If a parallelogram has diagonals of equal length, show that the lines joining the centres of opposite sides are axes of symmetry; and hence that the ▱ is a rectangle.

Baintear leas as cheist 10 thuas chun na ceisteanna seo a réiteach le compás agus riail:

## Ceist I

Ag pointe $P$ ar dhronlíne áirithe $\ell$, tarraing an dronlíne a ghníos uille áirithe leithi.

Tá Fíoghair anseo sa LSS, leathanach 47.

Abair gurb í $\angle XOY$ an uille áirithe a tugtar.

*Réiteach:*

Tarraing ⊙ ar bith gurb é $O$ a lár a ghearas géaga na h-uilleann in $X$ agus $Y$, abair.

Tarraing ⊙ cothrom ar lár dó $P$ agus tabhair $A$ ar phointe den dá phointe ina ngearrann sé $\ell$. Tarraing ⊙ eile ar lár dó $A$ gurb é fad an chórda $XY$ a ga, a ghearas an ⊙ eile in $B$ agus $C$.

'Sí $PB$ (nó $PC$) an dronlíne a fheileas.

*Cruthúnas:*

De réir na tógála, córdaí cómhfhada i gciorcail chothroma is ea $XY$ agus $AB$.

∴ Tá $\widehat{XOY} = \widehat{APB}$.

[Mar an gcéanna tá $\widehat{APC} = \widehat{XOY}$ freisin.]

You can use question 10 above to solve the following problems with ruler and compass:

## Question I

At a point $P$ on a certain straight line $\ell$, draw the straight line that makes a particular angle with it.

Suppose the given angle is $\angle XOY$.

*Solution:*

Draw any ⊙ with centre $O$ that cuts the arms of the angle at $X$ and $Y$, say.

Draw an equal ⊙ with centre $P$ and denote by $A$ one of the two points in which it cuts $\ell$. Draw another ⊙ with centre $A$ having the length of the chord $XY$ as radius, cutting the other ⊙ at $B$ and $C$.

Then $PB$ (or $PC$) is the straight line that meets the case.

*Proof:*

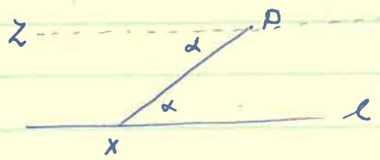
By construction, $XY$ and $AB$ are equal-length chords in a circle..

$\therefore \widehat{XOY} = \widehat{APB}$.

[Similarly $\widehat{APC} = \widehat{XOY}$ as well.]

## Ceist 2

Trí phointe áirithe P, tarraing an dronlíne a bheas parallélach le dronlíne áirithe $\underline{l}$




## Réiteach

Tóg pointe ar bith X san dronlíne $\underline{l}$ agus ceangail XP. Aimsigh (mar a rinneadh i gceist 1 é) an dronlíne trí P a ghníos an uille $\hat{\alpha}$ le PX, ach ní mór an dronlíne sin den phéire a thoghadh iontas gur uilleacha altéarnacha iad an dá uillinn chothroma.

De réir teoirme X tá ZP // le $\underline{l}$.

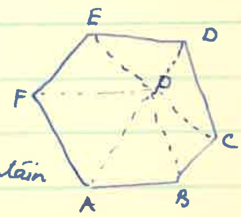
## Téarmaí

Tugtar poligón ar fhioghair iadta a gníntear le dronlínte. Is pentagón é má tá cúig sleasa ann, agus hexagón ea é má tá sé sleasa ann.

Poligón rialta a tugtar ar an bpoligón, má's cómhfhada na sleasa go léir agus má's cómhméad do na h-uilleacha go léir.

## Nóta ar uilleacha pholigóin

I bpoligón ar bith fána $\underline{n}$ cinn de shleasa, tá suim na n-uilleann istigh cothrom le $(n-2)180°$, nuair nach uille aisfhillteach í uille ar bith díobh.




Cruthúnas

Má ceangailtear reanna an pholigóin le pointe ar bith $\underline{O}$ istigh, gníntear $\underline{n}$ triantáin agus tá $180°$ i n-uilleacha gach triantáin acu.

Sin $n.180°$ i n-iomlán, ach ní mór na h-uilleacha ag O (gurb ionann le chéile iad agus $360°$) a dhealú uaidh.

$\therefore$ 'Sé $(n-2)180°$ suim na n-uilleann istigh.

Tosach leathanach 48 sa LSS.

### Ceist 2

Tré pointe áirithe $P$, tarraing an dronlíne a bheas parallélach le dronlíne áirithe $\ell$.

Tá Fíoghair anseo sa LSS, leathanach 48.

*Réiteach:*

Tóg pointe ar bith $X$ san dronlíne $\ell$ agus ceangail $XP$. Aimsigh (mar a rinneadh i gceist 1 é ) an dronlíne tré $P$ a ghníos an uille $\hat{\alpha}$ le $PX$, ach ní mór an dronlíne sin den péire a thoghadh ionas gur *uilleacha altéarnacha* iad an dá uillinn chothroma.

De réir Teoirme X tá $ZP \|$ le $\ell$.

### Téarmaí

Tugtar *poligón* ar an fhíoghair iata a gintear le dronlínte. Is *pentagón* é má tá cúig sleasa ann, agus *hexagón* má tá sé sleasa ann.

Poligón *rialta* a tugtar ar an bpoligón, má's cómhfhada na sleasa go léir agus má's cómhméad do na h-uilleacha go léir.

### Question 2

Through a given point $P$, draw the straight line that is parallel to a given line $\ell$.

*Solution:*

Select any point $X$ on the straight line $\ell$ and join $XP$. Find (as was done in Question 1) the straight line through $P$ making the angle $\hat{\alpha}$ with $PX$, but be careful to select that straight line of the pair such that the two equal angles are *alternate angles.*

By Theorem 10, we have $ZP \|$ to $\ell$.

### Definitions

A *polygon* is a closed figure make with straight lines. It is a *pentagon* if it has five sides and a *hexagon* if it has six sides.

A polygon is *regular* if all the sides have the same length and all the angles are of equal size.

### Nóta ar uilleacha pholigóin

I bpoligón ar bith fána $n$ cinn de shleasa, tá suim an n-uilleann istigh cothrom le $(n-2)180°$, nuair nach uille aisfhillteach í uille ar bith díobh.

Tá Fíoghair anseo sa LSS, leathanach 48.

*Cruthúnas:*

Má ceanglailtear reanna an pholigóin le pointe ar bith $O$ istigh, gintear $n$ triantáin agus tá 180° i n-uilleacha gach triantáin acu.

Sin $n \cdot 180°$ i n-iomlán, ach ní mór na h-uilleacha ag $O$ (gurb ionann le chéile iad agus 360°) a dhealú uaidh.

$\therefore$ 'Sé $(n-2)180°$ suim na n-uilleann istigh.

## A Note on the angles of a polygon

In any polygon having $n$ sides, the sum of the interior angles is equal to $(n-2)180°$, when none of them is a re-entrant angle.

*Proof:*

If the vertices of the polygon are joined to any point $O$ inside, then $n$ triangles are generated with 180° in the angles of each triangle.

That's $n \cdot 180°$ altogether, but we have to subtract the angles at $O$ (which together amount to 360°).

$\therefore$ $(n-2)180°$ is the sum of the interior angles.

## Cleachtaithe

1) Faigh (i) i gcóir phentagóin, agus (ii) i gcóir hexagóin, suim na n-uilleann istigh.

Má's fíoghracha rialta iad, faigh méad gach uilleann.

2) Tá dhá uillinn i dtriantán cothrom (leith ar leith) le dhá uillinn i dtriantán eile. Teaspáin gur cómhméad don dá uillinn eile freisin.

/p

3) Nuair castar dronlíne AB tré 180° timpeall pointe O amuigh, leagtar AB anuas go cruinn ar $A_1B_1$. Cruthuigh, (i) go bhfuil $B_1A_1$ // le AB, agus (ii) go bhfuil $AB_1$ = agus // le $BA_1$.

4) Sa triantán ABC 'siad L, M, lár na slios AB agus AC, agus castar an $\Delta$ ALM tré 180° timpeall M go leagtar ar an $\Delta$ CNM é.

Italics →

Cruthuigh (i) go bhfuil CN = agus // le BL ; (ii) go bhfuil LM // le BC agus = $\frac{1}{2}$BC.

5) Tré L lár an tsleasa AB sa triantán ABC, tarraingítear dronlíne atá // le BC agus is in M a ghearras sí AC.

Italics →

De bharr an $\Delta$ ALM a chasadh tré 180° timpeall L, cruthuigh go gcómhroinneann LM an slios AC freisin.

6) Pointí is ea A, B ar an taobh chéanna de dhronlíne áirithe $l$, agus 'sé X lár na dronlíne AB. Teaspáin go bhfuil an t-ingear ó X ar $l$ cothrom le leath-shuim na n-ingear ó A agus B uirthi.

7) Sa bparallélogram ABCD 'siad E, F lár na slios AD is BC. Teangmhaíonn BE agus DF leis an trasnán AC sna pointí X agus Y.

(New par.) → De bharr chasta timpeall lár AC cruthuigh go bhfuil DF // le EB.

Teaspáin freisin go bhfuil AX = XY = YC.

8) Sa gceathairshleasán ABCD, tá AB agus DC paralléalach gan a bheith cómhfhada, agus tá AD is BC cómhfhada gan a bheith paralléalach.

Cruthuigh go bhfuil $\hat{D} = \hat{C}$, agus $\hat{A} = \hat{B}$.

Tosach leathanach 49 sa LSS.

## Cleachtaithe

1. Faigh (i) i gcóir phentagóin, agus (ii) i gcóir hexagóin, suim na n-uillleann istigh.

   Má's fíogracha *rialta* iad, faigh méad gach uilleann.

2. Tá dhá uillinn i dtriantán cothrom (leith ar leith) le dhá uillinn i dtriantán eile. Teaspáin gur cómhméad den dá uillinn eile freisin.

3. Nuair castar dronlíne $AB$ tré 180° timpeall pointe $O$ amuigh, leagtar $AB$ anuas go cruinn ar $A_1B_1$. Cruthaigh (i) go bhfuil $B_1A_1 \parallel$ le $AB$ agus (ii) go bhfuil $AB_1$ = agus $\parallel$ le $BA_1$.

4. *Sa triantán $ABC$ 'siad $L, M$ láir na slios $AB$ agus $AC$, agus castar an $\Delta ALM$ tré* 180° *timpeall $M$ go leagtar ar an $\Delta CNM$* é.

   Cruthaigh (i) go bhfuil $CN$ = agus $\parallel$ le $BL$; (ii) go bhfuil $LM \parallel$ le $BC$ agus = $\frac{1}{2}BC$.

5. *Tré $L$ lár an tsleasa $AB$ sa triantán $ABC$, tarraingítear dronlíne atá* $\parallel$ le *$BC$ agus is in $M$ a ghearas sí $AC$.*

   De bharr an $\Delta ALM$ a chasadh tré 180° timpeall $L$, cruthuigh go gcómhroinneann $LM$ an slios $AC$ freisin.

6. Pointí is ea $A, B$ ar an taobh céanna de dhronlíne áirithe $\ell$, agus sé $X$ lárna dronlíne $AB$. Teaspáin go bhfuil an t-ingear ó $X$ ar $\ell$ cothrom le leath-suim na n-ingear ó $A$ agus $B$ uirthi.

7. Sa bparallélogram $ABCD$ 'siad $E, F$ láir na slios $AD$ is $BC$. Teagmhaíonn $BE$ agus $DF$ leis an treasnán $AC$ sna pointí $X$ agus $Y$.

   De bharr chasta timpeall láir $AC$ cruthuigh go bhfuil $DF \parallel$ le $EB$

   Teaspáin freisin go bhfuil $AX = XY = YC$.

8. Sa gceathairshleasán $ABCD$, tá $AB$ agus $DC$ parallélach, gan a bheith cómhfhada, agus tá $AD$ is $BC$ cómhfhada, gan a bheith parallélach.

   Cruthaigh go bhfuil $\hat{D} = \hat{C}$, agus $\hat{A} = \hat{B}$.

9) Nuair sìntear slios triantáin, má tá comhroinnteóir na h-uilleann amuigh parallélach leis an slios atá os cóir na h-uilleann, teaspáin gur triantán comhchosach é.

10) Sa Δ ABC teangmhaíonn comhroinnteóir na h-uilleann A leis an mbonn BC in X, agus pointí ar na sleasa AB is AC isea Y agus Z ionnus gur ▱ é AYXZ.

Bruthuigh (i) gur rhombus é AYXZ, agus (ii) go bhfuil ZY // le BC.

~~Is é M~~

11) Sa Δ ABC lár an t-sleasa BC isea M, agus 'be' an cineál triantáin é ABC go bhfuil MB = MA = MC.

Bruthuigh gur triantán dronuilleach é ABC.

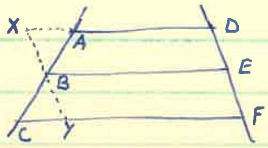
12) Pointe isea P idir dhá dhronlíne theangmhálacha l, m. Mínigh cé'n chaoi ina dtarraingítear dronlíne tré P a ghearras l, m sna pointí X agus Y, ionnus go mbeidh PX = PY.

[Lide: an fhioghair a chasadh tré 180° timpeall P].

## Teoirmí Breise

### Teoirim A

Má ghearrann trí dronlínte parallélacha (nó tuille) míreanna comhfhada ar threasnaí amháin, is míreanna comhfhada a ghearras siad ar gach treasnaí eile freisin.




**Hipotéis** Tá AD, BE, CF // le chéile, agus tá AB = BC

**Tátal** Tá DE = EF.

Tosach leathanach 50 sa LSS.

9. Nuair a síntear slios triantáin má tá cómhroinnteoir na huilleann amuigh parallélach leis an slios atá óscóir na huilleann, teaspáin gur triantán cómhchosach é.

10. Sa $\Delta ABC$ teagmhaíonn cómhroinnteóir na huilleann $A$ leis an mbonn $BC$ in $X$, agus pointí ar na sleasa $AB$ is $AC$ is ea $Y$ agus $X$ ionnus gur ▱ é $AYXZ$.

    Cruthaigh (i) gur rhombus é $AYXZ$, agus (ii) go bhfuil $ZY \parallel$ le $BC$.

11. Sa $\Delta ABC$ lár an t-sleasa $BC$ is ea $M$ agus 'sé an cineál triantáin é $ABC$ go bhfuil $MB = MA = MC$.

    Cruthaigh gur triantán dronuilleach é $ABC$.

12. Pointe is ea $P$ idir dhá dhronlíne theagmhálacha $\ell, m$. Mínigh cé'n chaoi ina dtarraingítear dronlíne tré $P$ a ghearas $\ell, m$ sna pointí $X$ agus $Y$, ionnus go mbeidh $PX = PY$.

    [Lide: an fhioghair a chasadh tré 180° timpeall $P$.]

## Exercises

1. Find (i) for a pentagon, and (ii) for a hexagon, the sum of the interior angles.

   If they are *regular* figures, find the size of each angle.

2. Two angles in a triangle are pairwise equal to two angles in another triangle. Show that the two other angles also have the same size.

3. When a straight line $AB$ is rotated through 180° about a point $O$ outside it, $AB$ ends up exactly on $A_1B_1$. Prove (i) $B_1A_1 \parallel$ to $AB$ and (ii) $AB_1$ = and $\parallel$ to $BA_1$.

4. *In the triangle $ABC$ the points $L, M$ are the centres of the sides $AB$ and $AC$, and the $\Delta ALM$ is rotated through* 180° *around $M$ and ends up on the $\Delta CNM$.*

   Prove (i) that $CN$ = and $\parallel$ to $BL$; (ii) that $LM \parallel$ to $BC$ and $= \frac{1}{2}BC$.

5. *Through $L$, the centre of the side $AB$ in the triangle $ABC$, a straight line is drawn that is* $\parallel$ to *$BC$ and cuts $AC$ at $M$.*

   By rotating the $\Delta ALM$ through 180° about $L$, prove that $LM$ bisects the side $AC$ as well.

6. $A, B$ are points on the same side of a certain straight line $\ell$, and $X$ is the centre of the straight line $AB$. Show that the perpendicular from $X$ on $\ell$ is equal to half the sum of the perpendiculars from $A$ and $B$ on it.

7. In the parallelogram $ABCD$, $E$ and $F$ are the centres of the sides $AD$ and $BC$. $BE$ and $DF$ imeet the diagonal $AC$ at the points $X$ and $Y$.

   By using a rotation about the centre of $AC$ prove that $DF\|$ to $EB$

   Show also that $AX = XY = YC$.

8. In the quadrilateral $ABCD$, $AB$ and $DC$ are parallel, but are not the same length, and $AD$ and $BC$ are the same length, but are not parallel.

   Prove that $\hat{D} = \hat{C}$, and $\hat{A} = \hat{B}$.

9. If when a side of a triangle is extended the bisector of the outside angle is parallel to the side opposite the angle, show that the triangle is isosceles.

10. In the $\Delta ABC$ the bisector of the angle $A$ meets the base $BC$ at $X$, and $Y$ and $X$ are points on the sides $AB$ and $AC$ such that $AYXZ$ is a ▱.

    Prove (i) that $AYXZ$ is a rhombus, and (ii) that $ZY\|$ to $BC$.

11. In the $\Delta ABC$ the centre of the side $BC$ is $M$ and $ABC$ is a triangle such that $MB = MA = MC$.

    Prove that $ABC$ is a right-angle triangle.

12. $P$ is a point between intersecting straight lines $\ell, m$. Explain how you would draw a straight line through $P$ that cuts $\ell, m$ in the points $X$ and $Y$, such that $PX = PY$.

    [Hint: rotate the figure through 180° around $P$.]

## 4.10 Teoirmí Breise

## 4.11 Teoirim A

*Má ghearrann trí dronlínte parallélacha (nó tuille) míreanna cómhfhada ar threasnaí amháin, is míreanna cómhfhada a ghearas siad ar gach treasnaí eile freisin.*

Tá Fíoghair anseo sa LSS, leathanach 50.

*Hipotéis:*
Tá $AD, BE, CF\|$ le chéile, agus tá $AB = BC$.

*Tátall:*
Tá $DE = EF$.

<u>Tógáil</u> Tarraing trí B an líne XBY atá // le DF.

<u>Cruthúnas</u>

Má castar an $\triangle$ BAX trí 180° timpeall B, cuirtear BA ar BC agus leagtar AX ~~fan~~ ar an bparalléil CY.

Ach is fan na líne BY a cuirtear BX.

∴ Tuiteann X ar Y, agus tá BX = BY.

Ach tá BX = ED de bhrí gur ▱ é XDEB ; mar an gceánna tá BY = EF.

Fágann sin go bhfuil DE = EF.

Q.E.D.

<u>Nóta 1</u> Is cuma cé mhéad línte // a tarraingítear, is féidir an teoirim a thagairt do gach trí cinn leantacha díobh.

<u>Nóta 2</u>

De thairbhe na teoirime seo, is féidir dronlíne chuimseach AB a roinnt in $\underline{n}$ cómhchoda mar seo a leanas.

Tarraing dronlíne eile trí A agus marcáil uirthi $\underline{n}$ línte cómhfhada leantacha, $AX_1, X_1X_2, X_2X_3, \ldots X_{n-1}X_n$.

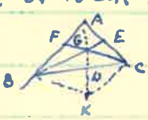
Ceangail $X_nB$ agus trí $X_1, X_2, \ldots X_{n-1}$ tarraing dronlínte atá // le $X_nB$. Má's in $Y_1, Y_2, \ldots Y_{n-1}$ a ghearras siad AB, tá

$AY_1 = Y_1Y_2 = Y_2Y_3 = \ldots\ Y_{n-1}B$.

<u>Téarma</u> Tugtar meánlíne thriantáin ar dhronlíne a cheanglaíos rinn ar bith le lár an t-sleasa ós a chóir.

<u>Teoirim B</u>

I dtriantán ar bith línte cómhreathacha ~~is~~ ea na meánlínte.



<u>Hipotéis</u> Meánlínte isea BE agus CF a ghearras a chéile in G.

<u>Tátall</u> 'Sé AG an mheánlíne eile.

<u>Tógáil</u> Sín AG go dtí K ~~ionnus~~ ar go mbeidh AG = GK. Ceangail CK, BK.

<u>Cruthúnas</u>

Sa triantán AKC láir na slios AC is AK isea F, G. ∴ Tá GE // le KC.

Mar an gceánna sa $\triangle$ ABK, tá FG // le BK.

Fágann sin gur ▱ é BGCK, agus ó suiméitreacht an ▱ tá DB = DC.

∴ Meánlíne i~~s~~ea AGD.

Q.E.D.

<u>Atora</u> Ó tharla AG = GK, GD = DK, tá AG = 2GD.
Mar an gceánna tá BG = 2GE, CG = 2GF.
Tugtar meánlár an triantáin ar an bpointe G.

Tosach leathanach 51 sa LSS.

*Tógáil:*
Tarraing tré $B$ an líne $XBY$ atá $\parallel$ le $DF$.

*Cruthúnas:*
Má castar an $\Delta BAX$ tré $180^\circ$ timpeall $B$, cuirtear $BA$ ar $BC$ agus leagfar $AX$ ar an bparalléil $CY$.
Ach is fan na líne $BY$ a cuirtear $BX$.
$\therefore$ Tuiteann $X$ ar $Y$, agus tá $BX = BY$.
Ach tá $BX = ED$ de bhrí gur ▱ é $XDEB$; mar an gcéanna tá $BY = EF$.
Fágann sin go bhfuil $DE = EF$. □

## 4.12 Extra Theorems

**Theorem A.** *If three (or more) parallel lines cut equal sections on any transversal, then they cut equal sections on any other transversal as well.*

*Hypothesis:*
$AD, BE, CF$ are $\parallel$ to one another, and $AB = BC$.

*Conclusion:*
$DE = EF$.

*Construction:*
Through $B$ draw the line $XBY \parallel$ to $DF$.

*Proof:*
If the $\Delta BAX$ is rotated through $180^\circ$ around $B$, $BA$ is laid on $BC$ and $AX$ is placed on the parallel $CY$.
But $BX$ is placed along the line $BY$.
$\therefore$ $X$ moves to $Y$, and $BX = BY$.
But $BX = ED$ since $XDEB$ is a ▱; Similarly, $BY = EF$.
It follows that $DE = EF$. □

**Tógáil** Tarraing tré B an líne XBY atá // le DF.

**Cruthúnas**

Má castar an $\Delta$ BAX tré $180°$ timpeall B, cuirtear BA ar BC agus leagtar AX ~~fan~~ ar an bparalléil CY.

Ach is fan na líne BY a cuirtear BX.

$\therefore$ Tuiteann X ar Y, agus tá BX = BY.

Ach tá BX = ED de bhrí gur $\square$ é XDEB ; mar an gceánna tá BY = EF.

Fágann sin go bhfuil DE = EF.

Q.E.D.

**Nóta 1** Is cuma cé mhéad línte // a tarraingítear, is féidir an teoirim a thagairt do gach tré cinn leantacha díobh.

**Nóta 2**

De thairbhe na teoirme seo, is féidir dronlíne chuimseach AB a roinnt in $\underline{n}$ comhchoda mar seo a leanas.

Tarraing dronlíne eile tré A agus marcáil uirthi $\underline{n}$ línte comhfhada leantacha, $AX_1$, $X_1X_2$, $X_2X_3$, ... $X_{n-1}X_n$.

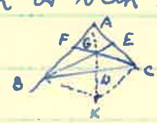
Ceangail $X_nB$ agus tré $X_1, X_2, .. X_{n-1}$ tarraing dronlínte atá // le $X_nB$. Má's in $Y_1, Y_2, .. Y_{n-1}$ a ghearras siad AB, tá

$AY_1 = Y_1Y_2 = Y_2Y_3 = ... Y_{n-1}B$.

**Téarma** Tugtar meánlíne thriantáin ar dhronlíne a cheanglaíos rinn ar bith le lár an t-sleasa ós a chóir.

**Teoirim B**

I dtriantán ar bith línte comhreathacha ~~isea~~ na meánlínte.




**Hipotésis** Meánlínte isea BE agus CF a ghearras a chéile in G.

**Tátall** 'Sé AG an mheánlíne eile.

**Tógáil** Sín AG go dtí K ~~ionnus~~ ar gu mbeidh AG = GK. Ceangail CK, BK.

**Cruthúnas**

Sa triantán AKC láir na slios AC is AK isea E, G. $\therefore$ Tá GE // le KC.

Mar an gceánna sa $\Delta$ ABK, tá FG // le BK.

Fágann sin gur $\square$ é BGCK, agus ó suiméitreacht an $\square$ tá DB = DC.

$\therefore$ Meanlíne ~~isea~~ AGD.

Q.E.D.

**Atora** Ó tharla AG = GK, GD = DK, tá AG = 2GD.
Mar an gceánna tá BG = 2GE, CG = 2GF.
Tugtar meánlár an triantáin ... an bpointe G.

Tosach leathanach 51 sa LSS.

### Nóta 1

is cuma cé mhéad línte ∥ a tarraingítear, is féidir an teoirim a thagairt do gach trí cinn leantacha díobh.

### Nóta 2

De thairbha na teoirme seo, is féidir dronlíne chuimseach $AB$ a roinnt in $n$ cómhchorda mar seo a leanas.

Tarraing dronlíne eile tré $A$ agus marcáil uirthi $n$ cómhfhada leantacha $AX_1, X_1X-2, X_2X_3, \ldots, X_{n-1}X_n$.

Ceangail $X_nB$ agus tré $X_1, X_2, \ldots, X_{n-1}$ tarraing dronlínte atá ∥ le $X_nB$. Má's in $Y_1, Y_2, \ldots, Y_{n-1}$ a ghearas siad $AB$, tá

$$AY_1 = Y_1Y_2 = Y_2Y_3 = \cdots Y_{n-1}B.$$

### Téarma

Tugtar *meánlíne* thriantáin ar dhronlíne a cheanglaíos rinn ar bith le lár an t-sleasa ós a chóir.

### Note 1

It doesn't matter how many ∥ lines are drawn, the theorem may be applied to any three consecutive lines among them.

### Note 2

As a result of this theorem, you can divide an arbitarary straight line $AB$ into $n$ equal pieces as follows:

Draw another straight line through $A$ and mark on it $n$ consecutive pieces $AX_1, X_1X-2, X_2X_3, \ldots, X_{n-1}X_n$.

Join $X_nB$ and through $X_1, X_2, \ldots, X_{n-1}$ draw straight lines that are ∥ to $X_nB$. If they cut $AB$ at $Y_1, Y_2, \ldots, Y_{n-1}$, then

$$AY_1 = Y_1Y_2 = Y_2Y_3 = \cdots Y_{n-1}B.$$

### Definition

A *median* of a triangle is a straight line that joins any one of the vertices to the middle of the side opposite to it.

## 4.13 Teoirim B

*I dtriantán ar bith línte cómhreathacha is ea na meáblínte.*

Tá Fíoghair anseo sa LSS, leathanach 51.

*Hipotéis:*
Meánlínte is ea $BE$ agus $CF$ a ghearas a chéile in $G$.

*Tátall:*
'Sé $AG$ an meánlíne eile.

*Tógáil:*
Sín $AG$ go dtí $K$ ionas go mbeidh $AG = GK$. Ceangail $CK, BK$.

*Cruthúnas:*
Sa triantán $AKC$ láir na slios $AC$ is $AK$ is ea $F, G$. ∴ Tá $GE\|$ le $KC$.
Mar an gcéanna sa $\Delta ABK$, tá $FG\|$ le $BK$.
Fágann sin gur ▱ é $BGCK$, agus ó suiméitreacht an ▱ tá $DB = DC$. .i. Meánlíne is ea $AGD$. □

**Theorem B.** *In any triangle at all the median lines are concurrent.*

*Hypothesis:*
$BE$ and $CF$ are medians that cut each other at $G$.

*Conclusion:*
$AG$ is the other median.

*Construction:*
Extend $AG$ to $K$ so that $AG = GK$. Join $CK, BK$.

*Proof:*
In the triangle $AKC$ the centres of the sides $AC$ and $AK$ are $F, G$. ∴ $GE\|$ to $KC$.
Similarly in the $\Delta ABK$, we have $FG\|$ to $BK$.
It follows that $BGCK$ is a ▱, and by the symmetry of the ▱ we have $DB = DC$. .i. $AGD$ is a median. □

### Atora

*Ó thárla $AG = GK$, $GD = DK$, tá $AG = 2GD$. Mar an gcéanna tá $BG = 2GE$, $CG = 2GF$.*

Tugtar *meánlár* an triantáin ar an bpointe $G$.

**Corollary.** *Since $AG = GK$ and $GD = DK$, we have $AG = 2GD$. Similarly, $BG = 2GE$, $CG = 2GF$.*

The point $G$ is called the *median* of the triangle.

# Caibidil V

## Fairsinge

Nuair a cuirtear gluaiseacht comhgrúach (.i. casadh, nó scáthú, nó aistriú) i bhfeidhm ar fhioghair phlánaigh, fanann méad agus deilbh na fioghaire sin buan; 'sé an t-ionad a hathraitear. Sa gcaibidil seo teaspáinfear gur féidir an deilbh a chlaochló gan an méad a athrú.

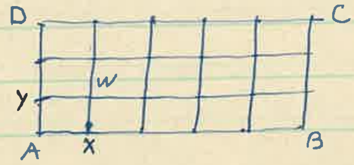
Tugtar fairsinge fhioghrach iadta ar mhéad an dromchla a thimpeallaíos a línte teorann.

Chun fairsinge a mheas ní mór aonad fairsinge a thoghadh i dtosach, agus is lena chur i gcóimhmheas leis an aonad sin a déantar fairsinge ar bith eile a thomhas.

Nuair ghnímid aonad faide d'ár rogha dronlíne (e.g. an t-órlach, an tslat, an centiméadar, etc.) cinneann sé sin aonad fairsinge san am chéanna .i. fairsinge na cearnóige a bhfuil aonad faide i ngach slios di. Ar an gcuma sin freagraíonn an t-órlach cearnach don órlach faide, agus freagraíonn an tslat chearnach don tslait, etc.

### Fairsinge Dhronuilleoige

Tóg dronuilleóg ar bith ABCD agus cuir i gcás go bhfuil cómhmhiosúr ag na sleasa AB agus AD .i. go bhfuil fad éigin ann (a toghfar mar aonad faide) gurb ionann slánuimhir $p$ díobh as a chéile agus an slios AB, agus go bhfuil slánuimhir eile $q$ díobh as a chéile cómhfhada le AD. Sa léaráid thíos tá $p=5$, $q=3$.




Roinn AB ina 5 cómhchoda, agus tarraing línte // le AD. Roinn AD ina 3 chomhchoda agus tarraing línte // le AB.

Is soiléir gur cearnóg í AXWY, agus gur féidir í a chur

# Caibidil 5

# Fairsinge

Tosach leathanach 52 sa LSS.

## Area

Nuar a cuirtear gluaiseacht chongríoch (.i. casadh, nó scáthú, nó aistriú) i bhfaidhm ar fhiogair phlánach, fanann méid agus deilbh na fioghaire sin buan; 'sé an t-ionad a hathraítear. Sa gcaibidil seo teaspáinfear gur féidir an deilbh a chlaochló gan an méad a athrú.

Tugtar *faisinge* fhioghrach iadta ar méad an drompla a thimpeallaíos a línte teorann.

Chun fairsinge a mheas ní mór aonad fairsinge a thoghadh i dtosach, agus is lena chur i gcóimhmeas leis an aonad sin a déantar fairsinge ar bith eile a thomhas.

Nuair ghnímid aonad faide dár rogha dronlíne (e.g. an t-órlach, an tslat, an ceintiméadar, etc) cinneann sé sin aonad fairsinge san am chéanna .i. fairsinge na cearnóige a bhfuil aonad faide i ngach slios di. Ar an gcuma sin freagraíonn an t-órlach cearnach don órlach faide, agus freagraíonn an tslat chearnach don tslait, etc.

When a congruent (isometric) movement (.i. a rotation, or a reflection, or a translation) is applied to a planar figure the size and shape of the figure remain constant; only its position is changed. In this chapter it will be shown that the shape can be changed without altering the size.

The *area* of a bounded figure is the name given to the size of the surface enclosed by the boundary lines.

To evaluate area we have to choose a unit of area first of all, and then by comparing with that unit we proceed to measure any other area.

When we choose a unit of length (e.g. the inch, the yard, the centimeter, etc) that determines a unit of area at the same time .i. the area of the square having one unit of length on each side. In that way the square inch corresponds to the inch of length, the square yard to the yard, etc.

anuas go cruinn (de hJaistriú) ar gach aon chearnóig de na $5 \times 3$ comhchoda ina ngeallta an dronuilleog ABCD.

Má glactar le AX mar aonad faide, ionnus gurb iad $p$ agus $q$ faid na slios AB, AD, agus má glactar leis an aonad fairsinge a fheagraíos do AX .i. an chearnóg AXWY, is follusach go bhfuil $p \times q$ aonaid fhairsinge san dronuilleóig.

∴ Tá <u>fairsinge dronuilleóige</u> = <u>fad</u> × <u>leithead</u>, nuair a fhreagraíos na h-aonaid faide agus fairsinge d'á chéile.

E.G. (1) Má tá an dronuilleog 5 ór. ar fad agus 3 ór. ar leithead, 'sé $5 \times 3 = 15$ ór. cearnacha an fhairsinge.

(2) Má 'siad $3\frac{1}{2}$ ór. agus $2\frac{3}{4}$ ór. an fhad is an leithead, ní miste $\frac{1}{4}$ ór. a thoghadh mar aonad faide i dtosach, ionnus go bhfuil 14 aonaid díobh sa bhfad agus 11 san leithead.

Fágann sin go bhfuil an fhairsinge = $14 \times 11$ de na cearnóga ar slios díobh $\frac{1}{4}$ ór.

Déanann $4 \times 4$ de na h-aonaid fhairsinge sin 1 ór. cearnach

∴ Tá fairsinge na dronuilleoige = $\frac{14 \times 11}{4 \times 4}$ = $3\frac{1}{2} \times 2\frac{3}{4}$ ór. cear.

.i. Feileann an fhormuil <u>fad</u> × <u>leithead</u> don chás.

## <u>Fairsinge Pharallélograim</u>

<u>Téarmaí</u>.

Ní miste an bonn a thabhairt ar shlios ar bith de shleasa an ▱. De bhrí go bhfuil an slios atá ós a <u>chóir</u> parallélach leis, is ~~comhfhad~~ comhfhada na h-ingir go léir a tarraingítear go dtí an bonn ó phointí an tsleasa atá ar a aghaidh. <u>Airde</u> <u>an pharallélograim</u> a tugtar ar ingear ar bith aca.

Má tógtar dhá ▱ fá'n airde chéanna ar aon bhonn amháin, is soiléir go mbeidh na sleasa atá ós cóir an bhoinn in aon dronlíne amháin, agus í ∥ leis an mbonn. Is ar an ábhar sin a tugtar ▱ <u>idir na parallélí céanna</u> ar dhá ▱ den chineál sin.

Tugtar <u>aoirde thriantáin</u> ar an ingear ó'n stuach ar an mbonn.

## 5.1 Fairsinge Dhronuilleoige

Tóg dronuilleóg ar bith $ABCD$ agus cuir i gcás go bhfuil cómhmhiosúr ag na sleasa $AB$ agus $AD$ .i. go bhfuil fad éigin ann (a toghfar mar aonad faide) gurb ionann slánuimhir $p$ díobh as a chéile agus an slios $AB$, agus go bhfuil slánuimhir eile $q$ díobh as a chéile cómhfhada le $AD$. Sa léaráid thíos tá $p = 5$, $q = 3$.

Tá Fíoghair anseo sa LSS, leathanach 52.

Roinn $AB$ ina 5 cómhchoda, agus tarraing línte ∥ le $AU$.
Roinn $AD$ ina 3 chomhchoda agus tarraing línte ∥ le $AB$.
Is soiléir gur cearnóg é $AXWY$, agus guri féidir é a chur

Tosach leathanach 53 sa LSS.

anuas go cruinn (le haistriú ) ar gach aon chearnóig de na $5 \times 3$ comhchoda ina ngeartar an dronuilleóg $ABCD$.

Má glactar le $AX$ mar aonad faide, ionas gurb iad $p$ agus $q$ faid na slios $AB, AD$, agus má glactar leis an aonad fairsinge a fhreagraíos do $AX$ .i. an chearnóg $AXWY$, is follasach go bhfuil $p \times q$ aonaid fhairsinge san dronuilleóig.

∴ Tá

$$\textit{fairsinge dhronuilleóige} = \textit{fad} \times \textit{leithead},$$

nuair a fhreagraíos na h-aonaid faide agus fairsinge d'á chéile.

## 5.2 The Area of a Rectangle

Take any rectangle at all $ABCD$ and suppose that the sides $AB$ agus $AD$ are commensurate .i. that there is some length (that shall be chosen as a unit of length), some whole number $p$ of which together have the same length as $AB$, and some other whole number $q$ of which together have the same length as $AD$. In the diagram below (MSS p52), we have $p = 5$, $q = 3$.

Divide $AB$ in 5 equal pieces, and draw lines ∥ to $AU$.

Divide $AD$ in 3 equal pieces and draw lines ∥ to $AB$.

It is clear that $AXWY$ is a square, and that it may be laid down exactly (by a translation) on each individual square of the $5 \times 3$ equal parts into which the rectangle $ABCD$ is divided.

If we accept $AX$ as the unit of length, so that $p$ and $q$ are the lengths of the sides $AB, AD$, and if we accept the unit of area that corrersponds to $AX$ .i. the square $AXWY$, it is obvious that there are $p \times q$ units of area in the rectangle.

$$\therefore \textit{area of a rectangle} = \textit{length} \times \textit{breadth},$$

when the units of length and area correspond to one another.

E.g. (1) Má tá an dronuilleóg 5 ór. ar fad agus 3 ór. ar leithead, 'sé $5 \times 3 = 13$ ór cearnacha an fhairsinge.

(2) Má 'siad $3\frac{1}{2}$ ór agus $2\frac{3}{4}$ ór. an fhad is an leithead, mí miste $\frac{1}{4}$ ór. a thoghadh mar aonad faife i dtosach, ionnus go bhfuil 14 aonaid díobh sa bhfad agus 11 san leithead.

Fágann sin go bhfuil an fhairsinge = $14 \times 11$ de na cearnóga ar slios díobh $\frac{1}{4}$ ór.

Déanann $4 \times 4$ de na h-aonaid fairsinge sin 1 ór cearnach.

∴ Tá fairsinge na dronuilleoige = $\frac{14 \times 11}{4 \times 4} = 3\frac{1}{2} \times 2\frac{3}{4}$ ór cear.

∴ Feileann an fhormuil *fad* × *leithead* don chás.

E.g. (1) If the rectangle is 5 inches long and 3 inches wide, then $5 \times 3 = 13$ square inches is the area.

(2) If $3\frac{1}{2}$ in. and $2\frac{3}{4}$ in. are the length and width, we have to choose $\frac{1}{4}$ in. as unit of length to start with, so that there are 14 of these units in the length and 11 in the width.

It follows that the area = $14 \times 11$ of the squares having side $\frac{1}{4}$ in.

$4 \times 4$ of those units make 1 square in.

∴ The area of the rectangle = $\frac{14 \times 11}{4 \times 4} = 3\frac{1}{2} \times 2\frac{3}{4}$ in. squared.

∴ The formula *length* × *width* agrees with the case.

## 5.3 Fairsinge Parallélograim

### Téarmaí

Ní miste an bonn a thabhairt ar shlios ar bith de shleasa an ▱. De bhrí go bhfuil an slios atá ós a chóir parallélach leis, is cómhfhada na h-ingir go léir a tarraingítear go dtí an bonn ó phointí an tsleasa atá ar a aghaidh. *Airde an pharallélograim* a tugtar ar ingear ar bith acu.

Má tógtar dhá ▱ fá'n airde chéanna ar aon bhonn amháin, is soiléir go mbeidh na sleasa atá ós cóir an bhoinn in aon dronlíne amháin, agus í ∥ leis an mbonn. Is ar an ábhar sin a tugtar ▱ *idir na paralléli* céanna ar dhá▱ den chineál sin.

Tugtar *aoirde* thriantáin ar an ingear ón stuach ar an mbonn.

## 5.4 The Area of a Parallelogram

### Definitions

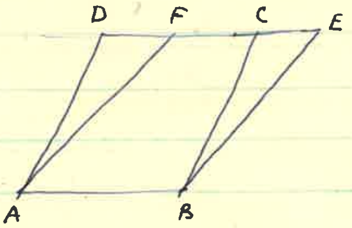
You are free to call any side of a parallelogram the base. Since the opposite side is parallel to it, all the perpendiculars on the base from points on the opposite side have the same length. Any one of them is called a *height of the parallelogram*

If you take two ▱ having the same height on a single base it is clear that the sides opposite the base lie on a single straight line which is ∥ to the base. That is why two such ▱ are called ▱ s *between the same parallels.*

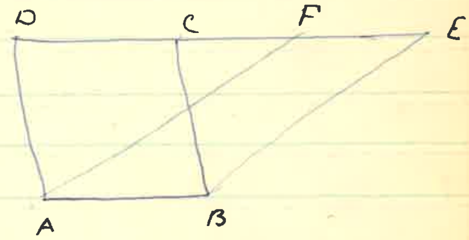
The *height* of a triangle is the perpendicular from the apex on the base.

## Teoirim XVI

Parallélograim chómhfhairsing isea parallélograim fá'n aoirde chéanna atá ar aon bhonn amháin.




Fíog. I




Fíog. II

Hipotéis  Parallélograim isea ABCD, ABEF, agus tá CD agus EF in aon dírlíne amháin.

Tátall  Tá ▱ ABCD cómhfhairsing le ▱ ABEF.

Cruthúnas

De bharr an plána a aistriú ó A go dtí B fan AB, cuirfear AD anuas ar BC, atá = agus // leis (~~Teoirim~~ Cuid IV)

Mar an gceanna, cuirfear AF ar BE.

Fágann sin go leagfar an Δ ADF ar an Δ cóngrúach BCE, ionnus go bhfuil an dá Δ sin cómhfhairsing.

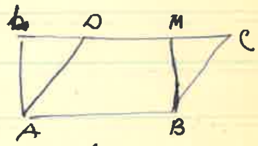
Ach tá an ▱ ABCD = an fhairsinge ABED — fairsinge an Δ BCE.

Agus tá an ▱ ABEF = an fhairsinge ABED − fairsinge an Δ ADF.

Dá bhrí sin, tá an dá ▱ ABCD agus ABEF cómhfhairsing.

Q.E.D.

Atora 1  Tá fairsinge ▱ = bonn × aoirde




Mar tá ▱ ABCD = fairsinge na dronuilleóige ABML, áit gurb iad AL is BM na h-ingir ó A is B ar an líne CD.

Atora 2  Is cómhfhairsing dhá ▱ a tógtar ar bhoinn chothroma agus iad ar aon aoirde.

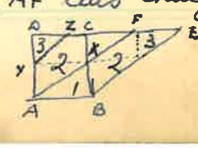
Mar déantar Fíogh I (nó II) díobh nuair leagtar bonn acu ar an mbonn cothrom eile.

Nóta.

Má's cárta é ABCD [fíog. I) is soiléir gur féidir deilbh an ▱ ABEF a chur air, lena ghearradh fan AF agus an Δ AFD a aistriú.

Déanfaidh gearradh ar bith, idir AB is CD, atá // le AF cúis chuige.

I bhFíog. II teastaíonn dhá ghearradh (ar a laghad); viz. fan na línte // AX is YZ.



Tosach leathanach 54 sa LSS.

## 5.5 Teoirim XVI

*Parallélogram chómhfhairsing is ea parallélograim fá'n aoirde chéanna atá ar aon bhonn amháin.*

Tá Fíoghair anseo sa LSS, leathanach 54.

*Hipotéis:*

Parallélograim is ea $ABCD$, $ABEF$, agus tá $CD$ agus $EF$ in aon dronlíne amháin.

*Tátall:*

Tá ▱ $ABCD$ cómhfhairsing le ▱ $ABEF$.

*Cruthúnas:*

De bharr an plána a aistriú ó $A$ go dtí $B$ fan $AB$, cuirfear $AD$ anuas ar $BC$, atá = agus ∥ leis (Caib. IV).

Mar an gcéanna, cuirfear $AF$ ar $BE$.

Fágann sin go leagfar an $\Delta ADF$ ar an $\Delta$ congruu ach $BCE$, ionas go bhfuil an dá $\Delta$ sin cómhfhairsing.

Ach tá an ▱ $ABCD$ = an fhairsinge $ABED-$ fairsinge an $\Delta BCE$.

Agus tá an ▱ $ABEF$ = an fhairsinge $ABED-$ fairsinge an $\Delta ADF$.

Dá bhrí sin tá an dá ▱ $ABCD$ agus $ABEF$ cómhfhairsing. □

**Theorem 16.** *All parallelograms on the same base and having the same height have the same area.*

*Hypothesis:*

$ABCD$, $ABEF$ are parallelograms, and $CD$ and $EF$ ilie in a single straight line.

*Conclusion:*

The ▱ $ABCD$ has the same area as the ▱ $ABEF$.

*Proof:*

By translating the plane from $A$ to $B$ along $AB$, $AD$ is placed on $BC$, which is = agus ∥ to it (Chapter 4).

Similarly, $AF$ may be placed on $BE$.

It follows that the $\Delta ADF$ is laid on the congruent $\Delta$ $BCE$, so that those two $\Delta$s have the same area.

But ▱ $ABCD$ = the area of $ABED-$ the area of $\Delta BCE$.

And ▱ $ABEF$ = the area of $ABED-$ the area of $\Delta ADF$.

Therefore the two ▱ $ABCD$ and $ABEF$ have the same area. □

## Atora 1

*Tá fairsinge ▱ = bonn × aoirde.*

Mar tá ▱ $ABCD$ = fairsinge na dronuilleóige $ABML$, áit gurb iad $AL$ is $BM$ na h-ingir ó $A$ is $B$ ar an líne $CD$.

Tá Fíoghair anseo sa LSS, leathanach 54.

## Atora 2

*Is cómhfhairsing dhá ▱ a tógtar ar bhoinn chothroma agus iad ar aon aoirde.*

Mar déantar Fiogh I (nó II) díobh nuair leagtar bonn acu ar an mbonn cothrom eile.

## Nóta

Má's cárta é $ABCD$ (Fiog I) is soiléir gur iféidir deilbh an ▱ $ABEF$ a chur air, lena ghearadh fan $AF$ agus an $\Delta AFD$ a aistriú .

Tá Fíoghair anseo sa LSS, leathanach 54, ag bun.

Déanfaidh gearadh ar bith, idir $AB$ is $CD$, atá ∥ le $AF$ cúis chuige. I bhFiog.II, teastaí onn dhá ghearadh (ar a laghad); viz. fan na línte ∥ $AX$ is $YZ$.

**Corollary 1.** *Area of ▱ = base × height.*

For ▱ $ABCD$ = area of the rectangle $ABML$, where $AL$ and $BM$ are the perpendiculars from $A$ and $B$ on the line $CD$.

**Corollary 2.** *Two ▱ constructed on equal bases and of equal height have the same area.*

For they become Figure I (or II) when one of the bases is laid on the other equal base.

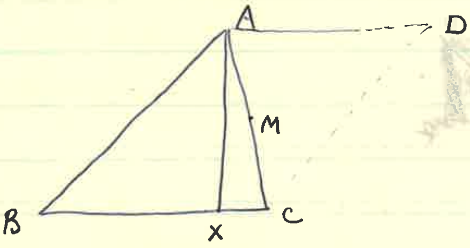
## Note

If $ABCD$ (Fig I) is a card, it is clearly possible to put impose the shape of the ▱ $ABEF$ on it, by cutting it along $AF$ and translating the $\Delta AFD$.

Any cut at all between $AB$ and $CD$, that is ∥ to $AF$ would do for this. In Fig II it needs two cuts (at least); viz. along the lines $AX$ and $YZ$.

## Teoirim XVII

Is ionann fairsinge thriantáin agus leath-fhairsinge pharallélograim ar bith a tógtar ar an mbonn céanna agus é ar aon aoirde leis an triantán




**Tógáil** Tarraing tré C agus A línte a bhéas // le BA is BC, ionnus gur ▱ é BCDA.

Má 'sé AX an t-ingear ó A ar BC is soiléir gurb é AX aoirde an ▱ BCDA, is aoirde an Δ ABC. Faigh M lár an tsleasa AC.

**Cruthúnas**

Le casadh an phlána tré 180° timpeall M, cuirtear an Δ ABC ar an Δ congrúach CDA (Caib IV), ionnus go bhfuil an dá Δ sin comhfhairsing.

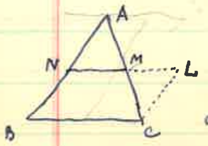
Fágann sin Δ ABC = ½ ▱ BCDA, gurb ionann é maidir le fairsinge agus ▱ ar bith eile a tógtar ar BC fá'n aoirde AX.

**Atora 1** Tá fairsinge Δ = ½ (bonn × aoirde)

**Atora 2** Is comhfhairsing dhá Δ fá'n aoirde chéanna atá ar aon bhonn amháin (nó atá ar bhoinn chomhfhada)

**Atora 3** Má tá triantáin chomhfhairsinge ar bhoinn chomhfhada, táid ar aon aoirde; agus má tá triantáin chomhfhairsinge ar comhaoirde táid ar bhoinn chomhfhada.

**Nóta 1** Triantán a aithdhealbhú ina pharallélogram (agus vice versa)




Má 'siad M, N láir na slios AC, AB sa triantán, gearr an Δ fan NM agus cas an píosa AMN tré 180° timpeall M.
[Fágtar fá'n léitheoir a chruthú gur ▱ é BCLN]

Tosach leathanach 55 sa LSS.

## 5.6 Teoirim XVII

*Is ionann fairsinge thriantáin agus leath-fhairsinge pharallélograim ar bith a tótar ar an mbonn céanna agus é ar aoun airde leis an triantán.*

Tá Fíoghair anseo sa LSS, leathanach 55.

*Tógáil:*

Tarraing tré $C$ agus $A$ línte a bhéas ∥ le $BA$ is $BC$, ionnus gur ▱ é $BCDA$.

Má 'sé $AX$ an t-ingear ó $A$ ar $BC$, is soiléir grub é $AX$ airde an ▱ $BCDA$, is airde an $\Delta ABC$. Faigh $M$ lár an tsleasa $AC$.

*Cruthúnas:*

Le casadh an phlána tré 180° timpeall $M$, cuirtear an $\Delta ABC$ ar an $\Delta$ concrúach $CDA$ (Caib. IV), ionas go bhfuil an dá $\Delta$ sin cómhfhairsing.

Fágann sin $\Delta$ $ABC = \frac{1}{2}$▱ $BCDA$, gurb ionann é maidir le fairsinge agus ▱ ar bith eile a tógtar ar $BC$ fá'n airde $AX$.

**Theorem 17.** *The area of a triangle is half the area of any parallelogram constructed on the same base and having bthe same height as the triangle.*

*Construction:*

Through $C$ and $A$ draw lines ∥ to $BA$ and $BC$, so that $BCDA$ is a ▱.

If $AX$ is the perpendicular from $A$ on $BC$, it is clear that $AX$ is a height of the ▱ $BCDA$, and the height of $ABC$. Find $M$, the centre of the side $AC$.

*Proof:*

When the plane is rotated through 180° about $M$, the $\Delta ABC$ is placed on the congruent $\Delta$ $CDA$ (Chapter 4), so those two $\Delta$ have the same area.

Thus $\Delta$ $ABC = \frac{1}{2}$▱ $BCDA$, which has the same area as any other ▱ constructed on the base $BC$ with height $AX$.

### Atora 1

*Tá fairsinge* $\Delta = \frac{1}{2}$(*bonn* × *aoirde*).

### Atora 2

*Is cómhfhairsing dhá* $\Delta$ *fá'n airde chéanna atá ar aon bhonn amháin (nó atá ar bhoinn chómhfhada).*

### Atora 3

*Má tá triantáin chómhfhairsinge ar bhoinn chómhfhada, táid ar aon aoirde; agus má tá triantáin chómhfhairsinge ar comhairde táid ar bhoinn chómhfhada.*

**Corollary 1.** *The area of* $\Delta = \frac{1}{2}($ *base* $\times$ *height*$)$.

**Corollary 2.** *Two* $\Delta$ *that have the same height and are on the same base (or are on bases of equal length) have the same area.*

**Corollary 3.** *If triangles have the same area and are on equally-long bases, then they have the same height; and if equal-area triangles have the same height, then their bases have the same length.*

### Nóta 1

*Triantán a aithdhealbhú ina pharallélogram (agus vice versa).*

Tá Fíoghair anseo sa LSS, leathanach 55.

Má 'siad $M, N$ láir na slios $AC, AB$ sa triantán, gearr an $\Delta$ fan $NM$ agus cas an píosa $AMN$ tré 180° timpeall $M$.

[Fágtar dá'n léitheoir a chruthú gur $\varodot$ é $BCLN$.

### Note 1

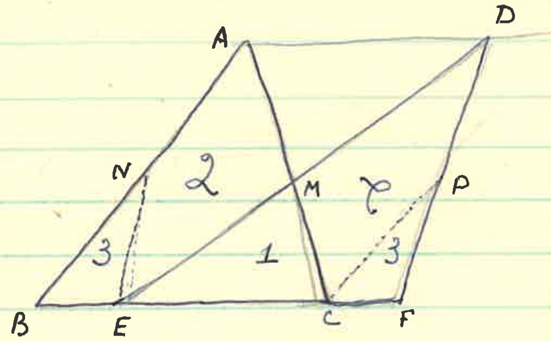
*To reshape a triangle as a parallelogram (and vice versa).*

If $M, N$ are the centres of the sides $AC, AB$ in the triangle, cut the $\Delta$ along $NM$ and rotate the piece $AMN$ through 180° around $M$.

[We leave it to the reader to prove that $BCLN$ is a $\varodot$.]

## Nóta 2

Triantán ABC a aithdhealbhú ina thriantán eile atá ar an mbonn céanna (agus, dá bhrí sin, atá ar aon airde leis).




~~Réiteach~~

Abair gurb iad M, N lár na sliosanna AC, AB. Sleamhnaigh bonn an triantán eile ar BC chun go dteighe an slios DE tré M.

Abair gurb é ~~Tá sé~~ P lár DF.

Tugtar dúinn EF = BC, AD // le BC.

Réiteach

Tá NM = $\frac{1}{2}$BC agus // le BC (bail IV), agus mar an gcéanna tá MP = $\frac{1}{2}$ EF agus // le EF.

Fágann sin MN = MP, agus aon dronlíne amháin isea NMP.

∴ tá NP = agus // le BC, ionnas go bhfuil BN = agus // CP, agus mar an gcéanna tá EN = agus // le FP.

Má geartar an △ ABC fan EM agus EN, cuirfear BEN ar CFP le haistriú, agus cuirfear ANEM ar CPDM le casadh tré 180° timpeall M.

### Cleachtaithe

1) Má haistrítear △ ABC go dtí an t-ionad $A_1B_1C_1$, taispeáin (a) gur ▱ a ghineas gach slios san imtheacht dó; (b) go bhfuil dhá ▱ acu le chéile comhfhairsing leis an tríú ceann.

2) ~~I dteoirim XV, Cuig II, má tá B~~ Pointe isea X ar shlios CD na dronuilleoige ABCD, agus 'sé BY an t-ingear ó B ar AX. Cruthaigh AB.AD = AX.BY.

3) Faigh airde an ▱ gur bonn dó 2" is go bhfuil 3 or. cear. ina fhairsinge. Má tá an slios eile 3" ar fad, línigh an ▱ agus tomhas na huilleacha.

Tosach leathanach 56 sa LSS.

## Nóta 2

*Triantán $ABC$ a aithdhealbhú ina thriantán eile atá ar an* mbonn céanna *(agus, dá bhrí sin, atá ar aon airde leis).*

Tá Fíoghair anseo sa LSS, leathanach 56.

Abair gurb iad $M, N$ láir na slios $AC, AB$. Sleamhnuigh bonn an triantáin eile ar $BC$ chun go dtéighe an slios $DE$ tré $M$.

Abair gurb é $P$ lár $DF$.

Tugtar dúinn $EF = BC$, $AD \|$ le $BC$.

*Réiteach:*

Tá $NM = \frac{1}{2}BC$ agus $\|$ le $BC$ (Caib. IV), agus mar an gcéanna tá $MP = \frac{1}{2}EF$ agus $\|$ le $EF$.

Fágann sin $MN = MP$, agus aon dronlíne amháin is ea $NMP$. $\therefore$ Tá $NP =$ agus $\|$ le $BC$, ionas go bhfuil $BN =$agus $\|$ le $CP$, agus mar an gcéanna tá $EN =$ agus $\|$ le $FP$.

Má gearrtar an $\Delta ABC$ fan $EM$ agus $EN$, cuirfear $BEN$ ar $CFP$ le haistriú , agus cuirfear $ANEM$ ar $CPDM$ le casadh tré 180° timpeall $M$

## Note 2

*To reshape a triangle $ABC$ into another triangle*[1] *that is on the* same base *(and that, hence, has the same height as it).*

Suppose that $M, N$ are the centres of the sides $AC, AB$. Slide the base of the other triangle on $BC$ until the side $DE$ passes through $M$.

Suppose that $P$ is the centre of $DF$.

We are given that $EF = BC$, $AD \|$ to $BC$.

*Solution:*

We have $NM = \frac{1}{2}BC$ and $\|$ to $BC$ (Chapter 4), and similarly $MP = \frac{1}{2}EF$ and $\|$ to $EF$.

It follows that $MN = MP$, and $NMP$ is a single straight line. $\therefore$ $NP =$ and $\|$ to $BC$, so that $BN =$and $\|$ to $CP$, and similarly $EN =$ and $\|$ to $FP$.

If we cut the $\Delta ABC$ along $EM$ and $EN$, then $BEN$ may be placed on $CFP$ by a translation, and $ANEM$ may be placed on $CPDM$ by a rotation through 180° about $M$.

[1] $\Delta DEF$, given, on an equal base (AOF)

4) Sa Δ ABC, 'sé M lár an bhoinn BC. Cruthuigh go bhfuil fairsinge an Δ AMB = $\frac{1}{2}$ Δ ABC.

5) Sa ▱ ABCD pointe ar an slios CD isea P. Teaspáin go bhfuil fairsinge an Δ PAB = $\frac{1}{2}$ ▱ ABCD.

6) Pointí isea P, Q ar shleasa AB, AC an Δ ABC i gcaoi go bhfuil PQ // le BC. Cruthuigh (i) fairsinge Δ PQB = fairsinge Δ PQC: (ii) fairsinge Δ AQB = fairsinge Δ APC.

7) Triantáin chómhfairsinge isea ABC, DBC, ceann acu ar gach taobh den bhonn BC. 'Siad AX, DY na h-ingir ó A is D ar BC, agus gearrann AD is BC in X.

Cruthuigh (i) AX = DY: (ii) go bhfuil Δ AZX agus Δ DZY congrúach: (iii) go bhfuil AZ = ZD.

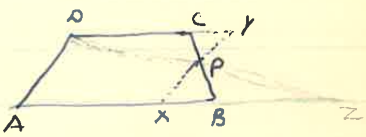
8) I dtriantán ABC isiad M, N láir na slios AB, AC, agus pointe ar bith in MN isea P. Tarraing dronlíne thré C atá // le BP a ghearras MN in Q. Cruthuigh go bhfuil an Δ ABC comhfhairsing leis an ▱ BCQP.

~~Tarraing~~ Ar bhonn triantáin ar bith tóg ▱ a bheas comhfhairsing leis an triantán, agus uilleann a bheith cothrom le h-uillinn áirithe.

9) I gceist 8, má tá P idir M agus N, agus má geartar an Δ fan MN agus BP, teaspáin gur féidir an ▱ BCQP a dhealbhú leis.

<u>Téarma</u>

Tugtar trapésium ar cheathairshleasán ina bhfuil dhá shlios // le chéile.




10) 'Sé P lár an tsleasa BC sa trapésium ABCD, agus tá XPY // le AD.

Cruthuigh (i) go bhfuil ABCD comhfhairsing leis an ▱ AXYD:
(ii) Má theangmhaíonn DP le AB in Z, go bhfuil ABCD comhfhairsing leis an Δ DAZ: (iii) de bhrí gurb é AB + DC bonn an Δ DAZ, cruthuigh:—

fairsinge trapésium = $\frac{1}{2}$ (suim na slios //) × (an airde ingearach eatortu)

## Cleachtaithe

1. Má haistrítear $\Delta\ ABC$ go dtí an t-ionad $A_1B_1C_1$, teaspáin (a) gur ▱ a ghineas gach slios san imtheacht dó ; (b) go bhfuil dhá ▱ acu le chéile cómhfhairsing leis an tríú ceann.

2. Pointe is ea $X$ ar shlios $CD$ na dronuilleóige $ABCD$, agus 'sé $BY$ an t-ingear ó $B$ ar $AX$. Cruthuigh $AB \cdot AD = AX \cdot BY$.

3. Faigh aoirde an ▱ gur bonn dó 2" is go bhfuil 3 or. cear. ina fhairsinge. Má tá an slios eile 3" ar fad, línigh an ▱ agus tomhas na huilleacha.

   Tosach leathanach 57 sa LSS.

4. Sa $\Delta\ ABC$, 'sé $M$ lár an bhoinn $BC$. Cruthuigh go bhfuil fairsinge an $\Delta\ AMB = \frac{1}{2}\Delta ABC$.

5. Sa ▱ $ABCD$ pointe an an slios $CD$ is ea $P$. Teaspáin go bhfuil fairsinge an $\Delta\ PAB = \frac{1}{2}$▱ $ABCD$.

6. Pointí is ea $P, Q$ ar shleasa $AB, AC$ an $\Delta ABC$ i gcaoi go bhfuil $PQ \|$ le $BC$. Cruthaigh (i) fairsinge $\Delta PQB$ = fairsinge $\Delta PQC$; (ii) fairsinge $\Delta AQB$ = fairsinge $\Delta APC$.

7. Triantáin chómhfhairsinge is ea $ABC$, $DBC$, ceann acu ar gach taobh den bhonn $BC$. 'Siad $AX, DY$ na h-ingir ó $A$ is $D$ ar $BC$, agus gearrann $AD$ is $BC$ in $X$

   Cruthaigh (i) $AX = DY$; (ii) go bhfuil $\Delta AZX$ agus $\Delta DZY$ congrúach; (iii) go bhfuil $AZ = ZD$.

8. I dtriantán $ABC$ is iad $M, N$ láir na slios $AB, AC$, agus pointe ar bith in $MN$ is ea $P$. Tarraing dronlíne tré$C$ atá$\|$ le $BP$ a ghearras $MN$ in $Q$. Cruthuigh go bhfuil an $\Delta\ ABC$ cómhfhairsing leis an ▱ $BCQP$

   Ar bhonn triantáin ar bith tóg ▱ a bheas cómhfhairsing leis an triantán, agus uilleann a bheith cothrom le h-uillinn áirithe.

9. I gceist 8, má tá $P$ idir $M$ agus $N$, agus má gearrtar an $\Delta$ fan $MN$ agus $BP$, teaspáin gur féidir an ▱ $BCQP$ a dhealbhú leis.

   **Téarma**

   Tugtar *trapésium* ar ceathairshleasán ina bhfuil dhá slios $\|$ le chéile.

10. 'Sé $P$ lár an tsleasa $BC$ sa trapésium $ABCD$, agus tá $XPY \|$ le $AD$.

    Tá Fíoghair anseo sa LSS, leathanach 57.

    Cruthaigh (i) go bhfuil $ABCD$ cómhfhairsing leis an ▱ $AXYD$; (ii) Má theagmhaíonn $DP$ le $AB$ in $Z$, go bhfuil $ABCD$ cómhfhairsing leis an $\Delta\ DAZ$; (iii) de bhrí gurb é $AB + DC$ bonn an $\Delta\ DAZ$, cruthuigh:=
    fairsinge trapésium = $\frac{1}{2}$(suim na slios $\|$) × (an airde ingearach eatorru).

## Exercises

1. If the $\Delta$ $ABC$ is translated to the position $A_1B_1C_1$, show (a) that each side generates a ▱ as it moves; (b) that two of those ▱ s together have the same area as the thrid one.

2. $X$ is a point on the side $CD$ of the rectangle $ABCD$, and $BY$ is the perpendicular from $B$ on $AX$. Prove that $AB \cdot AD = AX \cdot BY$.

3. Find the height of the ▱ which has base 2" and area 3 inches squared. If the other side is 3" long, draw the ▱ and measure the angles.

4. In the $\Delta$ $ABC$, the point $M$ is the centre of the base $BC$. Prove that the area of the $\Delta$ $AMB = \frac{1}{2}\Delta ABC$.

5. In the ▱ $ABCD$ the point $P$ is on the side $CD$. Show that the area of $\Delta$ $PAB$ = $\frac{1}{2}$▱ $ABCD$.

6. $P, Q$ are points on the sides $AB, AC$ of the $\Delta ABC$ such that $PQ\|$ le $BC$. Prove (i) area $\Delta PQB$ = area $\Delta PQC$; (ii) area $\Delta AQB$ = area $\Delta APC$.

7. The triangles $ABC$, $DBC$ have the same area, and one of them lies on each side of the base $BC$. The perpendiculars from $A$ and $D$ on $BC$ are $AXandDY$, and $AD$ and $BC$ meet at $X$

   Prove (i) $AX = DY$; (ii) that $\Delta AZX$ and $\Delta DZY$ are congruent; (iii) that $AZ = ZD$.

8. In a triangle $ABC$ the points $M, N$ are the centres of the sides $AB$, $AC$, and $P$ is any point at all in $MN$. Draw a straight line through $C$ that is ∥ to $BP$ and cuts $MN$ at $Q$. Prove that $\Delta$ $ABC$ has the same area the ▱ $BCQP$.

   On the base of any triangle construct a ▱ that will have the same area as the triangle and that has an angle equal to a given angle.

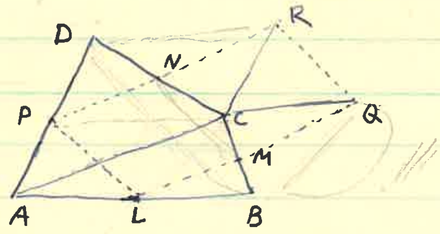
9. In question 8, if $P$ is between $M$ and $N$, and if the $\Delta$ is cut along $MN$ and $BP$, show that it is possible to construct the ▱ $BCQP$ with it.

   **Definition**

   A *trapezium* is a quadrilateral having two sides ∥ to one another.

10. $P$ is the centre of the side $BC$ in the trapezium $ABCD$, and $XPY\|$ to $AD$. Prove (i) that $ABCD$ has the same area as the ▱ $AXYD$; (ii) if $DP$ meets $AB$ at $Z$, that $ABCD$ has the same area as the $\Delta$ $DAZ$; (iii) using the fact that $AB + DC$ is the base of the $\Delta$ $DAZ$, prove:—
    Area of a trapezium = $\frac{1}{2}$(sum of the ∥ sides) × (the perpendicular distance between them).

Ceist I    Parallélogram a thógáil a bheas cómhfhairsing le ceathairshleasan a tugtar.




Abair gurb é ABCD an 4-shleasán, agus gurb iad L, M, N, P láir na slios.

Réiteach

Tré C tarraing CR // le AD, agus tarraing CQ // le AB. Ceangail QR.

Parallélogram a fheilfas isea LQRP.

Cruthúnas

▭ isea ACRP atá cómhfhairsing leis an △ DAC (Teoirim XVI, nóta 1).

Mar an gcéanna ▭ isea ACQL atá cómhfhairsing leis an △ BAC.

∴ Tá ABCD cómhfhairsing le suim an dá ▭ ACRP agus ACQL.

Ach, ó thárla AP = agus // le CR, agus ó thárla AL = agus // le CQ, leagtar an △ APL ar an △ CRQ le h-aistriú fan AC.

Fágann sin:–

(i) go bhfuil RQ = agus // le PL, ionnas gur ▭ é LQRP;

(ii) go bhfuil an ▭ LQRP = suim an dá ▭ ACRP is ACQL = an 4-shleasán ABCD.

Atora    Tá fairsinge 4-shleasáin ar bith cothrom le leath an ▭ go bhfuil a shleasa cómhfhada agus parallélleach le trasnáin an 4-shleasáin.

Mar tá PR = agus // le AC, PL // le BD agus $= \frac{1}{2}$ BD.

Tosach leathanach 58 sa LSS.

## Ceist 1

*Parallélogram a thógáil a bhéas cómhfhairsing le ceathairshleasán a tugtar.*

Tá Fíoghair anseo sa LSS, leathanach 58.

Abair gurb é $ABCD$ an 4-shleasán, agus gurb iad $L, M, N$ láir na slios.

*Réiteach:*
Tré $C$ tarraing $CR\|$ le $AD$[2] agus tarraing $CQ\|$ le $AB$[3]. Ceangail $QR$.
Parallélogram a fheileas is ea $LQRP$.

*Cruthúnas:*
/
–

▱ is ea $ACRP$ atá cómhfhairsing leis an $\Delta\ DAC$ (Teoirim XIV, nóta 1).
Mar ac gcéanna ▱ is ea $ACQL$ atá cómhfhairsing leis an $\Delta\ BAC$.
∴ Tá $ABCD$ cómhfhairsing le suim an dá ▱ $ACRP$ agus $ACQL$.
Ach, ó thála $AP$ = agus $\|$ le $CR$, agus ó thárla $AL$ = agus $\|$ le $CQ$, leagtar an $\Delta\ APL$ ar an $\Delta\ CRQ$ le haistriú fan $AC$.
Fágann sin:=
(i) go bhfuil $RQ$ = agus $\|$ le $PL$, ionas gur ▱ é $LQRP$;
(ii) go bhfuil an ▱ $LQRP$ = suim an dá ▱ $ACRP$ is $ACQL$ = an 4-shleasán $ABCD$.

## Question 1

*To construct a parallogram that has the same area as a given quadrilateral.*

Suppose $ABCD$ is the quadrilateral, and that $L, M, N$ are the centres of the sides.

*Solution:*
Through $C$ draw $CR\|$ to $AD$[4] and draw $CQ\|$ to $AB$[5]. Join $QR$.
A parallelogram that meets the case is $LQRP$.

*Proof:*
$ACRP$ is a ▱ that has the same area as the $\Delta\ DAC$ (Theorem 14, Note 1).
Similarly, $ACQL$ is a ▱ that has the same area as the $\Delta\ BAC$.
∴ $ABCD$ has the same area as the sum of the two ▱ $ACRP$ and $ACQL$.
But, since $AP$ = and $\|$ to $CR$, agus ó thárla $AL$ = agus $\|$ le $CQ$, the translation along $AC$ lays the $\Delta\ APL$ on the $\Delta\ CRQ$.
It follows:—

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[2]ag teaghmháil le $PN$ ag $R$ (AOF)
[3]ag teaghmháil le $LM$ ag $Q$ (AOF)
[4]meeting $PN$ at $R$
[5]meeting $LM$ at $Q$

(i) that $RQ$ = and ∥ to $PL$, so that $LQRP$ is a ▱;
(ii) that the ▱ $LQRP$ = sum of the two ▱ $ACRP$ and $ACQL$ = the quadrilateral $ABCD$.

## Atora

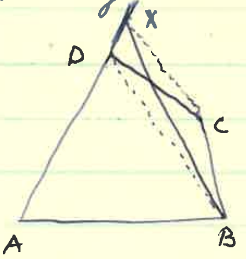
*Tá fairsinge 4-shleasáin ar bith cothrom le leath an ▱ go bhfuil a shleasa cómhfhada agus parallèileach le treasnáin an 4-shleasáin.*

Mar tá $PR$ = agus ∥ le $AC$, $PL$∥ le $BD$ agus = $\frac{1}{2}BD$.

**Corollary.** *The area of any quadrilateral is equal to half the ▱ that has its sides the same length and parallel to the diagonals of the quadrilateral.*

For we have $PR$ = and ∥ to $AC$, $PL$∥ to $BD$ and = $\frac{1}{2}BD$.

Ceist 2  Triantán a thógáil ar shlios cheathairshleasáin a bheas comhfhairsing leis an gceathairshleasán féin.




Abair gurb é ABCD an 4-shleasán a tugtar.

Réiteach

Bhun Δ ar AB a thógáil tarraing trí C parallél le BD, a ghearras síneadh AD in X, abair. Ceangail BX.

Tá an Δ ABX comhfhairsing le ABCD.

Cruthúnas

Ó tharla XC∥le BD, triantáin fán airde chéanna atá ar aon bhonn amháin DB, isea an Δ XDB agus an Δ CDB.

∴ Tá an dá Δ XDB is CDB comhfhairsing (Teoirim XVI)

De bharr fairsinge an Δ DAB a shuimiú leo, gheofar

Δ XAB = Δ DAB + Δ CDB = an 4-shleasán ABCD.

## Cleachtaithe

1) Sa bhFíoghair i gceist 1, cruthuigh gur ▭ é LMNP atá = $\frac{1}{2}$ ABCD.

2) Má ghearrtar an 4-shleasán ABCD fan NP, PL, LM, teaspáin gur féidir an ▭ PLQR a dhealbhú leis na 4 píosaí.

3) I 4-shleasán ABCD tarraingítear dhá dhronlíne trí B agus D atá ∥ leis an trasnán AC; agus tarraingítear péire eile trí A agus C atá ∥ leis an trasnán BD.

Cruthuigh fán ▭ a geintear uatha, (i) go bhfuil sé = 2ABCD ina fhairsinge; (ii) gur féidir ABCD a dhealbhú leis na 4 triantáin ag na cúinní.

4) I gceathairshleasán ABCD tá X, lár an trasnáin DB taobh istigh den Δ DAC, agus siad N, P láir na slios CD, DA. Gearrtar an 4-shleasán sin fan NX, DX, PX, AC.

Teaspáin go ngeintear ▭ leis na 4 píosaí nuair castar DPX trí 180° timpeall P, nuair castar DNX timpeall N, agus nuair a haistrítear ABC fan BX.

Tosach leathanach 59 sa LSS.

## Ceist 2

*Triantán a thógáil ar shlios cheathairshleasáin a bheas cómhfhairsing leis an gceathair-shleasán féin.*

Tá Fíoghair anseo sa LSS, leathanach 59.

Abair gurb é $ABCD$ an 4-shleasán a tugtar.

*Réiteach:*

Chun $\Delta$ ar $AB$ a thógáil tarraing tré $C$ parallél le $BD$, a ghearras síneadh $AD$ in $X$, abair. Ceangail $BX$.

Tá an $\Delta\ ABX$ cómhfhairsing le $ABCD$.

*Cruthúnas:*

Ó thárla $XC \|$ le $BD$, triantán fá'n airde chéanna atá ar aon bhonn amáin $DB$, is ea an $\Delta\ XDB$ agus an $\Delta\ CDB$.

$\therefore$ Tá an dá $\Delta\ XDB$ is $CDB$ cómhfhairsing (Teoirim XVI).

De bharr fairsinge an $\Delta DAB$ a shuimiú leo, gheofar

$\Delta XAB = \Delta DAB + \Delta CDB =$ an 4-shleasán $ABCD$.

## Question 2

*To construct a triangle on one side of a quadrilateral that will have the same area as the quadrilateral itself.*

Suppose that $ABCD$ is the given quadrilateral.

*Solution:*

To construct a $\Delta$ on $AB$ draw a straight line through $C$ parallel to $BD$, cutting $AD$ at $X$, say. Join $BX$.

The $\Delta\ ABX$ has the same area as $ABCD$.

*Proof:*

Since $XC \|$ to $BD$, the triangles $\Delta\ XDB$ and $\Delta\ CDB$ have the same height and are on the same base $DB$.

$\therefore$ $\Delta\ XDB$ and $CDB$ have the same area (Theorem 16).

By adding the area of the $\Delta DAB$ to them, we get

$\Delta XAB = \Delta DAB + \Delta CDB =$ the quadrilateral $ABCD$.

5) Tarraing $\Delta$ ar AB a bheas cómhfhairsing le 4-shleasán ABCD, agus a mbeidh uille ann cothrom le $A\hat{B}C$.

6) Tóg ar AB $\Delta$ cómhchosach a bheas cómhfhairsing le ABCD.

7) Má ghearrann BC is AD i bpointe Y, cruthuigh go bhfuil an $\Delta$ BYX = an $\Delta$ CYD i bhfairsinge [Féogh. i gceist II]

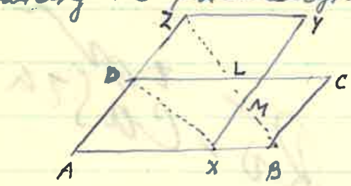
8) I gcearthairshleasán ABCD 'siad L, M, N, P láir na slios AB, BC, CD, DA, agus ceangailtear LM, LN, LP. Castar NLPD tré 180° timpeall N, agus castar LMB tré 180° timpeall M.

De bharr ALP a aistriú fan AC cruthuigh go ngeibhtear $\Delta$ a bhfuil dhá shlios ann cothrom agus parallíleach leis na trasnáin AC, BD.

9) Sa 4-shleasán ABCD i gceist I, 'sé K pointe teagmhála PL agus AC, agus geartar an 4-shleasán fan PL, NK, MK. Cruthuigh gur féidir $\Delta$ a dhealbhú leis na 4 píosaí.

10) Cén chaoi a mba chóir ABCD i gceist II a ghearradh chun an $\Delta$ XAB a dhealbhú [Teoirim XV nóta 2, a úsáid]

Ceist 3 Parallélogram a thógáil ar dhronlíne áirithe a bheas cómhfhairsing le parallélogram eile a tugtar.




Réiteach Ar shlios AB den ▱ ABCD a tugtar, marcáil AX atá cómhfhada leis an dronlíne áirithe. ~~Tarrtar~~

Tarraing tré B paralléil le XD, a ghearras síneadh AD in Z.

Tá an ▱ AXYZ, atá sleasa cómhgaracha dhó AX is AZ, cómhfhairsing le ▱ ABCD.

Cruthúnas

Sá ▱ BLDX tá BL = XD, agus sa ▱ DXMZ tá MZ = XD.

Fágann sin BL = MZ, agus tá BM = LZ freisin.

Ó tharla ZY ∥ le LC, agus YM ∥ CB, cuirtear an $\Delta$ CLB anuas go cruinn ar an $\Delta$ YZM ~~le~~ le (a) aistriú ó B go dtí M fan BM.

Mar an gceánna leagtar an $\Delta$ MXB ar an $\Delta$ ZDL de bharr aistrithe ó B go dtí L fan BL.

Fágann sin gurb ionann an ▱ ABCD = AXML + $\Delta$ CLB + $\Delta$ MXB
= AXML + $\Delta$ YZM + $\Delta$ ZDL = an ▱ AXYZ.

## Cleachtaithe

1. Sa bhFioghair i gceist 1, cruthuigh gur ▱ é $LMNP$ atá = $\frac{1}{2}ABCD$.

2. Má gearrtar an 4-shleasán $ABCD$ fan $NP, PL, LM$, teaspáin gur féidir an ▱ $PLQR$ a dhealbhú leis na 4 píosaí.

3. I 4-shleasán $ABCD$ tarraingítear dhá dhronlíe tré $B$ agus $D$ atá ∥ leis an treasnán $AC$; agus tarraingítear péire eile tré $A$ agus $C$ atá ∥ leis an treasnán $BD$.

   Cruthuigh fán ▱ a geintear uatha, (i) go bhfuil sé = 2 $ABCD$ ina fhairsinge; (ii) gur féidir $ABCD$ a dhealbhú leis na 4 traintáin ag na coirnéil.

4. I gceathairshleasán $ABCD$ tá $X$, lár an treasnáin $DB$ taobh istigh den $\Delta DAC$, agus 'siad $N, P$ láir na slios $CD, DA$.
   Gearrtar an 4-shleasán sin fan $NX, DX, PX, AC$.

   Teaspáin go ngintear ▱ leis na 4 píosaí nuair castar $DPX$ tré 180° timpeall $P$, nuair castar $DNX$ timpeall $N$, agus nuair a haistrítear $ABC$ fan $BX$.

   Tosach leathanach 60 sa LSS.

5. Tarraing Δ ar $AB$ a bheas cómhfhairsing le 4-shleasán $ABCD$, agus a mbeidh uille ann cothrom le $\widehat{ABC}$.

6. Tóg ar $AB$ Δ cómhchosach a bheas cómhfhairsing le $ABCD$.

7. Má ghearrann $BC$ is $AD$ i bpointe Y, cruthuigh go bhfuil an $\Delta BYX$ = an $\Delta CYD$ i bhfairsinge [Fiogh. i gceist 3 ].

8. I gceathair shleasán $ABCD$ 'siad $L, M, N, P$ láir na slios $AB, BC, CD, DA$, agus ceangailtear $LM, LN, LP$. Castar $NLPD$ tré 180° timpeall $N$, agus castar $LMB$ tré 180° timpeall $M$.

   De bharr $ALP$ a aistriú fan $AC$ cruthuigh go ngintear Δ a bhfuil dhá shlios ann cothrom agus paralléileach leis na treasnáin $AC, BD$.

9. Sa 4-shleasán $ABCD$ i gceist I, 'sé $K$ pointe teagmhála $PL$ agus $AC$, agus gearrtar an 4-shleasán fan $PL, NK, MK$. Cruthuigh gur féidir Δ a dhealbhú leis na 4 píosaí.

10. Cé'n chaoi a mba chóir $ABCD$ i gceist II a ghearradh chun an $\Delta XAB$ a dhealbhú [Teoirim XV nóta2, a úsáid].

## Exercises

1. In the figure in Question 1, prove that $LMNP$ is a ▱ that = $\frac{1}{2}ABCD$.

2. If we cut the quadrilateral $ABCD$ along $NP, PL, LM$, show that it is possible to assemble the ▱ $PLQR$ with the 4 pieces.

3. In a quadrilateral $ABCD$ two straight lines are drawn through $B$ and $D$ that are $\parallel$ to the diagonal $AC$; and another pair through $A$ and $C$ that are $\parallel$ to the diagonal $BD$.

   Prove this about the ▱ they generate: (i) that it has twice the area of $ABCD$; (ii) that $ABCD$ can be assembled from the four triangles at the corners.

4. In a quadrilateral $ABCD$ the centre $X$ of the diagonal $DB$ is inside the $\Delta DAC$, and $N, P$ are the centres of the sides $CD, DA$.
   The quadrilateral is cut along $NX, DX, PX, AC$.

   Show that the 4 pieces make a ▱ when $DPX$ is rotated through 180° around $P$, when $DNX$ is rotated around $N$, and when $ABC$ is translated along $BX$.

   Tosach leathanach 60 sa LSS.

5. Draw a $\Delta$ on $AB$ that will have the same area as a quadrilateral $ABCD$, and that will have an angle equal to $\widehat{ABC}$.

6. Construct on $AB$ an isosceles $\Delta$ having the same area as $ABCD$.

7. If $BC$ and $AD$ cut at the point Y, prove that the $\Delta BYX =$ the $\Delta CYD$ in area [Fig. in Question 3].

8. In a quadrilateral $ABCD$, $L, M, N, P$ are the centres of the sides $AB, BC, CD, DA$, and $LM, LN, LP$ are joined. $NLPD$ is rotated through 180° around $N$, and $LMB$ s rotated through 180° around $M$.

   By translating $ALP$ along $AC$ prove that a $\Delta$ is generated that has two of its sides equal and parallel to the diagonals $AC, BD$.

9. In the quadrilateral $ABCD$ in Question 1, $K$ is the point where $PL$ meets $AC$, and the quadrilateral is cut along $PL, NK, MK$. Prove that the $\Delta$ can be assembled from the 4 pieces.

10. How should one cut $ABCD$ in Question 2 in order to assemble the $\Delta XAB$? [Use Theorem 15, Note 2]

## Ceist 3

*Parallélogram a thógáil ar dhronlíne áirithe a bheas cófhairsing le parallélogram eile a tugtar.*

Tá Físeán anseo sa LSS, leathanach 60.

*Réiteach:*
Ar shlios $AB$ den ▱ $ABCD$ a tugtar, marcáil $AX$ atá cómhfhada leis an dronlíne áirithe.

Tarraing tré $B$ paralléil le $XD$, a ghearras síneadh $AD$ in $Z$.
Tá an $\varparallelogram\ AXYZ$, ar sleasa cómhgaracha dhó $AX$ is $AZ$, cómhfhairsing le $\varparallelogram\ ABCD$.

*Cruthúnas:*

Sa $\varparallelogram\ BLDX$ tá $BL = XD$, agus sa $\varparallelogram\ DXMZ$ tá $MZ = XD$. Fágann sin $BL = MZ$, agus tá $BM = LZ$ freisin.

Ó thárla $YZ \|$ le $LC$ agus $YM \|$ le $CB$, cuirtear an $\Delta CLB$ anuas go cruinn ar an $\Delta YZN$ le haistriú ó $B$ go dtí $M$ fan $BM$.

Mar an gcéanna leagtar an $\Delta MXB$ ar an $\Delta ZDL$ de bharr aistrithe ó $B$ go dtí $L$ fan $BL$.

Fágann sin go bhfuil an $\varparallelogram\ ABCD = AXML + \Delta CLB + \Delta MXB = AXML + \Delta YZM + \Delta ZDL =$ an $\varparallelogram\ AXYZ$.

## Question 3

*To construct a parallelogram on a given straight line that will have the same area as another given parallelogram.*

*Solution:*

On the side $AB$ of the given $\varparallelogram\ ABCD$, mark $AX$ having the same length as the given straight line .

Draw a line through $B$ parallel to $XD$, cutting the extension of $AD$ at $Z$.
The $\varparallelogram\ AXYZ$, on the adjacent sides $AX$ and $AZ$, has the same area as the $\varparallelogram\ ABCD$.

*Proof:*

In the $\varparallelogram\ BLDX$ we have $BL = XD$, and in the $\varparallelogram\ DXMZ$ we have $MZ = XD$. It follows that $BL = MZ$, and also that $BM = LZ$.

Since $YZ \|$ to $LC$ and $YM \|$ to $CB$, the $\Delta CLB$ is laid exactly on the $\Delta YZN$ by the translation from $B$ to $M$ along $BM$.

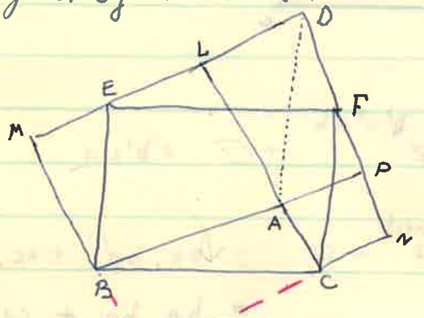
Similarly, the $\Delta MXB$ is laid on the $\Delta ZDL$ by translation from $B$ to $L$ along $BL$.

It follows that the $\varparallelogram\ ABCD = AXML + \Delta CLB + \Delta MXB = AXML + \Delta YZM + \Delta ZDL =$ the $\varparallelogram\ AXYZ$.

Téárma I dtriantán dronuilleach ~~sé~~ is é an hipoteniús an slios atá ar aghaidh na dronuilleann.

## Teoirim XVIII (Teoirim Phutagorais)

Is ionann an chearnóg ar hipoteniús thriantáin dronuilligh agus suim na gcearnóg ar an dá shlios eile.




Hipotesis. Dronuille is ea A sa Δ ABC.

Tógáil Ar AB is AC tóg na cearnóga BALM, ACNP. Déan ME = AC, NF = AB, ionnus gurb iad BME agus FNC ionaid an Δ BAC arna chasadh tré 90° timpeall B agus C. Ceangail EF, AD.

Tátal Tá na cearnóga ar AB is AC le chéile comhfhairsing leis an gcearnóig ar BC.

Cruthúnas

'Sé BE ionad BC arna chasadh tré 90° timpeall B.

∴ Tá BE = BC, agus tá $C\hat{B}E = 90°$.

Mar an gcéanna tá CB = CF, agus tá BCF = 90°.

Fágann sin gurb í BCFE an chearnóg ar BC, agus de thairbhe EF agus // le BC, ED // le BA, FD // le CA, tá an Δ DEF comhréach leis an Δ BAC san aistriú ó B go dtí E (nó, ó C go dtí F).

Is léir anois go bhfuil

an chearnóg BCEF = an ▱ BEDA + an ▱ CFDA

= an chear. BALM + an chear. ACNP (Teoirim XV)

∴ Tá $BC^2 = AB^2 + AC^2$.

Q.E.D.

Tosach leathanach 61 sa LSS.

### Téarma

I dtriantán dronuilleach is é an *hipotenús* an slios atá ar aghaidh na dronuilleann.

## 5.7 Teoirim XVIII (Teoirim Phutagorais)

*Is ionann an chearnóg ar hipotenús thriantáin dronuilligh agus suim na gcearnóg ar an dá shlios eile.*

Tá Fíoghair anseo sa LSS, leathanach 61.

*Hipotéis:*
Dronuille is ea $A$ sa $\Delta$ $ABC$.

*Tógáil:*
Ar $AB$ is $AC$ tóg na cearnóga $BALM, ACNP$. Déan $ME = AC$, $NF = AB$, ionas gur iad $BME$ agus $FNC$ ionaid an $\Delta BAC$ arna chasadh tré 90° timpeall $B$ agus $C$. Ceangail $EF$, $AD$.

*Tátall:*
Tá na cearnóga ar $AB$ is $AC$ le chéile cómhfhairsing leis an gcearnóg ar $BC$.

*Cruthúnas:*
'Sé $BE$ ionad $BC$ arna chasadh tré 90° timpeall $B$.
$\therefore$ Tá $BE = BC$, agus tá $\widehat{CBE} = 90°$.
Mar an gcéanna, tá $CB = CF$, agus tá $BCF = 90°$.
Fágann sin gurb í $BCFE$ an chearnóg ar $BC$, agus de thairbhe $EF$ = agus $\parallel$ le $BC$, $ED\parallel$ le $BA$, $Fd\parallel$ le $CA$, tá an $\Delta DEF$ concrúach leis an $\Delta BAC$ san aistriú ó $B$ go dtí $E$ (nó , ó $C$ go dtí $F$).
Is léir anois go bhfuil
an chearnóg $BCEF =$ an ▱BEDA+ an ▱$CFDA$
$=$ an cear. $BALM +$ an cear. $ACNP$ (Teoirim XV).
.i. Tá $BC^2 = AB^2 + AC^2$. □

### Definition

In a right-angle triangle the *hypotenuse* is the side that is opposite the right angle.

**Theorem 18** (Pythagoras' Theorem). *The square on the hypotenuse of a right-angle triangle is equal to the sum of the squares on the other two sides.*

*Hypothesis:*
$\hat{A}$ is a right angle in the $\Delta$ $ABC$.

*Construction:*

On $AB$ and $AC$ construct the squares $BALM$, $ACNP$. Make $ME = AC$, $NF = AB$, so that $BME$ and $FNC$ are the positions of the $\Delta BAC$ after rotation through 90° around $B$ and $C$. Join $EF$, $AD$.

*Conclusion:*

The squares on $AB$ and $AC$ together have the same area as the square on $BC$.

*Proof:*

$BE$ is the position of $BC$ after rotation through 90° about $B$.

$\therefore$ $BE = BC$, and $\widehat{CBE} = 90°$.

Similarly, $CB = CF$, and $BCF = 90°$.

It follows that $BCFE$ is the square on $BC$, and since $EF$ = and ∥ to $BC$, $ED$∥ to $BA$, $Fd$∥ to $CA$, the $\Delta DEF$ is congruent to the $\Delta BAC$ by the translation from $B$ to $E$ (or, from $C$ to $F$).

It is clear now that

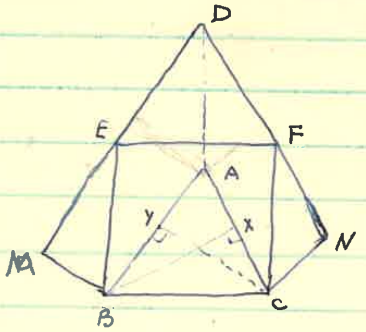
the square $BCEF = \varparallelogram$BEDA$+\varparallelogram CFDA$

= the square $BALM+$ the square $ACNP$ (Theorem 15).

.i. $BC^2 = AB^2 + AC^2$. □

## Nota

Má's géaruille i A san Δ ABC, abair gurb iad BY is CX na h-ingir ó B is C ar na sleasa CA agus AB.




Má 'siad BME agus CNF ionaid na dtriantán dronuilleach BYC agus CXB arna gcasadh tré 90° timpeall B agus C leith ar leith, gheofar amach go bhfuil $BC^2 = BA \times BY + CA \times CX$ $= BA^2 + CA^2 - 2t$

Fágann sin go bhfuil $BC^2$ níos lú ná $AB^2 + AC^2$ sa gcás sin.

Má's maoluille i A áfach, is amhlaí go bhfuil $BC^2$ níos mó ná $AB^2 + AC^2$.

$t = AY \cdot AB = AX \cdot AC$

## Cleachtaithe

1) I gceist 3, teaspáin gur féidir an □ AXYZ a dhealbhú ó'n □ ABCD arna ghearradh i dtrí píosaí. Leis na píosaí sin dealbhuigh □ ar an mbonn BL freisin.

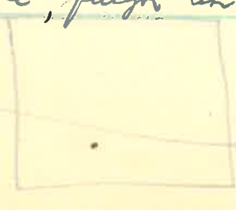
2) I gceist 2 má theagmhaíonn AD is BC le chéile in Z, cruthuigh go bhfuil an Δ BZX = Δ ZDC.

Mínigh cé'n chaoi ina dtarraingítear dronlíne tré phointe ar bith B i slios ZC an triantáin ZDC, a ghníos Δ le BZ agus ZD a bheas comhfhairsing leis an Δ ZDC.

4) Sa Δ ABC pointe ar bith sa slios AB is ea X. Tarraing tré X líne a ghníos dhá leith d'fhairsinge an triantáin.

[Ās a bhaint as ceist 3 sa Δ ABM, áit gurb é M lár BC]

5) I dtriantán dronuilleach 'siad 5" agus 12" na sleasa; faigh an hipoteniús. Má tá hipoteniús triantáin dronuilligh 25" ar fad, agus má tá 7" i slios eile, faigh an tríú slios.



Tosach leathanach 62 sa LSS.

## Nóta

Má's *géaruille* í $A$ san $\Delta\ ABC$, abair gurb iad $BY$ is $CZ$ na hingir ó $B$ is $C$ ar na sleasa $CA$ agus $AB$.

Tá Fíoghair anseo sa LSS, leathanach 62.

Má 'siad $BME$ agus $CNF$ ionaid na dtriantáin dronuilleach $BYC$ agus $CXB$ arna gcasadh tré 90° timpeall $B$ agus $C$ leith ar leith, gheofar amach go bhfuil
$BC^2 = BA \times BY + CA \times CX = BA^2 + CA^2 - 2t$.

Fágann sin go bhfuil $BC^2$ níos lú ná $AB^2 + AC^2$ sa gcás sin.

Má's *maoluille* í $A$, áfach, is amhlaí go bhfuil $BC^2$ níos mó ná $AB^2 + AC^2$.

$t = AY \cdot AB = AX \cdot AC$.

## Note

If $A$ is an *acute angle* in the $\Delta\ ABC$, suppose $BY$ and $CZ$ are the perpendiculars from $B$ and $C$ on the sides $CA$ and $AB$.

If $BME$ and $CNF$ are the positions of the right-angle triangles $BYC$ and $CXB$ after a rotation through 90° aroundl $B$ and $C$, respectively, it will be found that
$BC^2 = BA \times BY + CA \times CX = BA^2 + CA^2 - 2t$.

It follows that $BC^2$ is smaller than $AB^2 + AC^2$ in that case.

If $A$ is an *obtuse angle*, however, it turns out that $BC^2$ is greater than $AB^2 + AC^2$.

$t = AY \cdot AB = AX \cdot AC$.

6) Tá dréimire ina luighe i gcoinne bhalla. Tá bun an dréimire 8' amach ón mballa agus is $31\frac{1}{2}$' suas an balla a shroiseas sé. Coinnitear bun an dréimire socair ach iompaítear go dtí taobh eile na sráide an dréimire féin. Má tá an tsráid 36' ar leithead, cá fhaid suas atá barr an dréimire anois?

7) Sa bhFiogh. i dteoirim XVIII, má theagmhaíonn síneadh DA le BC in X, cruthuigh, (i) go bhfuil AX ⊥ le BC; (ii) $BA^2 = BC.BX$; (iii) $CA^2 = BC.CX$.

8) Má's é FZ an t-ingear ó F ar CA, teastáin (i) gurb é DFZ an cheamóg ar AC; (ii) gurb é FZC ionad ina an △EMB ama aistriú fan BC.

9) Má cuirtear dhá cheamóg pháipéir ABLM agus FDKZ taobh le taoibh, mínigh cé'n chaoi a ndealbhaitear an cheamóg BCFE leo ama ngearradh fan BE agus EF.

10) Má's uimhreacha ar bith iad p agus q, teastain gur triantán dronuilleach é an triantán ar sleasa dho $p^2-q^2$, $2pq$, $p^2+q^2$. Cé ceann sin an hipoteniús?

## Cleachtaithe

1. I gceist 3, teaspáin gur féidir an ▱ $AXYZ$ a dhealbhú ó'n ▱ $ABCD$ arna ghearadh i dtrí píosaí . Leis na píosaí sin dealbhuigh ▱ an an mbonn $BL$ freisin.

2. I gceist 2 má teagmhaíonn $AD$ is $BC$ le chéile in $L$, cruthuigh go bhfuil an $\Delta BZX = \Delta ZDC$.

   Mínigh cé'n chaoi ina dtarraingítear dronlíne tré hointe ar bith $B$ i slios $ZC$ an triantáin $ZDC$, a ghníos $\Delta$ le $BZ$ agus $ZD$ a bhéas comhfhairsing leis an $\Delta$ $ZDC$.

3. [6]

4. Sa $\Delta$ $ABC$ pointe ar bith sa slios $AB$ is ea $X$. Tarraing tré $X$ líne a ghníos dhá leith d'fhairsinge an triantáin.

   [Áis a bhaint as ceist 3 sa $\Delta ABM$, áit gurb é $M$ lár $BC$]

5. I dtriantán dronuilleach 'siad 5" agus 12" na sleasa; faigh an hipotenús. Má tá hipotenús triantáin dronuilligh 25" ar fad, agus mátá 7" is slios eile, faigh an tríú slios.

   Tosach leathanach 63 sa LSS.

6. Tá dréimire ina luí i gcoinne bhalla. Tá bun an dréimire $8'$ amach ón mballa agus is $31\frac{1}{2}'$ suas an balla a shroiseas sé.

   Coinnítear bun an dréimire socair ach iompaítear go dtí taobh eile na sráide an dréimire féin. Má tá an tsráid $36'$ ar leithead, cá fhaid suas atá barr an dréimire anois?

7. Sa bhFiog i dTeoirim XVIII, má teagmhaíonn síneadh $DA$ le $BC$ in $X$, cruthuigh, (i) go bhfuil $AX \perp$ le $BC$; (ii) $BA^2 = BC \cdot BX$; (iii) $CA^2 = BC \cdot CX$.

8. Má's é $FZ$ an t-ingear ó $F$ ar $CL$, teaspáin (i) gurb í $LDFZ$ an chearnóg ar $AC$; (ii) gurb é $FZC$ ionad nua an $\Delta EMB$ arna aistriú fan $BC$.

9. Má cuirtear dhá chearnóg pháipéir $ABLM$ agus $FDLZ$[7] taobh le taoibh, mínigh cé'n chaoi a ndealbhaítear an chaernóg $BCFE$ leo arna ngearradh fan $BE$ agus $EF$.

10. Má's uimhreacha ar bith iad $p$ agus $q$, $p > q$, teaspáin gur traintán dronuilleach é an triantán ar sleasa dhó $p^2 - q^2, 2pq, p^2 + q^2$. Cé acu sin an hipotenús?

[6] Níl aon uimhir a 3 ann.
[7] LSS deacair a léamh (AOF)

## Exercises

1. In Question 3, show that it is possible to assemble the $\varparallelogram\ AXYZ$ from pieces of $\varparallelogram\ ABCD$ by cutting it into three pieces. Using those pieces, assemble a $\varparallelogram$ on the base $BL$, too.

2. In Question 2, if $AD$ and $BC$ meet at $L$, prove that $\Delta BZX = \Delta ZDC$.

   Explain how to draw a straight line through any given point $B$ in the side $ZC$ of the triangle $ZDC$, which will make a $\Delta$ with $BZ$ agus $ZD$ having the same area as the $\Delta\ ZDC$.

3. [8]

4. In the $\Delta\ ABC$, $X$ is an arbitrary point in the side $AB$. iDraw a line through $X$ that divides the area of the triangle into two halves.

   [Apply Question 3 to the $\Delta ABM$, where $M$ is the centre of $BC$]

5. In a right-angle triangle the sides are 5" agus 12"; Find the hypotenuse. If the hypotenuse of a right-angle triangle is 25" long, and another side is 7", find the third side.

6. A ladder is standing against a wall. The foot of the ladder is $8'$ out from the wall, and it reaches $31\frac{1}{2}'$ up the wall.

   Without moving the bottom of the ladder it is swung over to the other side of the street. If the street is $36'$ wide, how far up is the top of the ladder now?

7. In the figure in Theorem 18, if the extension of $DA$ meets $BC$ at $X$, prove (i) that $AX \perp$ to $BC$; (ii) $BA^2 = BC \cdot BX$; (iii) $CA^2 = BC \cdot CX$.

8. If $FZ$ is the perpendicular from $F$ on $CL$, show (i) that $LDFZ$ is the square on $AC$; (ii) that $FZC$ is the position to which $\Delta EMB$ is moved by the translation along $BC$.

9. If two paper squares $ABLM$ and $FDLZ$[9] are placed side by side, explain how to construct the square $BCFE$ from them by cutting along $BE$ and $EF$.

10. If $p$ and $q$ are any two numbers such that $p > q$, show that a triangle with sides $p^2 - q^2, 2pq, p^2 + q^2$ is right-angled. Which side is the hypothenuse?

[8] There is no exercise number 3.
[9] LSS deacair a léamh (AOF)

(8)

# Caibidil VI

## Cóimhionannas Triantán. Éagcudromóidí

Cé go bhfuil trí sleasa agus trí h-uilleacha i dtriantán, ní gá na neithe sin go léir a thabhairt chun méad agus deilbh an triantáin a chinneadh: e.g. is leor dhá uillinn a bheith ar eolas chun an ceann eile a áireamh, de bhrí go bhfuil 180° sna trí h-uilleacha le chéile. Teaspáinfear sa gcaibidil seo gur leor cnuais áirithe de thrí bhaill den triantán a thabhairt chun gur cinnteach don triantán idir mhéad is deilbh.

Tá fiogracha géométreacha cóimhionann le chéile (nó congrúach) má's feidir ceann acu a leagan anuas ar an gceann eile le h-aistriú, nó le casadh, nó le scáthú.

### Teoirim ~~XVIII~~ XIX

Chun go mbeadh dhá thriantán congrúach le chéile, is leor:

I go mbeadh dhá shlios i dtriantán acu cómhfhada le dhá shlios sa gceann eile, agus na h-uilleacha a chríoslaíonn gach péire acu sin a bheith ar cómhméad.

Nó,

II go mbeadh na trí sleasa i dtriantán acu cómhfhada le trí sleasa an triantáin eile.

Nó

III go mbeadh dhá uillinn i dtriantán acu cothrom le dhá uillinn sa triantán eile, agus slios amháin sa gcéad triantán a bheith cómhfhada leis an slios a fhreagraíos dó sa gceann eile.

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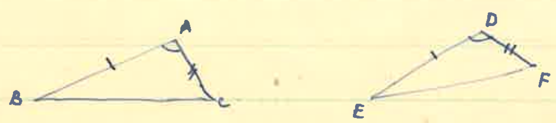



Cás I

Hipotéis Tá AB = DE, AC = DF, $B\hat{A}C = E\hat{D}F$.

Tátall Triantáin chongrúacha is ea iad.

# Caibidil 6

# Cóimhionannas Triantán. Éagcudromóidí

Tosach leathanach 64 sa LSS.

## Identity of Triangles. Inequalities

Cé go bhfuil trí sleasa agus trí h-uilleacha i dtriantán, ní gá na neithe sin go léir a thabhairt chun méid agus deilbh an triantán a chinneadh: e.g. is leor dhá uillinn a bheith ar eolas chun an ceann eile a áireamh, de bhrí go bhfuil 180° sna trí h-uilleacha le chéile. Teaspáinfear sa gcaibidil seo gur leor cnuas áirithe de thrí bhaill den triantán a thabhairt chun gur cinnteach don triantán idir mhéad agus deilbh.

Tá fiogracha geómétracha cómhioann le chéile (nó congrúach) má's féidir ceann acu a leagan amach ar an gceann eile le h-aistriú , nó le casadh, nó le scáthú.

Even though a triangle has three sides and three angles, you don't have to give all those in order to determine the size and shape of the triangle: e.g. it is enough to know two of the angles in order to calculate the other one, since there are 180° in the three angles together. In this chapter it will be shown that to determine a triangle's size and shape it is enough to specify any one of a particular collection of triples of elements of the triangle.

Two geometrical figures are identical with one another (or congruent) if it is possible to lay one of them on the other by means of a translation, or a rotation, or a reflection.

Cruthúnas

Má cuirtear DE fan AB conas go dtuiteann D ar A, is ar an bpointe B a thuiteas E de bhrí go bhfuil DE = AB.

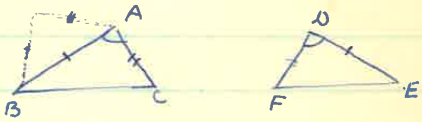
Ó tharla $\hat{B} = \hat{A}$ leagtar DF fan AC; agus de thairbhe DF = AC, is ar an bpointe C a cuirtear F.

∴ Is anuas go cruinn ar an Δ ABC a leagtar an Δ DEF.

Q.E.D.

Nota 1. Fágann sin go bhfuil $\hat{E} = \hat{B}$, $\hat{F} = \hat{C}$, EF = BC.

Nota 2. Má tá na h-uilleacha cothroma BÂC agus EDF i dtreónna contrárdha, is soiléir gur anuas ar scáth an Δ ABC sa slios AB a leagtar an Δ DEF.




Tabhair fá deara gurb iad na sleasa atá ós cóir na n-uilleann gcothrom atá cómhfhada, agus gurb iad na h-uilleacha atá ar aghaidh na slios gcómhfhada atá ar cómhméad. Sin é is ciall le h-uilleacha freagarthacha agus le sleasa freagarthacha.

Cás II

Hipotéis Tá AB = DE, AC = DF, BC = EF.

Tátal Δ chongrúacha iad ABC agus DEF


Cruthúnas

Leag an ~~Δ DEF~~ slios EF ar an slios cómhfhada BC i gcaoi go gcuirtear E ar C, agus go gcuirtear F ar B.

Abair gurb é XCB an t-ionad nua a ghabhas an Δ DEF.

Fágann sin BX = AC, XC = AB, conas gur ▱ é BACX.

De thairbhe teóirme XV ansin is féidir an Δ XCB a leagan anuas ar an Δ ABC le casadh trí 180° timpeall lár BC.

∴ Tá Δ DEF congrúach le Δ XCB, atá congrúach le Δ ABC.

Q.E.D.

## 6.1 Teoirim XIX

*Chun go mbeadh dhá thriantán congrúach le chéile, is leor:*
***I*** *go mbeadh dhá shlios i dtriantán acu cómhfhada le dhá shlios sa gceann eile, agus na huilleacha achrioslaíonn gach péire acu sin a bheith ar cómhméad.*
*Nó*
***II*** *go mbeadh na trí sleasa i dtriantán acu cómhfhada le trí sleasa an triantáin eile.*
*Nó*
***III*** *go mbeadh dhá uillinn i dtriantán acu cothrom le dhá uillinn sa triantán eile, agus slios amháin sa gcéad triantán a bheith cómhfhada leis an slios a fhreagraíos dó sa gceann eile.*

Tá Fíoghair anseo sa LSS, leathanach 64.

### Cás I

*Hipotéis:*
Tá $AB = DE$, $AC = DF$, $\widehat{BAC} = \widehat{EDF}$.

*Tátall:*
Triantáin congrúacha is ea iad.

Tosach leathanach 65 sa LSS.

*Cruthúnas:*
Má cuirtear $DE$ fan $AB$ ionas go dtiteann $D$ ar $A$, is ar an bpointe $B$ a thiteas $E$ de bhrí go bhfuil $DE = AB$.
Ó thárla $\hat{D} = \hat{A}$ leagtar $DF$ fan $AC$; agus de thairbhe $DF = AC$, is ar an bpointe $C$ a cuirtear $F$.
∴ Is anuas go cruinn ar an $\Delta\ ABC$ a leagtar an $\Delta\ DEF$. □

**Theorem 19.** *For two triangles to be congruent, it is enough:*
***I*** *That two sides in one triangle be the same length as two sides in the other one, and the angles embraced by each pair be the same size.*
*Or*
***II*** *that the three sides in one of the triangles be the same length as the three sides of the other triangle.*
*Or*
***III*** *that two angles in one of the triangles be equal to two angles in the other triangle, and one side in the first triangle the same length as the corrersponding side in the other one.*

### Case I

*Hypothesis:*
$AB = DE$, $AC = DF$, $\widehat{BAC} = \widehat{EDF}$.

*Conclusion:*

The triangles are congruent.

*Proof:*

If we place $DE$ along $AB$ so that $D$ falls on $A$, then the point $B$ iis where $E$ falls, since $DE = AB$.

Since $\hat{D} = \hat{A}$, $DF$ lies along $AC$; and since $DF = AC$, the point $C$ iis where $F$ is placed.

∴ It is exactly on top of the $\Delta$ $ABC$ that the $\Delta$ $DEF$ is laid. □

## Nóta 1

Fágann sin go bhfuil $\hat{E} = \hat{B}$, $\hat{F} = \hat{C}$, $EF = BC$.

## Nóta 2

Má tá na huilleacha cothoma $\widehat{BAC}$ agus $\widehat{EOF}$ is dtreóanna contrárdha, is soiléir gur anuas ar scáth an $\Delta$ $ABC$ sa slios $AB$ a leagtar an $\Delta DEF$.

Tá Fíoghair anseo sa LSS, leathanach 65.

Tabhair fá deara gurb iad na sleasa atá ós cóir na n-uilleann gcothrom atá cómhfhada, agus gurb iad na huilleacha atá ar aghaidh na slios gcómhfhada atá ar cómhméad. Sin é is ciall le *huilleacha freagarthacha* agus le sleasa freagathacha.

## Note 1

It follows that $\hat{E} = \hat{B}$, $\hat{F} = \hat{C}$, $EF = BC$.

## Note 2

If the equal angles $\widehat{BAC}$ and $\widehat{EOF}$ are in contrary directions, it is clear that the $\Delta DEF$ is laid on the reflection of $ABC$ in the side $AB$.

Notice that it is the sides opposite the equal angles that are the same length, and that the angles opposite the equal sides that are the siame size. That is what we mean by *corresponding angles* and by corresponding sides.

## Cás II

*Hipotéis:*

Tá $AB = DE$, $AC = DF$, $BC = EF$.

*Tátall:*

Triantáin congrúacha is ea iad.

Tá Fíoghair anseo sa LSS, leathanach 65.

*Cruthúnas:*

Leag an slios $EF$ ar an slios cómhfhada $BC$ i gcaoi go gcuirtear $E$ ar $C$, agus go gcuirtear $F$ ar $B$.

Abair gurb é $XCB$ an t-ionad nua a ghabhas an $\Delta$ $DEF$.

Fágann sin $BX = AC$, $XC = AB$, ionas gur ▱ é $BACX$.

De thairbhe teoirme XV ansin is féidir an $\Delta XCB$ a leagan anuas ar an $\Delta$ $ABC$ le casadh tré 180° timpeall lár $BC$.

.i. Tá $\Delta$ $DEF$ congrúach lr $\Delta$ $XCB$, atá congrúach le $\Delta$ $ABC$. □

## Case II

*Hypothesis:*

$AB = DE$, $AC = DF$, $BC = EF$.

*Conclusion:*

They are congruent triangles.

*Proof:*

Lay the side $EF$ on the equally-long side $BC$ in such a way that $E$ is placed on $C$, and $F$ on $B$.

Suppose that $XCB$ is the new location of the $\Delta$ $DEF$.

It follows that $BX = AC$, $XC = AB$, so that $BACX$ is a ▱.

Then by Theorem 15 the $\Delta XCB$ can be laid on the $\Delta$ $ABC$ by a rotation through 180° about the centre of $BC$.

.i. $\Delta$ $DEF$ is congruent to $\Delta$ $XCB$, which is congruent to $\Delta$ $ABC$. □

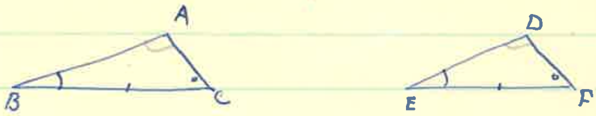
**Nóta** Má tá treóanna na n-uilleann bhfreagarthach $B\hat{A}C$ agus $E\hat{D}F$ contrárdha dá chéile, gheofar amach gur anuas ar scáth an Δ ABC san dronlíne BC a leagtar an Δ DEF.

**Cás III**

**Hipotesis** Tá $\hat{B} = \hat{E}$, $\hat{C} = \hat{F}$, BC = EF.

**Tátal** Δ chongrúacha iad ABC agus DEF.

~~bruth~~




**Cruthúnas**

Má cuirtear EF fan na líne BC ionnas go bhfuil E ar B, is ar an bpointe C a thuiteas F de bhrí go bhfuil EF = BC.

Ó tharla $\hat{B} = \hat{E}$ is fan na dronlíne BA a leagtar ED, agus mar an gcéanna cuirtear FD fan na dronlíne CA.

Fágann sin go leagtar D, pointe teagmhála ED is FD ar phointe theagmhála BA is CA; ∴ ar A.

∴ Leagtar an Δ DEF ar an Δ ABC.

Q.E.D.

**Nóta 1.** Má 's é $\hat{B} = \hat{E}$, $\hat{A} = \hat{D}$, BC = EF a tugtar 's é an cás céanna é, de bhrí go gcaithfidh $\hat{C} = \hat{F}$ ansin de bharr teóirme XIII.

**Nóta 2** Is féidir triantán a thógáil nuair is eol dúinn, (i) dhá shlios agus an uille a chrioslaíonn siad, nó (ii) na trí sleasa, nó (iii) dhá uillinn agus slios.

Mar, ar dhronlíne ar bith sa bhplána lig linn mír BC a ghearradh atá cómhfhada le slios ar bith a tugtar, agus is féidir an triantán a thógáil ar BC ar bhealach a bheas soiléir don léitheoir gan míniú ar bith. Mar gheall ar an éigcinnteacht fá ionad BC níl aon chuimse lena bhfuil de thriantáin éagsúla a chóimhlíonas na coinníollacha, ach is macasamhla d'á chéile iad uilig de barr na teoirme.

Tosach leathanach 66 sa LSS.

### Nóta

Má tá treóanna na n-uilleann bhfreagarthach $\widehat{BAC}$ agus $\widehat{EDFi}$ contrárdha dá chéile, gheofar amach gur ar scáth an $\Delta\ ABC$ san dronlíne $BC$ a leagtar $\Delta\ DEF$.

### Note

If the direction of the corresponding angles $\widehat{BAC}$ and $\widehat{EDFi}$ are contrary to one another, it will be found that $\Delta\ DEF$ is laid on the reflection of the $\Delta\ ABC$ in the straight line $BC$.

### Cás III

*Hipotéis:*

Tá $\hat{B} = \hat{E}$, $\hat{C} = \hat{F}$, $BC = EF$.

*Tátall:*

Triantáin congrúacha is ea iad.

Tá Fíoghair anseo sa LSS, leathanach 66.

*Cruthúnas:*

Má cuirtear $EF$ fan na líne $BC$ ionas go bhfuil $E$ ar $B$, is ar an bpointe $C$ a thuiteas $F$ de bhrí go bhfuil $EF = BC$.

Ó thárla $\hat{B} = \hat{E}$ is fan na dronlíne $BA$ a leagtar $ED$, agus mar an dcéanna cuirtear $FD$ fan na dronlíne $CA$.

Fágann sin go leagtar $D$, pointe teagmhála $ED$ is $FD$, ar phointe teagmhála $BA$ is $CA$; .i. ar $A$.

∴ Leagtar an $\Delta\ DEF$ ar an $\Delta\ ABC$. □

### Case III

*Hypothesis:*

$\hat{B} = \hat{E}$, $\hat{C} = \hat{F}$, $BC = EF$.

*Conclusion:*

The triangles are congruent.

*Proof:*

If we place $EF$ along the line $BC$ so that $E$ is on $B$, then $F$ falls on the point $C$, because $EF = BC$.

Since $\hat{B} = \hat{E}$, $ED$, is laid along the straight line $BA$, and in the same way $FD$ is placed along the straight line $CA$.

It follows that $D$, the point where $ED$ meets $FD$, is placed on the point where $BA$ meets $CA$; .i. on $A$.

∴ The $\Delta\ DEF$ is laid on the $\Delta\ ABC$. □

## Nóta 1

Má sé $\hat{B} = \hat{E}$, $\hat{A} = \hat{D}$, $BC = EF$ a tugtar, sé an cás céanna é , de bhrí go gcaithfidh $\hat{C} = \hat{F}$ ansin de bharr teoirme XIII.

## Nóta 2

Is féidir triantán a thógáil nuair is eol dúinn, (i) dhá shlios agus an uille a chrioslaíonn siad, *nó* (ii) na trí sleasa, *nó* (iii) dhá uillinn agus slios.

Mar, ar dhronlíne ar bith sa phlána tig linn mír $BC$ a ghearradh atá cómhfhada le slios ar bith a tugtar, agus is féidir an triantán a thógáil ar $BC$ ar bhealach a bheas soiléir don léitheoir gan míniú ar bith. Mar gheall ar an éigcinnteacht fá ionad $BC$ níl aon chuimse lena bhfuil de thriantáin éagsúla a chóimhlíonas na coinníollacha, ach is macasamhla d'á chéile iad uilig de bharr na teoirme..

## Note 1

If we are given that $\hat{B} = \hat{E}$, $\hat{A} = \hat{D}$, $BC = EF$, then it is the same situation, since it must follow that $\hat{C} = \hat{F}$ becasue of Theorem 8.

## Note 2

A triangle can be constructed when we know (i) two sides and the angle between them, *or* (ii) the three sides *or* (iii) two angles and a side.

For , on any straight line in the plane we can cut a segment $BC$ that has the same length as any given side, and then it is possible to construct the triangle on $BC$ is an way that should be obvious to the reader without any explanation. Because the position of $BC$ is arbitrary, there is no limit to the number of different triangles that satisfy the conditions, but they are all facsimiles of one another because of the theorem.

Cleachtaithe

1) Tarraing Δ ar sleasa dhó 3 ór. agus $2\frac{1}{2}$ ór., agus 30° a bheith san uillinn eatorru.

2) Tarraing Δ ar sleasa dhó $1\frac{1}{2}$", 2", $2\frac{1}{2}$". Cad í an uille atá ós cóir an tsleasa is fuide? Cad í an uille is lú?

3) Tarraing Δ fá na h-uilleacha 40°, 60°, 80° nuair is é 3" fad an tsleasa atá ós cóir na h-uilleann is mó.

Triantáin i gcóimhréir.

In áit a bheith cómhfhada, má's i gcóimhréir atá na sleasa a luaitear sa teoirim, tá uilleacha triantáin acu ar cómhméad le h-uilleacha an triantáin eile, ach is i gcóimhréir (de réir an chóimhmeasa chéanna k:1, abair) atá na sleasa. Fágann sin go bhfuil Δ acu ina chóip de réir an scála k:1 den cheann eile.

Gheofar chruthúnas don tora úd i gcaib. IX, ach ní miste é a luadh annseo mar gheall ar a thábhachtaí is atá sé. Is iomdha leas a bhaineas suirbhéara as.

Cleachtaithe

1) Tá 100 tr., 72 tr., agus 80 tr. i sleasa Δ áirithe. Línigh an Δ de réir an scála 40 tr. in aghaidh an órlaigh, agus tomhas an uille is mó.

2) Crann bratáigh isea XY, agus pointe ar an talamh cothrom isea P i gcaoi go bhfuil 30 sl. idir P agus bun an chrainn bhratáigh X. Áirítear 20° don uillinn XPY. Línigh an Δ dronuilleach PXY de réir an scála 15 sl. in aghaidh 1 ór., agus faigh áirde XY.

3) Trí bailte isea A, B, C. Tá B 20 míle thiar ó A go díreach, ach tá C thiar ó thuaidh de A agus é 35 míle uaidh. Aimsigh fad agus treo na líne BC.

4) Chuaidh duine 10 míle soir ó phointe A. Tar éis dó 6 mhíle soir ó thuaidh a chur de ina dhiaidh sin d'athrúigh sé (a) treo arís agus chuaidh sé 5 mhíle siar. Cá fhaide ó bhaile atá sé anois?

Tosach leathanach 67 sa LSS.

## Cleachtaithe

1. Tarraing Δ ar sleasa dhó 3 ór. agus $2\frac{1}{2}$ ór., agus 30° a bheith san uillinn eatorru.
2. Tarraing Δ ar stuanna dhó $1\frac{1}{2}$", 2", $2\frac{1}{2}$". Cad é an uille atá ós cóir an tsleasa is fuide? Cad é an uille is lú?
3. Tarraing Δ fá na h-uilleacha 40°, 60°, 80° nuair 'sé 3" fad an tsleasa atá ós cóir na huilleann is mó.

## Exercises

1. Draw a Δ having sides 3 in. and $2\frac{1}{2}$ in., and with 30° in the angle between them.
2. Draw a Δ with sides $1\frac{1}{2}$", 2", $2\frac{1}{2}$". What is the angle opposite the longest side? What is the least angle?
3. Draw a Δ with angles 40°, 60°, 80° where the side opposite the greatest angle is 3" long.

## Tríantáin i gCóimhréir

In áit a bheith cómhfhada, má's i gcóimhréir atá na sleasa a luaitear sa teoirim, tá uilleacha triantáin acu ar cómhméad le h-uilleacha an triantáin eile, ach is i gcóimhréir (de réir an chóimhmeasa chéanna $k:1$, abair) atá na sleasa. Fágann sin go bhfuil Δ acu ina chóip de réir an scála $k:1$ den cheann eile.

Gheofar chruthúnas don tora úd i gcaib IX[1], ach ní miste é a lúa anseo mar gheall ar a thábhachtaí is atá sé . Is iomdha leas a bhaineas *silbhe*ára as.

## Triangles in Proportion

If, instead of having the same length, the sides mentioned in the theorem are in proportion, then the angles in one triangle are the same size as the angles in the other triangle, but all the sides are in proportion (according to the same ratio $k:1$, say). It follows that one triangle is a scale copy of the other in the ratio $k:1$.

You will find the proof of that result in Chapter 9 [2], but it is as well to mention it here because of its importance.

Surveyers make much use of it.

[1] nár ann dó san LSS

[2] This refers to a non-existent chapter in the MSS.

5) Pólla telegrafa isea AB, agus ó phointe X ar an mbóthar tuglar fá deara go bhfuil $A\hat{X}B = 30°$. Ag pointe eile Y ar an mbóthar atá níos goire don phólla agus é 30 tr. ó X, tá 50° san uillinn $A\hat{Y}B$. Faigh áirde an phólla go h-athchomair le léaráid a tarraingítear de réir an scála 20 tr. in aghaidh 1".

6) Tá dhá long X is Y ar farraige, agus pointí ar an gcladach isea A is B ionnus go bhfuil B míle bealaigh thoir ó A go díreach. Ag an bpointe céanna tuglar fá deara go bhfuil $X\hat{B}A = 60°$, $Y\hat{B}A = 40°$, $Y\hat{A}B = 80°$, $X\hat{A}B = 45°$.

Faigh fad agus treo na líne ~~AB~~ XY.

## An Cás Dá-bhrítheach

7) Má tá dhá shlios agus uille (atá ós cóir sleasa acu ~~nach í an uille a chrioslaíonn an péire~~) i dtriantán cothrom le dhá shlios agus leis an uillinn fhreagarthaí i dtriantán eile, ní gá don dá thriantán sin a bheith congrúach, cé go bhféadfadh sé sin a thárlú.

Mara bhfuil siad congrúach áfach, cruthuigh de thairbhe na léaráide thíos gur uilleacha fórlíontacha iad na h-uilleacha atá ós cóir an dá shlios chómhfhada eile.


8) Sa gcás dá-bhrítheach má's eol nach bhfuil maoluille i dtriantán ar bith den phéire, cruthuigh go bhfuil na Δ congrúach le chéile.

9) Má's dronuilleacha iad na h-uilleacha cothroma a luaitear i gceist 7, cruthuigh go bhfuil na triantáin congrúach.

## Cleachtaithe

1. Tá 100 tr., 72 tr., agus 80 tr. i sleasa $\Delta$ áirithe. Línigh an $\Delta$ de réir an scála 40 tr. in aghaidh an órlaigh, agus tomhas an uille is mó.

2. Crann brataigh is ea $XY$, agus pointe ar an talamh cothrom is ea $P$ i gcaoi go bhfuil 30 sl. idir $P$ agus bun an chrainn bhrataigh $X$. Áirítear 20° don uillinn $\widehat{XPY}$. Línigh an $\Delta$ dronuilleach $PXY$ de réir an scála 15 sl. in aghaidh 1 ór., agus faigh áirde $XY$.

3. Trí bailte is ea $A, B, C$. Tá $B$ 20 míle thiar ó $A$ go díreach, agus tá $C$ thiar ó thuaidh de $A$ agus é 30 mhíle uaidh. Aimsigh fad agus treo na líne $BC$.

4. Chuaidh duine 10 míle soir ó phointe $A$. Tar éis dó 6 mhíle soir ó thuaidh a chur de ina dhiaidh sin d'athruigh sé a threo arís agus chuaidh sé 5 mhíle siar. Cá fhaide ó bhaile atá sé anois?

   Tosach leathanach 68 sa LSS.

5. Pólla telegrafa is ea $AB$, agus ó phointe $X$ ar an mbóthar tugtar fá deara go bhfuil $\widehat{AXB} = 30°$. Ag pointe eile $Y$ ar an mbóthat atá níos goire don phólla agus é 30 tr. ó $X$, tá 50° san uillinn $\widehat{AYB}$. Faigh áirde an phólla go h-athchomair le léaráid a tarraingítear de réir an scála 20 tr. in aghaidh 1".

6. Tá dhá loing $X$ is $Y$ ar farraige, agus pointí ar an gcladach is ea $A$ is $B$ ionas go bhfuil $B$ míle bealaigh thoir ó $A$ go díreach. Ag an bpointe ama céanna tugtar fá deara go bhfuil $\widehat{XBA} = 60°$, $\widehat{YBA} = 40°$, $\widehat{YAB} = 80°$, $\widehat{XAB} = 45°$.

   Faigh fad agus treo na líne $XY$.

### An Cás Dá-bhrítheach

7. Má tá dhá shlios agus uille (atá ós cóir shleasa acu) i dtriantán cothrom le dhá shlios agus leis an uillinn fhreagarchaí i dtriantán eile, ní gá don dá thriantán sin a bheith congrúach, cé go bhféadfadh sé sin a thárlú.

   Mora bhfuil siad congrúach áfach, cruthuigh de thairbhe na léaráide thíos gur uilleacha fóirlíontacha iad na h-uilleacha atá ós cóir an dá shlios chómhfhada eile.

   Tá Fíoghair anseo sa LSS, leathanach 68.

8. Sa gcásdá-bhrítheach má's eol nach bhfuil maoluille i dtriantán ar bith den péire, cruthuigh go bhfuil na $\Delta$ congúach le chéile.

9. Má's dronuilleacha iad na h-uilleacha cothroma a lúaitear i gceist 7, cruthuigh go bhfuil na triantáin congrúach.

## Exercises

1. The sides of a particular triangle have 100 ft., 72 ft., agus 80 ft.. Draw the triangle to the scale of 40 ft. to the inch, and measure the larget angle.

2. $XY$ is a flagpole, and $P$ is a point on the level ground such that there are 30 yd. between $P$ and the bottom $X$ of the flagpole. The angle $\widehat{XPY}$ is reckoned at 20°. Draw the right-angle $\Delta\, PXY$ at a scale of 15 yards to 1 inch, and find the height of $XY$.

3. $A, B, C$ are three towns. $B$ is 20 miles directly west of $A$, and $C$ is northwest of $A$ and 30 miles away from it. Find the length and direction of the line $BC$.

4. A person went 10 miles east from point $A$. After travelling a further 6 miles north-east he changed direction again and went 5 miles west. How far away from home is he now?

5. $AB$ is a telegraph pole, and from a point $X$ on the road it is observed that $\widehat{AXB} = 30°$. At another point $Y$ on the road, closer to the pole and 30 ft. from $X$, the angle $\widehat{AYB}$. neasures 50°. Find the height of the pole approximately by drawing a diagram to the scale of 20 ft. to the inch.

6. Two ships $X$ and $Y$ are at sea, and $A$ and $B$ are points on the shore, with $B$ a mile due east from $A$. At a certain moment it is observed that $\widehat{XBA} = 60°$, $\widehat{YBA} = 40°$, $\widehat{YAB} = 80°$, $\widehat{XAB} = 45°$.

   Find the length and direction of the line $XY$.

### The Ambiguous Case

7. If two sides and an angle (that is opposite one of them) in a triangle are equal to two sides and the corresponding angle in another triangle, it is not necessarily the case that those two traingles are congruent, even though that might be the case.

   If, however, they are not congruent, prove using the diagram below that the angles opposite the other two equal sides are complementary.

8. In the ambiguous case, if it is given that neither of the pair of triangles has an obtuse angle, prove that the $\Delta$ are congruent to one another.

9. If the equal angles mentioned on exercise 7 are right angles, prove that the triangles are congruent.

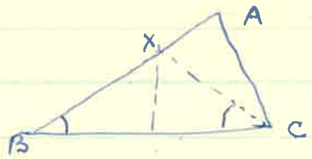
## Éagcudromóidí

San alt seo breithnítear éagcudromóidí áirithe a shíolraíos den tsonrú gurb í an dronlíne an fhad is giorra idir dhá phointe.

Scríobhtar > in áit "níos mó ná...", agus < in áit "níos lú ná..."

### Teoirim ~~XIX~~ XX

I dtriantán ar bith má's mó uille áirithe ná uille eile, is faide an slios atá ar aghaidh na h-uilleann is mó ná an slios atá ar aghaidh na h-uilleann eile; agus, is fíor an coinvéarsa.



Hipotesis  Tugtar $\hat{C} > B$.

Tátall  Tá $AB > AC$.

Tógáil  Déan $B\hat{C}X = \hat{B}$.

Cruthúnas

Tá $CX + XA > AC$ (sonrú dronlíne): agus, tá $CX = BX$ (teoirim V)

$\therefore$ Tá $BX + XA > AC$;  $\therefore$ $AB > AC$

Q.E.D.

Nóta  Is ionann le chéile a rá go bhfuil $\hat{C} > \hat{B}$, nó go bhfuil A agus C ar aon taobh amháin de ais shuiméitreachta BC.

Bhéarfar cruthúnas neamhdhíreach i gcóir an choinvéarsa.

An Coinvéarsa

Hipotesis  Tugtar $AB > AC$

Tátall  Tá $\hat{C} > \hat{B}$

Cruthúnas  Ní fhéadfadh A a bheith ar a.s. BC, gan AB a bheith = AC.

Ní fhéadfadh A is B a bheith ar an taobh chéanna de a.s. BC gan AC a bheith > AB (de réir na teoirime)

Ach ní thagann tátall acu súd leis an hipotesis $AB > AC$.

$\therefore$ Is ar aon taobh amháin le C atá A  $\therefore$ $\hat{C} > \hat{B}$  Q.E.D

Aliora  I dtriantán dronuilleach 'sé an hipotenúise an slios is faide.

Tosach leathanach 69 sa LSS.

## 6.2 Éagcudromóidí

San alt seo breithnítear éagcudromóidí áirithe a shíolraíos den tsonnrú gurb í an dronlíne an fhad is goire idir dhá phointe.

Scríobhtar > is áit “níos mó ná", agus < in áit “níos lú ná".

## 6.3 Inequalities

In this section we shall study inequalities that follow from the fact that the straight line is the shortest distance between two points.

We write > for “greater than" and < for “less than".

## 6.4 Teoirim XX

*I dtriantán ar bith má's mó uille áirithe ná uille eile, is fuide an slios atá ar aghaidh na h-uilleann is mó ná an slios atá ar aghaidh ne h-uilleann eile; agus is fíor an coinvéarsa.*

Tá Fíoghair anseo sa LSS, leathanach 69.

*Hipotéis:*
Tugtar $\hat{C} > \hat{B}$.

*Tátall:*
Tá $AB > AC$.

*Tógáil:*
Déan $\widehat{BCX} = \hat{B}$.

*Cruthúnas:*
Tá $CX + XA > AC$ (sonnrí dronlíne); agus tá $CX = BX$ (teoirim V).
∴ Tá $BX + XA > AC$; ∴ $AB > AC$. □

Bhéarfar crutúnas neamhdíreach i gcóir an choinvéarsa.

### An Coinvéarsa

*Hipotéis:*
Tugtar $AB > AC$.

*Tátall:*
Tá $\hat{C} > \hat{B}$.

*Cruthúnas:*
Ní fhéadfadh $A$ a bheith ar a.s. $BC$, gan $AB$ a bheith $= AC$.

Ní fhéadfadh $A$ is $B$ a bheith ar an taobh chéanna de a.s. $BC$ gan $AC$ a bheith $> AB$ (de réir na teoirme).

Ach ní thagann tátall acu siúd leis an hipotesis $AB > AC$.

$\therefore$ Is ar aon taobh amháin le $C$ atá $A$ .i. $\hat{C} > \hat{B}$. □

## Atora

*I dtriantán dronuilleach 'sé an hipotenúse an slios is faide.*

**Theorem 20.** *In any triangle if a particular angle is greater than another angle, then the side opposite the greater angle is longer than the side opposite the other angle; and the converse is also true.*

*Hypothesis:*

We are given $\hat{C} > \hat{B}$.

*Conclusion:*

$AB > AC$.

*Construction:*

Make $\widehat{BCX} = \hat{B}$.

*Proof:*

We have $CX + XA > AC$ (definition of a line); and $CX = BX$ (Theorem 5).

$\therefore BX + XA > AC$; $\therefore AB > AC$. □

We shall give an indirect proof for the converse.

## The Converse

*Hypothesis:*

We are given $AB > AC$.

*Conclusion:*

$\hat{C} > \hat{B}$.

*Proof:*

$A$ could not lie on the a.s. of $BC$, unless $AB$ is $= AC$.

$A$ and $B$ could not be on the sme side of the a.s. of $BC$ without $AC$ being $> AB$ (by the theorem).

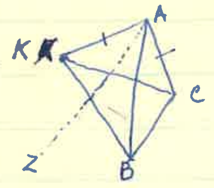
But neither of these conclusions agree with the hypothesis $AB > AC$.

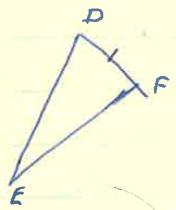
$\therefore A$ lies on the same side as $C$ .i. $\hat{C} > \hat{B}$. □

**Corollary.** *In a right-angle triangle the hypotenuse is the longest side.*

## Teoirim XX

Tá dhá shlios i dtriantán comhfhada le dhá shlios i dtriantán eile, ach is mó an uille a chrioslaíonn an chéad phéire ná an uille a chrioslaíonn an péire eile. ~~Is faide bonn~~ Is é an triantáin gurb ann atá an uille is mó an triantán is fáide bonn.




Hipotesis Tá AB = DE, AC = DF, $E\hat{D}F > B\hat{A}C$.

Tátall Tá EF > BC.

Cruthúnas Leag an slios DE ar an slios comhfhada AB, agus le seáthú in AB (má's gá é), cuir △DEF san ionad ABK.

Fágann sin AK = AC, KB = EF, $K\hat{A}B > B\hat{A}C$.

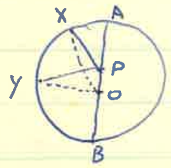
Ó tharla $K\hat{A}B > B\hat{A}C$, tá AZ comhroinnteóir na huilleann KAC, idir AK agus AB.

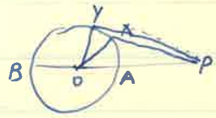
Ach sé AZ a.s. na líne KC (teoirim IV), agus de bhrí go bhfuil B agus C ar an taobh chéanna dhí, tá BK > BC (teoirim XIX)

∴ Tá EF > BC

Q.E.D.

Atora I Ó phointe P, má tarraingítear dronlínte difriúla go dtí imlíne ~~ciorcail~~ chiorcail gurb é O a lár, teigheann an dronlíne is fáide agus an dronlíne is giorra díobh ~~siúd~~ tré O. Maidir le líne ar bith eile PX de na línte tré P, d'á laghad í an uille $P\hat{O}X$ ~~sea~~ is ea is giorra an líne sin.




Tá PX > PA, de bhrí go bhfuil OP+PX > OX ∴ OP+PX > OP+PA.

Tá PY > PX, de bhrí go bhfuil OP agus OX comhfhada le OP agus OY, ach is mó an uille $P\hat{O}Y$ ná an uille $P\hat{O}X$.

Tosach leathanach 70 sa LSS.

An tarna leagan de Teoirim XX:

## Teoirim XX

*Tá dhá shlios i dtriantán cómhfhada le dhá shlios i dtriantán eile, ach is mó an uille a chrioslaíonn an chéad péire ná an uille a chríoslaíonn an péire eile. Is é an triantán gurb ann atá an uille is mó an triantán is faide bonn.*

Tá Fíoghair anseo sa LSS, leathanach 70.

*Hipotéis:*

Tá $AB = DE$, $AC = DF$, $\widehat{EDF} > \widehat{BAC}$.

*Tátall:*

Tá $EF > BC$.

*Cruthúnas:*

Leag an slios $DE$ ar an slios cómhfhada $AB$, agus le scáthú in $AB$ (má's gá é ), cuir $\Delta DEF$ san ionad $ABK$.

Fágann sin $AK = AC$, $KB = EF$, $\widehat{KAB} > \widehat{BAC}$.

Ó thárla $\widehat{KAB} > \widehat{BAC}$, tá $AZ$ cómhroinnteóir na h-uilleann $KAC$, idir $AK$ agus $AB$.

Ach 'sé $AZ$ a.s. na líne $KC$ (teoirim IV), agus de bhrí go bhfuil $B$ agus $C$ ar an taobh chéanna dhi, tá $BK > BC$ (teoirim III)

.i. Tá $EF > BC$. □

**Theorem.** *Suppose two sides in a triangle have the same lengths as two sides in another triangle, but the angle between the first pair is greater then the angle between the other pair. Then the triangle with the greater angle has the greater base.*

— the second version of Theorem 20

*Hypothesis:*

$AB = DE$, $AC = DF$, $\widehat{EDF} > \widehat{BAC}$.

*Conclusion:*

$EF > BC$.

*Proof:*

Lay the side $DE$ on the equal-length side $AB$, and by reflection in $AB$ (if necessary), put $\Delta DEF$ iin the position $ABK$.

It follows that $AK = AC$, $KB = EF$, $\widehat{KAB} > \widehat{BAC}$.

Since $\widehat{KAB} > \widehat{BAC}$, the straight line $AZ$, bisector of the angle $KAC$, lies between $AK$ and $AB$.

But $AZ$ s the a.s. of the line $KC$ (Theorem 4), and since $B$ and $C$ are on the same side of it, we have $BK > BC$ (Theorem 3) .i. $EF > BC$. □

## Atora 1

*Ó phointe $P$, má tarraingítear dronlínte difriúla go dtí imlíne chiorcail gurb é $O$ a lár, téigheann an dronlíne is faide agus an dronlíne is giorra díobh tré $O$. Maidir le líne ar bith eile $PX$ de na línte tré $P$, d'á i laghad í an uille $\widehat{POX}$ is ea is giorra an líne sin.*

Tá Fíoghair anseo sa LSS, leathanach 70.

Tá $PX > PA$, de bhrí go bhfuil $OP + PX > OX$ .i. $OP + PX > OP + PA$.

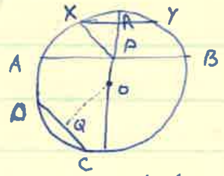
Tá $PY > PX$, de bhrí go bhfuil $OP$ agus $OX$ cómhfhada le $OP$ agus $OY$, ach is mó an uille $\widehat{POY}$ ná an uille $\widehat{POX}$.

**Corollary 1.** *If, from a point $P$, the various straight lines are drawn to the perimeter of a circle having centre $O$, then the longest and shortest of them pass through $O$. As far as any other line $PX$ through $P$, the smaller the angle $\widehat{POX}$ the shorter the line.*

We have $PX > PA$, because $OP + PX > OX$ .i. $OP + PX > OP + PA$.

$PY > PX$, because $OP$ and $OX$ have the same length as $OP$ and $OY$, but the angle $\widehat{POY}$ is greater than the angle $\widehat{POX}$.

Atoria 2 — I gciorcal ar bith má's goire don lár córda amháin ná córda áirithe eile, 'sé an chéad chórda acu siúd is faide.




Tugtar OQ > OP. Tá le cruthú go bhfuil AB > CD.

Cruthúnas

Cas an córda CD timpeall O go gcuirtear OQ ar OR, agus go dtiteann CD ar an gcórda XY.

Tá PA > PX (atoria 1), agus tá PX > XR (teoirim III, atoria)

∴ Tá PA > XR, ionnas go bhfuil 2PA > 2XR ∴ AB > XY.

## Cleachtaithe

1) Cruthuigh go bhfuil slios triantáin níos faide ná an difríocht idir an dá shlios eile.

2) Pointe is ea P atá taobh istigh den Δ ABC, agus teagmhaíonn BP le AC in X. Cruthuigh go bhfuil (i) AB+AC > BX+XC, (ii) BX+XC > BP+PC, (iii) AB+AC > BP+PC.

3) I gceathairshleasán ar bith taispeáin (i) gur mó leath-shuim na slios ná trasnán ar bith den dá thrasnán; (ii) gur mó leath-shuim na slios ná suim an dá thrasnán.

4) 'Siad P is Q pointí teagmhála dhá chiorcal ar lár dóibh O agus $O_1$, agus córda dúbailte trí P is ea LPM. Tarraingítear na h-ingir OX agus $O_1Y$ ar LPM. Cruthuigh (a) go bhfuil LM = 2XY: (b) go bhfuil $XY < OO_1$, mara bhfuil LM ∥ le $OO_1$.
Taispeáin cén chaoi a n-aimsítear an córda dúbailte is faide.

5) Má's pointe é P atá taobh istigh de chiorcal gur lár dó O, cruthuigh gurb é an córda atá ⊥ le OP an córda is giorra trí P.

Tosach leathanach 71 sa LSS.

## Atora 2

*I gciorcal ar bith má's goire don lár córda amháin ná córda áirithe eile, 'sé an chéad chórda acu siúd is faide.*

Tá Fíoghair anseo sa LSS, leathanach 71.

Tugtar $OQ > OP$. Tá le cruthú go bhfuil $AB > CD$.

*Cruthúnas:*

Cas ancórda $CD$ timpeall $O$ go gcuirtear $OQ$ ar $OR$, agus go dtiteann $CD$ ar an gcórda $XY$.

Tá $PA > PX$ (atora 1), agus tá $PX > XR$ (teoirim III atora).

∴ Tá $PA > XR$, ionas go bhfuil $2PA > 2PX$ .i. $AB > XY$.

**Corollary 2.** *In any circle, if one chord is closer to the centre than a second chord, then the first chord is the longer.*

We are given $OQ > OP$. We have to prove that $AB > CD$.

*Proof:*

Rotate the chord $CD$ around $O$ to place $OQ$ on $OR$, so that $CD$ lies on the chord $XY$.

We have $PA > PX$ (Corollary 1), and $PX > XR$ (Theorem 3, Corollary).

∴ $PA > XR$, so that $2PA > 2PX$ .i. $AB > XY$.

## Cleachtaithe

1. Cruthuigh go bhfuil slios triantán níos faide ná an difríocht idir an dá shlios eile.

2. Pointe is ea $P$ atá taobh istigh den $\Delta ABC$, agus teagmhaíonn $BP$ le $AC$ in $X$. Cruthuigh go bhfuil (i) $AB + AC > BX + XC$, (ii) $BX + XC > BP + PC$, (iii) $AB + AC > BP + PC$.

3. I gceathairshleasán ar bith teaspáin (i) gur mó leath-shuim na slios ná treasnán ar bith den dá threasnán; (ii) gur mó leath-shuim na slios ná suim an dá threasnán.

4. 'Siad $P$ is $Q$ pointí teagmhála dhá chiorcail ar láir dóibh $O$ agus $O_1$, agus córda dúbailte tré $P$ is ea $LPM$. Cruthuigh (a) go bhfuil $LM = 2XY$; (b) go bhfuil $XY < OO_1$, mara bhfuil $LM \parallel$ le $OO_1$.

   Teaspáin cé'n chaoi a n-aimsítear an córda dúbailte is faide.

5. Má's pointe é $P$ atá taobh istigh de chiorcal gur lár dó $O$, cruthuigh gurb é an córda atá $\perp$ le $OP$ an córda is giorra tré $P$.

6 Sa bparallélogram ABCD maoluille isea an uille A istigh, ionnas gur géaruille í a fórlíon B. Cruthuigh de chabhair teoirme II go bhfuil an trasnán BD > AC.

7. I dtriantán ar bith ABC, is é M lár BC. Cruthuigh AB + AC > 2AM. Le suimiú teaspáin gur faide suim na dtrí slios ná suim na dtrí meánlíne.

8 Sa léaráid i dTr. B caib IV cruthuigh $BG = \frac{2}{3}BE$, $CG = \frac{2}{3}CF$. Teaspáin gur faide suim na meánlínte faoi dhó ná suim na slios fá trí.

9. Pointí isea A, B ar an taobh chéanna de dhronlíne áirithe l, agus pointe ar bith den líne l isea P. Más é $A_1$ scáth A in l, agus má ghearrann $A_1B$ an líne l ag C, cruthuigh go bhfuil $AP + PB > A_1B$ (.i. $AP + PB > AC + CB$).

10. Sa △ ABC is é X lár an tsleasa BC. Má tá AX > BX cruthuigh $\hat{B} + \hat{C} > \hat{A}$ agus gur géaruille í A dá réir.

Má tá AX < BX, cruthuigh gur maoluille í A.

Céard é an tátall nuair AX = BX?

11 Sa △ ABC gearrann comhroinnteoir $\hat{A}$ an bonn BC in X. Cruthuigh (i) AB > BX ; AC > XC.

Más faide AB ná AC cruthuigh (le scáthú in AX) gur faide BX ná XC.

Tosach leathanach 72 sa LSS.

6. Sa bparallélogram $ABCD$ maoluille is ea an uille $A$ istigh, ionas gur géaruille í a fóirlíon $B$. Cruthuigh de thairbhe teoirme II go bhfuil an treasnán $BD > AC$.

7. I dtriantán ar bith $ABC$, 'sé $M$ lár $BC$. Cruthuigh $AB + AC > 2AM$. Le suimiú teaspáin gur faide suim na dtrí slios ná suim na dtrí meánlíne.

8. Sa léaráid i dTr. B Caib IV cruthuigh $BG = \frac{2}{3}BE$, $CG = \frac{2}{3}CF$. Teaspáin gur faide suim na meánlínte faoi dhó ná suim na slios fá trí'

9. Pointí is ea $A, B$ ar an taobh chéanna de dhronlíne áirithe $\ell$, agus pointe ar bith den líne $\ell$ is ea $P$. Má 'sé $A_1$ scáth $A$ in $\ell$, agus má ghearrann $A_1B$ an líne $\ell$ ag $C$, cruthuigh go bhfuil $AP + PB > A_1B$ (.i. $AP + PB > AC + CB$).

10. Sa $\Delta ABC$ 'sé $X$ lár an tsleasa $BC$. Má tá $AX > BX$ cruthuigh $\hat{B} + \hat{C} > \hat{A}$ agus gur géaruille í $A$ d'á réir.

    Má tá $AX < BX$, cruthuigh gur maoluille í $A$.

    Céard é an tátall nuair $AX = BX$?

11. Sa $\Delta$ $ABC$ gearrann cómhroinnteoir $\hat{A}$ an bonn $BC$ in $X$. Cruthuigh (i) $AB > BX$; $AC > XC$.

    Má's faide $AB$ ná $AC$ cruthuigh (le scáthú in $AX$) gur faide $BX$ ná $XC$.

## Exercises

1. Prove that a side of a triangle is longer than the difference between the other two sides.

2. $P$ is a point inside the $\Delta ABC$, and $BP$ meets $AC$ at $X$. Prove that (i) $AB + AC > BX + XC$, (ii) $BX + XC > BP + PC$, (iii) $AB + AC > BP + PC$.

3. In any quadrilateral prove (i) that half the perimeter is greater than either diagonal; (ii) that half the perimeter is greater than the sum of the two diagonals.

4. $P$ and $Q$ are the common points of two circles having centres $O$ and $O_1$, and $LPM$ is a double chord through $P$. Prove (a) that $LM = 2XY$; (b) that $XY < OO_1$, unless $LM \parallel$ le $OO_1$.

   Show how to find the longest double chord.

5. If $P$ is a point inside a circle with centre $O$, prove that the chord that is $\perp$ to $OP$ is the shortest chord through $P$.

6. In the parallelogram $ABCD$ the inside angle $A$ is obtuse, so that its complement $B$ is an acute angle. Prove by using Theorem 2 that the diagonal $BD > AC$.

7. In any triangle $ABC$, let $M$ be the centre of $BC$. Prove that $AB + AC > 2AM$. By adding, show that the sum of the three sides is greater than the sum of the three medians.

8. In the diagram in Theorem B of Chapter 4, prove that $CG = \frac{2}{3}CF$. Show that twice the sum of the medians is greater than three times the sum of the sides.

9. $A, B$ are points on the same side of a certain straight line $\ell$, and $P$ is any point of the line $\ell$. If $A_1$ is the reflection of $A$ in $\ell$, and if $A_1B$ cuts the line $\ell$ at $C$, prove that $AP + PB > A_1B$ (.i. $AP + PB > AC + CB$).

10. In the $\Delta ABC$ the centre of the side $BC$ is $X$. i If $AX > BX$, prove that $\hat{B} + \hat{C} > \hat{A}$ and that consequently $A$ is an acute angle.

    If $AX < BX$, prove that $A$ is an obtuse angle.

    What is the conclusion when $AX = BX$?

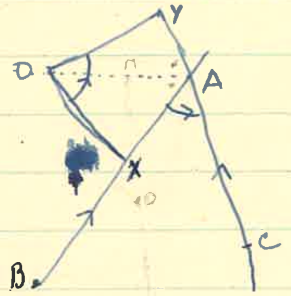
11. In the $\Delta$ $ABC$ the bisector of $\hat{A}$ cuts the base $BC$ at $X$. Prove (i) $AB > BX$; (ii) $AC > XC$.

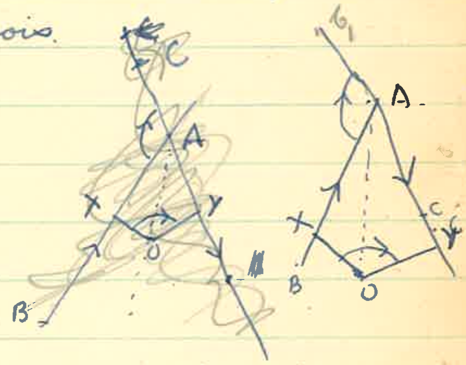
    If $AB$ is longer than $AC$, prove (by reflecting in $AX$) that $BX$ is longer than $XC$.

# Caibidil VII

## Uilleacha i gCiorcal Tadlaithe

I gCaib II. nuair leagtar dronlíne $l$ ar cheann eile $m$ le casadh an phlána timpeall a bpointe teagmhála A, is léir go bhfanann A féin socair, agus go gcuirtear pointe B den líne $l$ ar an bpointe C den líne eile ionnas go bhfuil AB = AC. Is mian linn anois tabhairt faoin gcasadh a bhreithniú a chuirfeas AB fán dronlíne eile AC ionnas go dtuitfidh B ar C, nuair nach bhfuil AB = AC. Ní h-é A lár an chasta anois.




Feicfimíd go bhfuil dhá chasadh dhifriúla a fheileas don cás de réir an treo ar leith ina gcuirtear an casadh i ngníomh.

Má 'sé O lár an chasta is eol dúinn (Teoirim VIII atóra 3) go bhfuil sé ar chómhroinnteoir éigin de dhá chómhroinnteoir na n-uilleann A. Má siad OX, OY na h-ingir uaidh ar na línte, is soiléir go gcuirfear OX ar OY ionnas gurb í $X\hat{O}Y$ uille an chasta.

Ach sa gceathairshleasán OXAY 'sé 360° suim na n-uilleann istigh (Caib IV), agus is dronuilleacha iad X agus Y, fágann sin $X\hat{O}Y$ = forlíon $X\hat{A}Y$ = $B\hat{A}C$ (Teoirim I) [= BAC (∴ II)]

Sin í go díreach an uille ~~idir na línte atá~~ den dá uillinn idir na línte atá in aon treo le h-uillinn an chasta.

∴ Má leagtar líne $l$ fán líne $m$ le casadh an phlána, is ionann uille an chasta (idir méad is treo) agus an uille idir $l$ agus $m$ a fhreagraíos di

# Caibidil 7

# Uilleacha i gCiorcal. Tadlaithe

Tosach leathanach 73 sa LSS.

## Angles in Circles. Tangents

I gCaib II nuair leagtar dronlíne $\ell$ ar cheann eile $m$ le casadh an phlána timpeall a bpointe teagmhála $A$, is léir go bhfanann $A$ féin socair, agus go gcuirtear pointe $B$ den líne $\ell$ ar an bpointe $C$ den líne eile ionas go bhfuil $AB = AC$. Is mian linn anois tabhairt fá'n gcasadh a bhreithiú a chuirfeas $AB$ fan dronlíne eile $AC$ ionas go dtitfidh $B$ ar $C$, nuair nach bhfuil $AB = AC$.
Ní hé $A$ lár an chasta anois

Tá Fíoghair anseo sa LSS, leathanach 73.

Feicfimid go bhfuil dhá chasadh dhifriúla a fheileas don cás de réir an treo ar leith ina gcuirtear an casadh i ngníomh.

Má 'sé $O$ lár an chasta is eol dúinn (Teoirim VIII atora 3) go bhfuil sé ar chómhroinnteoir éigin de dhá chómhroinnteoir na huilleann $A$. Má siad $OX$,$OY$ na hingir uaidh ar na línte, is soiléir go gcuirfear $OX$ ar $OY$ ionas gurb í $\widehat{XOY}$ uille an chasta.

Ach sa gceathairshleasán $OXAY$ 'sé 360° suim na nuilleann istigh (Caib IV), agus ósdronuilleacha iad $X$ agus $Y$, fágann sin $\widehat{XOY}$ = fóirlíon $\widehat{XAY} = \widehat{BAC}$ ( Fiogh I)

Sin í go díreach an uille den dá uillinn [$BAC$ (Fiogh II) ][1] idir na línte atá in aon treo le h-uillinn an chasta.

*Má leagtar líne $\ell$ fan líne $m$ le casadh an phlána, is ionann uille an chasta (idir méad is treo) agus an uille idir $\ell$ agus $m$ a fhreagraíos di.*

[1]LSS doiléir

'S furasta anois an dá chasadh a chinneadh a leagas dronlíne BA ar dhronlíne CA i gcaoi go dtuitfidh B ar C.

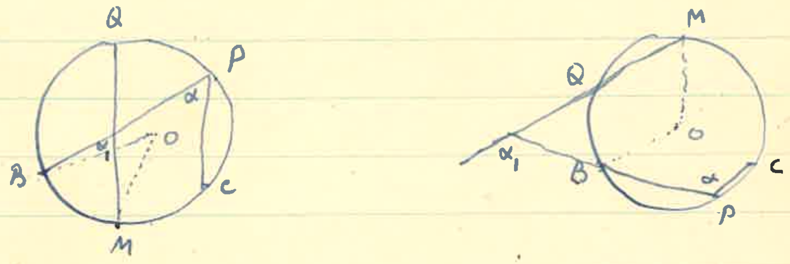
Mar, de bhrí go gcaithfidh B agus C a bheith cómhfhada ó lár an chasta, luíonn an lár úd ar ais shuiméitreacha BC. Ach tá sé ar chómhroinnteoir den uillinn A é freisin

∴ Siad pointí teagmhála a.Δ BC le dhá chómh-roinnteoir na h-uilleann A, lár an dá chasadh a fheileas.

Léiríonn an dá theóirim seo a leanas cé'n chaoi a bhfuil siad i ngaol leis an Δ ABC.

## Teóirim XXI

Is ionann an uille a ghabhas stuagh ciorcail ag an imlíne agus an uille a ghabhas leath an stuagh sin ag lár an chiorcail.




<u>Hipotéis</u>: 'Sé M lár an stuagh BC, agus pointe den imlíne idea P.

<u>Tátall</u>: Tá $B\hat{P}C = B\hat{O}M$.

<u>Cruthúnas</u> Cas an plána timpeall O tré'n uillinn $B\hat{O}M$, ionnus go dtéann B go dtí M, agus go ngluaiseann P ar imlíne an O go dtí Q

∴ Tá stuagh BM = stuagh PQ, agus sé MQ ionad nua BP.
'Sé sin, tá uille an chasta $B\hat{O}M = \hat{\alpha}_1$.

Ach tugtar stuagh BM = stuagh MC.

∴ Tá stuagh MC = stuagh PQ, ionnas gurb í an láirlíne chéanna is ais shuiméitreachta do PC agus QM.

Fágann sin go bhfuil PC // QM, agus tá $\hat{\alpha}_1 = \alpha$ (Tr. XI)

∴ $B\hat{O}M = B\hat{P}C$ Q.E.D.

Tosach leathanach 74 sa LSS.

Is furusta anois an dá chasadha chinneadh a leagas dronlíne $BA$ ar dhronlíne $CA$ i gcaoi go dtuitfidh $B$ ar $C$.

Mar, de bhrí go gcaithfidh $B$ agus $C$ a bheith cómhfhada ó lár an chasta, luíonn an lár úd ar ais shuiméitreachta $BC$. Ach tá sé ar chómhroinnteoir den uillinn $A$ é freisin.

Siad pointí teagmhála a.s. $BC$ le dhá chómhroinnteoir na huilleann $A$, lár an dá chasadh a fheileas.

Léiíonn an dá theoirim seo a leanas cé'n chaoi a bhfuil siad i ngaol leis an $\Delta\ ABC$.

In Chapter II, when a straight line $\ell$ s placed on another $m$ by a rotation of the plane about their common point $A$, it is clear that $A$ itself stays fixed and a point $B$ of the line $\ell$ is placed on the point $C$ of the other line in such a way that $AB = AC$. Now we want to consider the rotation that will place $AB$ along another straight line $AC$ so that $B$ moves to $C$, when it is not the case that $AB = AC$.
This time, $A$ is not the centre of the rotation.

We shall see that there are two different rotations that meet the case, depending on the particular direction in which the rotation is made.

If $O$ is the centre of the rotation, then we know (Theorem 8, Corollary 3) that it lies on some bisector of the angle $A$. If $OX$,$OY$ are the perpendiculars from it on the lines, it is clear that $OX$ is placed on $OY$ so that $\widehat{XOY}$ is the angle of the rotation.

But in the quadrilateral $OXAY$ the sum of the inside angles is 360° (Chapter IV), and since $X$ and $Y$ are right angles, it follows that $\widehat{XOY}$ = the complement of $\widehat{XAY} = \widehat{BAC}$ (Fig I)

That is exactly the angle of the two angles [$BAC$ (Fig II) $a$] between that are in the same direction as the rotation angle.

*If a line $\ell$ s laid along a line $m$ by a rotation of the plane, then the angle of rotation is the same (in size and direction) as the corresponding angle between $\ell$ and $m$.*

It is easy now to determine the two rotations that will lay the straight line $BA$ on the straight line $CA$ and place $B$ on $C$.

For , since $B$ and $C$ have to be the same distance from the centre of the rotation, that centre lies on the axis of symmetry of $BC$. But it is also on a bisector of the angle $A$ as well.

The points where the a.s. of $BC$ meets the two bisectors of the angle $A$, are the centres of the two rotations that suit.

The following two theorems show their relationship to the $\Delta\ ABC$.

## 7.1 Teoirim XXI

*Is ionann an uille a ghabhas stua[2] ciorcail ag an imlíne agus an uille a ghabhas leath an stua sin ag lár an chiorcail.*

Tá Fíoghair anseo sa LSS, leathanach 74.

[2]LSS: stuagh; ceartaithe go stua go minic sa LSS, agus anseo agamsa (AOF).

*Hipotéis:*
'Sé $M$ lár an stua $BC$, agus pointe den imlíne is ea $P$.

*Tátall:*
Tá $\widehat{BPC} = \widehat{BOMi}$.

*Cruthúnas:*
Cas an plána timpeall $O$ tré'n uille $\widehat{BOM}$,ionas go dtéann $B$ go dtí $M$, agus go ngluaiseann $P$ ar imlíne an $\odot$ go dtí $Q$.
$\therefore$ Tá stua $MB$ = stua $PB$ agus sé $MQ$ ionad nua $BP$.
'Sé sin 'sé uille an chasta $\widehat{BOM} = \hat{\alpha}_1$.
Ach tugtar stua $BM$ = stua $MC$.
$\therefore$ Tá stua $MC$ = stua $PQ$, ionas gurb í an lárlíne chéanna is ais shuiméitreachta do $PC$ agus $QM$.
Fágann sin go bhfuill $PC\|$ le $QM$ agus tá $\hat{\alpha}_1 = \hat{\alpha}$ (Tr. XI). $\widehat{BOM}$=$\widehat{BPC}$. □

**Theorem 21.** *The angle subtended by an arc of a circle at the perimeter is equal to the angle subtended by half that arc at the centre.*

*Hypothesis:*
$M$ is the centre of the arc $BC$, and $P$ is a point on the perimeter.

*Conclusion:*
$\widehat{BPC} = \widehat{BOM}$.

*Proof:*
Rotate the plane around $O$ through the angle $\widehat{BOM}$,so that $B$ goes to $M$, and so that $P$ moves on the perimeter of the circle to $Q$.
$\therefore$ The arc $MB$ = the arc $PB$ and $MQ$ is the new position of $BP$.
That is, the angle of the rotation is $\widehat{BOM} = \hat{\alpha}_1$.
But we are given that the arc $BM$ = the arc $MC$.
$\therefore$ arc $MC$ = arc $PQ$, so that the same diameter is an axis of symmetry for $PC$ and $QM$.
It follows that $PC\|$ to $QM$ and $\hat{\alpha}_1 = \hat{\alpha}$ (Theorem 11). $\widehat{BOM}$=$\widehat{BPC}$. □

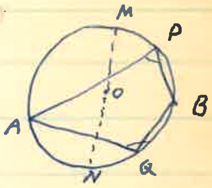
Téarmaí Gineann córda ciorcail agus stuagh ar bith den dá cheann a ghearras sé den imlíne, fíoghair iadta d'a ngoirtear teascán ciorcail.

Pointí cómhchiorcalacha isea ceithre pointí (nó tuille) a ngabhann ciorcal tríothu uilig, agus tugtar ceathairshleasán cómhchiorcalach ar an gceathairshleasán a ghineas siad.

Atora 1 Is buan don uillinn $B\hat{P}C$ cibé cé'n áit ar an stuagh BPC a bhfuil an pointe P.
Mar is ionann é agus BOM atá seasmhach.
Tugtar uille an teascáin APB ar an uillinn sin.

Atora 2 Is dronuille í uille leithchiorcail.

Atora 3 I gceathairshleasán chómhchiorcalach uilleacha fóirlíontacha isea na h-uilleacha atá ar aghaidh a chéile.




Mar, is leithchiorcal é an tsuim nuair suimítear leath an stuagh $A\hat{P}B$ le leath an stuagh $A\hat{Q}B$.

Atora 4 I gciorcal ar bith (nó i gciorcail chómhionanna) is cómhfhada na stuaghanna a gabhas uilleacha cothroma ag an imlíne, agus is fíor an coinvéarsa.

Mar, ~~córda~~ stuaghanna isea iad a ghabhas uilleacha cothroma ag an lár freisin, de réir na teoirme.

Tosach leathanach 75 sa LSS.

## Téarmaí

Gineann córda ciorcail agus stua ar bith den dá cheann a ghearas sé den imlíne, fioghair iadta d'a ngoirtear *teascán* ciorcail.

Pointí *cóimhchioralacha* is ea ceithre pointí (nó tuille) a ngabhann ciorcal triothu uilig, agus tugtar ceathairshleasán *cóimhchioralach* ar an gceathairshleasán a ghineas siad.

## Atora 1

*Is buan don uillinn* $\widehat{BPC}$ *cébi cé'n áit ar an stua* $BPC$ *a bhfuil an pointe* $P$.

Mar is ionann í agus $BOM$ atá seasmhach.

Tugtar *uille an teascáin* $APB$ ar an uillinn sin.

## Atora 2

*Is dronuille í uille leithchiorcail.*

## Atora 3

*I gceathairshleasán chóichiorcalach uilleacha fóirlíontacha is ea na huilleacha atá ar aghaidh a chéile.*

Tá Fíoghair anseo sa LSS, leathanach 75.

Mar , is leithchiorcal é an ttsuim nuair suimítear leath an stua $\widehat{APB}$ le leath an stua $\widehat{AQB}$.

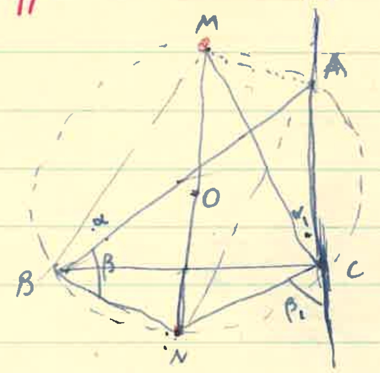
## Atora 4

*I gciorcal ar bith (nó i gciorcail chóimhionanna) is cómhfhada na stua a gabhas uilleacha cothroma ag an implíne, agus is fíor an coinvéarsa.*

Mar , stuanna is ea iad a ghabhas uilleacha cothroma ag an lár freisin, de réir na teoirme.

# Teoirim XXII

Má cuirtear dronlíne BA fan dronlíne eile CA i gcaoi go dtiteann B ar C le casadh, comhroinneann lár an chasta stuagh den dá stuagh BC ar an gciorcal ABC.




**Hipotéis** 'Siad M, N pointí teagmhála a.s. BC le comhroinnteoirí na huilleann A.

**Tátal** Tá M, N ar an ⊙ ABC, agus lár na stuagh BC iseadh iad.

**Tógáil** Faigh O, lár MN.

**Cruthúnas** *

[* Footnote Bé gur fusa cruthúnas neadíreach is ionspéisiúla an ceann seo]

De bhrí go gcomhroinneann AN, AM uilleacha comhgharacha ag A, dronuille iseadh MAN, agus sé O ~~lár M~~ iomlár an Δ MAN.

Sa gcasadh timpeall M, cuirtear $\hat{\alpha}$ ar $\hat{\alpha}_1$, ∴ $\hat{\alpha} = \hat{\alpha}_1$.

Sa gcasadh timpeall N, cuirtear $\beta$ ar $\hat{\beta}_1$, ∴ $\hat{\beta} = \hat{\beta}_1$.

∴ $\hat{\alpha} + \hat{\beta} = \hat{\alpha}_1 + \hat{\beta}_1$

Ach tá $\hat{\alpha} + \hat{\beta} = M\hat{C}N$ (scátha a chéile in MN)

∴ Tá $M\hat{C}N$ = a fóirlíon $\hat{\alpha}_1 + \hat{\beta}_1$, ∴ dronuille iseadh MCN (agus MBN)

∴ Sé O iomlár na Δ MNC agus MNB chomh maith.

Gabhann an ⊙ ar lár dó O agus ar ga dó OM tré na pointí M, A, C, N, B go léir

De bhrí gurb í MN a.s. BC, 'siad M, N láir na stuagh BC.

**Aiora** ~~Nuair castar plána timpeall O, má thuiteann cuirtear~~ B ar

Má cuirtear pointe B ar phointe C de bharr casta ar bith, leagtar dronlíne ar bith tré B fan na dronlíne sin tré C a theagmhaíos leis an gréad cheann ar an gciorcal tré B, C agus lár an chasta.

[Tuigfear i dteoirim XXV céard á tharlaíos don líne BC féin]

[illegible] ag C

Tosach leathanach 76 sa LSS.

### Definitions

A chord of a circle and any arc of the two into which it cuts the perimeter make a closed figure that is called a *segment* of the circle.

*Concyclic points* are any four (or more) points through all of which some circle passes, and the quadrilateral they generate is called a *cyclic quadrilateral* .

**Corollary 1.** *The angle $\widehat{BPC}$ is constant, no matter where the point $P$ lies on the arc $BPC$.*

For it is equal to the fixed $\widehat{BOM}$.

That angle is called *the angle of the segment $APB$*.

**Corollary 2.** *The angle in a semicircle is a right angle.*

**Corollary 3.** *In a cyclic quadrilateral opposite angles are complementary.*

For , we get a semicircle when we add half the arc $\widehat{APB}$ and half the arc $\widehat{AQB}$.

**Corollary 4.** *In any circle (or in equal circles) arcs that subtend equal angles at the perimeter have the same length, and conversely.*

For , they are also arcs that subtend equal angles at the centre, by the theorem.

## 7.2 Teoirim XXII

*Má cuirtear dronlíne $BA$ fan dronlíne eile $CA$ i gcaoi go dtiteann $B$ ar $C$ le casadh, cómhroinneann lár an chasta stua den dá stua $BC$ ar an gciocal $ABC$.*

Tá Fíoghair anseo sa LSS, leathanach 76.

*Hipotéis:*
'Siad $M, N$ pointí teagmhála a.s. $BC$ le cómhroinnteoirí na huilleann $A$.

*Tátall:*
Tá $M, N$ ar an $\odot ABC$, agus lár na stua $BC$ is ea iad.

*Tógáil:*
Faigh $O$, lár $MN$.

*Cruthúnas:*
[3]

De bhrí go gcómhroinneann $AN, AM$ uilleacha cómhgharacha ag $A$, dronuille is ea $MAN$, agus sé $O$ iomlár na $\Delta MAN$.

[3]Cé gur fusa cruthúnas neadrach is ionspéisiúla an ceann seo.

Sa gcasadh timpeall $M$, cuirtear $\hat{\alpha}$ ar $\hat{\alpha}_1 \therefore \hat{\alpha} = \hat{\alpha}_1$.
Sa gcasadh timpeall $N$, cuirtear $\hat{\beta}$ ar $\hat{\beta}_1 \therefore \hat{\beta} = \hat{\alpha}_1$.

$$\hat{\alpha} + \hat{\beta} = \hat{\alpha}_1 + \hat{\beta}_1.$$

Ach tá $\hat{\alpha} + \hat{\beta} = \widehat{MCN}$ (scátha a chéile in $MN$).
$\therefore$ Tá $\widehat{MCN}$= a fóirlíon $\hat{\alpha}_1 + \hat{\beta}_1$ .i. dronuille is ea $MCN$ (agus $MBN$).
.i. Sé $O$ iomlár na $\Delta$ $MNC$ agus $MNB$ chómh maith.
Gabhann an $\odot$ ar lár dó $O$ agus ar ga dó $OM$ tré na pointí $M, A, C, N, B$ go léir.
De bhrí gurb í $MN$ a.s. $BC$, 'siad $M, N$ lár na stua $BC$.

**Theorem 22.** *If a straight line $BA$ is moved by a rotation to another straight line $CA$ in such a way that $B$ is moved to $C$, then the centre of the rotation bisects one of the two arcs $BC$ on the circle $ABC$.*

*Hypothesis:*
$M, N$ are the points where the a.s. of $BC$ meets the bisectors of the angle $A$.

*Conclusion:*
$M, N$ are on the circle $ABC$, and are the centres of cthe arcs $BC$.

*Construction:*
Find $O$, the centre of $MN$.

*Proof:*
[4]

Since $AN, AM$ bisect adjacent angles at $A$, the angle $MAN$ is a right angle, and $O$ is the incentre of the $\Delta MAN$.
The rotation about $M$ places $\hat{\alpha}$ on $\hat{\alpha}_1 \therefore \hat{\alpha} = \hat{\alpha}_1$.
The rotation about $N$ places $\hat{\beta}$ on $\hat{\beta}_1 \therefore \hat{\beta} = \hat{\alpha}_1$.

$$\hat{\alpha} + \hat{\beta} = \hat{\alpha}_1 + \hat{\beta}_1.$$

But $\hat{\alpha} + \hat{\beta} = \widehat{MCN}$ (reflections of one another in $MN$).
$\therefore$ $\widehat{MCN}$= its complement $\hat{\alpha}_1 + \hat{\beta}_1$ .i. $MCN$ is a straight line (as is $MBN$).
.i. $O$ is the circumcentre of the $\Delta$ $MNC$ and of $MNB$ as well.
The $\odot$ with centre $O$ and radius $OM$ passes through all the points $M, A, C, N, B$.
Since $MN$ is the a.s. of $BC$, the points $M, N$ are the centres of the arcs $BC$.

## Atora

*Má cuirtear pointe $B$ ar phointe $C$ de bharr chasta ar bith, leagtar dronlíne ar bith tré $B$ fan na dronlíne sin tré $C$ a theagmhaíos leis an gcéad cheann ar an gciorcal tríd $B, C$ agus lár an chasta.*

[ Feicfear i dteoirim XXV céard a tharlaíos don líne $BC$ féin.][5]

[4]Even though a proof by contradition would be easier, this one is particularly interesting.
[5]Tadhlí ag $C$ (san imeall)

**Corollary.** *If a point $B$ is placed on a point $C$ by any rotation, any straight line through $B$ is moved to that straight line through $C$ that meets the first one on the circle that passes through $B, C$ and the centre of the rotation.*

[ It will be seen in Theorem 25 what happens to the line $BC$ itself.][6]

[6] A marginal note in the MSS says: Tangent at $C$.

## Ceisteanna

1) Cruthuigh gur dronuilleóg é gach ▱ comhchiorcalach

2) Má síntear síos ceathairshleasán chómhchiorcalaigh, taispeán gurb ionann an uille amuigh agus an uille atá ós a cóir istigh. Luaidh an casadh a chuireas uille acu ar an gceann eile.

3. Má's pointí iad M agus A atá ar aon taobh amháin den bhonn BC ionnus go bhfuil $M\hat{B}A = M\hat{C}A$, cruthuigh nach bhféadfadh an ⊙ MBA an líne AC a ghearradh i bpointe ar bith eile thairis C gan teoirim XXI a bhréagnú.

Tabhair, ar an gcuma sin, cruthúnas neadíreach i gcóir Tr XXII

4 Uilleacha forlíontacha iseadh dhá uillinn atá ar aghaidh a chéile i gceathairshleasán. Cruthuigh gur ceathair-shleasán comhchiorcalach é.

5. Siad BL, CM na hingir ó B, C ar shleasa AC agus BA an Δ ABC. Cruthuigh gur pointí comhchiorcalacha iad B, C, L, M.

6 Uilleacha sa teascán céanna iad BPC, BQC. Luaidh casadh a leagfas uille acu ar an gceann eile.

Tosach leathanach 77 sa LSS.

## 7.3 Cleisteanna

1. Cruthuigh gur dronuilleóg é gach ▱ cóimhchiorcalach.
2. Má síntear slios ceathairshleasáin chóimhchiocalaigh, teaspáin gurb ionann an uille amuigh agus an uille atá ós a cóir istigh. Luaidh an casadh a chuireas ceann acu ar an gceann eile.
3. Má;s pointí iad $M$ agus $A$ atá ar aon taobh amháin den bhonn $BC$ ionas go bhfuil $\widehat{MBA}=\widehat{MCA}$, cruthuigh nach féidir an $\odot MBA$ an líne $AC$ a ghearradh i bpointe ar bith eile thairis $C$ gan teoirim XXI a bhréagnú'

   Tabhair, ar an gcuma sin, cruthúnas neadréach i gcóir Tr. XXII.
4. Uilleacha fóirlíontacha is ea uillinn atá ar aghaidh a chéile i gceathairshleasán. Cruthuigh gur 4-shleasán cóimhchiorcalach é.
5. Siad $BL, CM$ na hingir ó $B, C$ ar shleasa $AC$ agus $BA$ an $\Delta\ ABC$. Cruthuigh gur pointí cóimhchiorcalacha iad $B, C, L, M$.
6. Uilleacha sa teascán céanna iad $BPC, PQC$. Luaidh casadh a leagfas uille acu ar an gceann eile.

## 7.4 Exercises

1. Prove that each cyclic ▱ is a rectangle.
2. If a side of a cyclic quadrilateral is extended, show that the external angle is equal to the opposite inside angle. State the rotation that will move one of them on the other one.
3. If the points $M$ and $A$ are on the same side of the base $BC$ so that $\widehat{MBA}=\widehat{MCA}$, prove that the $\odot MBA$ and the line $AC$ can only cut at $C$ and at no other point, without contradicting Theorem 21.

   In that way, give a proof by contradiction for Theorem 21.
4. Opposite angles in a certain quadrilateral are complementary. Prove that it is a cyclic quadrilateral .
5. $BL, CM$ are the perpendiculars from $B, C$ one the sides $AC$ and $BA$ of the $\Delta\ ABC$. Prove that $B, C, L, M$ are concyclic points.
6. $BPC, PQC$ are angles in the same segment. Give a rotation that lays one of them on the other.

*Nach ceart a chur leis nach dteagmhann sé leis an imlíne arís nuair a síntear í?*

**Téarma** Tugtar tadhlaí ar dhronlíne a theagmhaíos leis an imlíne in aon phointe amháin (an pointe tadhaill). agus teagmháil eile leis fé threó ina síntear an tadhlaí.


Ag pointe P den imlíne má tarraingítear an dronlíne thré P atá ingearach leis an nga OP, is follusach go bhfuil gach pointe X den líne sin (cé's moite de P féin) taobh amuigh den imlíne, mar OX > ga an ⊙ OP. (teoirim XIX)

∴ Tadhlaí iseadh PX

Mar an gcéanna Is é PX an t-aon tadhlaí amháin thré P, de bhrí gurb é OP an fhad is giorra idir O agus tadhlaí ar bith thré P, agus fágann sin gurb é OP an t-ingear ó O ar an tadhlaí.

Tosach leathanach 78 sa LSS.

## Téarma

Tugtar *tadhlaí* ar dhronlíne a teagmhaíos leis an imlíne in aon phointe amháin (an pointe tadhaill) agus nach bhfuil teagmháil eile leis pé treó ina síntear an tadhlaí.

Tá Físoghair anseo sa LSS, leathanach 78.

Ag pointe $P$ den imlíne má tarraingítear an dronlíne tré $P$ atá ingearach leis an nga $OP$, is follasach go bhfuil gach pointe $X$ den líne sin (cé's moite de $P$ féin) taobh amuigh den imlíne, mar $OX >$ ga an $\odot OP$ (teoirim XIX).

$\therefore$ Tadhlaí is ea $PX$.

Mar an gcéanna 'sé $PX$ an t-aon tadhlaí amháin tré $P$, de bhrí gurb é $OP$ an fhad is giorra idir $O$ agus tadhlaí ar bith tré $P$, agus fágann sin gurb é $OP$ an t-ingear ó $O$ ar an tadhlaí.

## Definition

A *tangent* is a straight line that meets the perimeter in a single point (the *point of tangency*) and does not meet it again no matter how far or in which direction it may be extended.

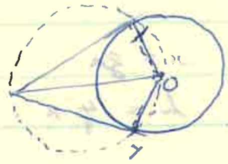
At a point $P$ of the perimeter if we draw the straight line through $P$ that is perpendiculer to the radius $OP$, it is clear that each point $X$ of that line (apart from $P$ itself) outside the perimeter, for $OX >$ the radius of the $\odot OP$ (Theorem 19).

$\therefore$ $PX$ is a tangent.

Similarly, $PX$ is the only tangent through $P$, because $OP$ is the shortest distance between $O$ and any tangent through $P$, and it follows that $OP$ is the perpendicular from $O$ on the tangent.

## Teoirim XXIII

Is féidir dhá thadhlaí a tharraint go dtí cioreal ó phointe ar bith taobh amuigh.




Abair gurb é O lár an ⊙, agus gurb é P an pointe a tugtar.

**Tógáil** ~~Bruthúnas~~

Línigh an ⊙ gurb í OP a lárlíne agus tabhair X is Y ar na pointí ina ngearrann sé an ⊙ a tugtar.
∴ 'Siad PX agus PY an dá thadhlaí ó P.

**Bruthúnas**

Uille leithchioreail isea $O\hat{X}Y$, ionnas gur dronuille í (Teoirim XXI).

Fágann sin PX ⊥ leis an nga OX.

∴ 'Sé PX an tadhlaí ag X, agus mar an gcéanna 'sé PY an tadhlaí ag Y.

**Atora 1** Scátha a chéile in OP isea PX agus PY.

**Atora 2** Má cuirtear dronlíne ar dhronlíne eile le casadh timpeall pointe O, tadhlann gach dronlíne acu ⊙ áirithe gurb é O a lár.

Mar pointe ar chómhroinnteoir na ∠i uilleann eatorra isea O, agus is cómfhada na ⊥ingir uaidh ar an dá líne.

**Téarma** Má thagann dhá chioreal le chéile ag pointe P i gcaoi go dtadhlann dronlíne tré P an dá chioreal, deirtear go dtadhlann an dá chioreal sin a chéile ag P.

## Teoirim XXIV

Má thadhlann dhá chioreal a chéile, tá an pointe tadhaill ar dhronlíne cheangail na lár.


**Hipotesis** Tadhlann PZ an dá chioreal.

**Tátal** Luigheann P ar an dronlíne $OO_1$.

**Bruthúnas** De bhrí gur tadhlaí é PZ don dá ⊙, luigheann O agus $O_1$ ar an ingear ~~do~~ ar PZ ag P.
∴ Atá dronlíne amháin isea $POO_1$. Q.E.D

Tosach leathanach 78 + 1 (gan uimhir) sa LSS.

## 7.5 Teoirim XXIII

*Is féidir dhá thadhlaí a thabhairt go dtí ciorcal ó phointe ar bith taobh amuigh.*

Tá Fíoghair anseo sa LSS, leathanach 78+1.

Abair gurb é $O$ lár an ⊙, agus gurb é $P$ an pointe a tugtar.

*Tógáil:*

Línigh an ⊙ gurb í $OP$ a láirlíne agus tabhair $X$ is $Y$ ar na pointí ina ngearrann sé an ⊙ a tugtar.

'Siad $PX$ agus $PY$ an dá thadhlaí ó $P$.

*Cruthúnas:*

Uille leithchiorcail is ea $\widehat{OXY}$, ionas gur dronuille é (Teoirim XXI).

Fágann sin $PX \perp$ leis an nga $OX$.

∴ 'Sé $PX$ an tadhlaí ag $X$, agus mar an gcéanna 'sé $PY$ an tadhlaí ag $Y$.

**Theorem 23.** *It is possible to draw two tangents to a circle from any point outside it.*

Suppse $O$ is the centre of the ⊙, and that $P$ is the given point.

*Construction:*

Draw the ⊙ with diameter $OP$ and call the points where it cuts the given circle $X$ and $Y$.

$PX$ and $PY$ are the two tangents from $P$.

*Proof:*

$\widehat{OXY}$ is a half-circle angle, so that it is a right angle (Theorem 21).

It follows that $PX \perp$ to the radius $OX$.

∴ $PX$ is the tangent at $X$, and in the same way $PY$ is the tangent at $Y$.

### Atora 1

*Scátha a chéile in $OP$ is ea $PX$ agus $PY$.*

### Atora 2

*Má cuirtear dronlíne ar dhronlíne eile le casadh timpeall phointe $O$, tadhlann gach dronlíne acu ⊙ áirithe gurb é $O$ a lár.*

Mar pointe ar chómhroinnteoir na huilleann eatoru is ea $O$, agus is cómhfhada na hingir uaidh ar an dá líne.

### Téarma

Má thagann dhá chiorail le chéile ag pointe $P$ i gcaoi go dtadhlann dronlíne tré $P$ an dá chiorcal, deirtear go dtadhlann an dá chiorcal sin a chéile ag $P$.

**Corollary 1.** *$PX$ and $PY$ are reflections of one another in $OP$.*

**Corollary 2.** *If a rotation about the point $O$ places one straight line on another, then both straight lines are tangent to a certain $\odot$ with centre $O$.*

For $O$ is a point on the bisector of the angle bwteen them, and the perpendiculars from it onto the two lines have the same length.

### Definition

If two circles meets at a point $P$ in such a way that some straight line through $P$ is tangent to both circles, then we say that those two circles are tangent to one another at $P$.

## 7.6 Teoirim XXIV

*Má thadhlann dhá chiorcail a chéile, tá an pointe tadhaill ar dhronlíne cheangail na lár.*

Tá Fíoghair anseo sa LSS, leathanach 78+1, bun.

*Hipotéis:*Tadhlann $PZ$ an dá chiorcal.

*Tátall:*
Luigheann $P$ ar an dronlíne $OO_1$.

*Cruthúnas:*
De bhrí gur tadhlaí é $PZ$ don dá $\odot$, luigheann $O$ agus $O_1$ ar an ingear le $PZ$ ag $P$.
∴ Aon dronlíne amháin is ea $POO_1$. □

**Theorem 24.** *If two circle are tangent to one another, then the point of contact is on the straight line joining the centres.*

*Hypothesis:*Tadhlann $PZ$ an dá chiorcal.

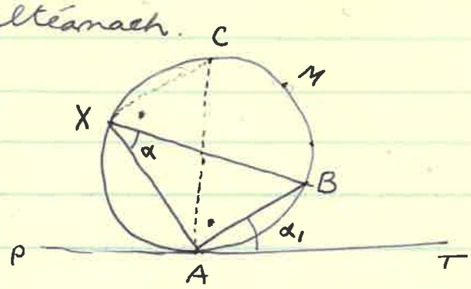
*Conclusion:*
$P$ lies on the straight line $OO_1$.

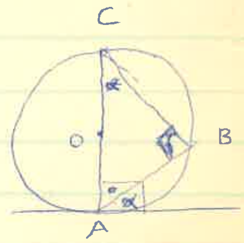
*Proof:*
Since $PZ$ is tangent to the two $\odot$, $O$ and $O_1$ lie on the perpendicular to $PZ$ ag $P$.
∴ $POO_1$i is a single straight line . □

C.

Teoirim ~~XXIV~~ XXV

Tá an uille idir thadhlaí ciorcail agus córda ar bith tré'n bpointe tadhaill, cóimhionann leis an uillinn san teascán alteánach.




Hípotesis Tadhlaí agus córda isea AT agus AB, agus pointe ar bith sa teascán ACB isea X.

Tátall Tá $\hat{\alpha}_1 = \hat{\alpha}$.

Tógáil Tarraing an láirlíne AC, agus faigh M lár an stuaigh ACX.

Cruthúnas

Dronuilleacha isea $C\hat{X}A$ (uille leithchiorcail), agus $C\hat{A}T$ (idir láirlíne agus an tadhlaí).

$\therefore$ Tá $C\hat{X}A = C\hat{A}T$

De bharr chasta tré'n uillinn $X\hat{M}A$ timpeall M, cuirtear X ar A, agus leagtar XC fan AC (teoirim XXI).

Agus de thairbhe $C\hat{X}A = C\hat{A}T$, is fan ~~na líne~~ an tadhlaí AT a leagtar XA.

Ach is fan na líne AB a leagtar XB (teoirim XXI)

$\therefore$ Cuirtear $\hat{\alpha}$ anuas go cruinn ar $\hat{\alpha}_1$. $\therefore$ Tá $\hat{\alpha}_1 = \hat{\alpha}$

Q.E.D.

Atoradh

Tá $P\hat{A}B$ (fórlíon $\hat{\alpha}_1$) = uille an mhionteascáin AB.

## Cleachtaithe

1) Cruthuigh gur dronuilleóg é gach paralléilogram cóimhchiorcalach

2) Má síntear slios ceathairshleasáin chómhchiorcalaigh, teaspáin gurb ionann an uille amuigh agus an uillinn atá ós a cóir istigh.

Tosach leathanach 79 sa LSS.

## 7.7 Teoirim XXV

*Tá an uille idir thadhlaí ciorcail agus córda ar bith tré'n bpointe tadhaill, cóimhionann leis an uillinn san teascán altéarnach.*

Tá Fíoghair anseo sa LSS, leathanach 79.

*Hipotéis:*Tadhlaí agus córda is ea $AT$ agus $AB$, agus pointe ar bith sa teascán $ACB$ is ea $X$.

*Tátall:*
Tá $\hat{\alpha}_1 = \hat{\alpha}$.

*Tógáil:*Tarraing an lárlíne $AC$; agus faigh $M$ lár an stua $ACX$.

*Cruthúnas:*
Dronuilleacha is ea $\widehat{CXA}$ (uille leithchiorcail), agus $\widehat{CAT}$ (idir láirlíne agus an tadhlaí.)
∴ Tá $\widehat{CXA}$=$\widehat{CAT}$.
De bharr chasta tré'n uillinn $\widehat{XMA}$ timpeall $M$, cuirtear $X$ ar $A$, agus leagtar $XC$ fan $AC$ (teoirim XXI).
Agus de thairbhe $\widehat{CXA}$=$\widehat{CAT}$, is fan an tadhlaí $AT$ a leagtar $XA$.
Ach is fan na líne $AB$ a leagtar $XB$ (teoirim XXI).
.i. Cuirtear $\hat{\alpha}$ anuas go cruinn ar $\hat{\alpha}_1$. ∴ Tá $\hat{\alpha}_1 = \hat{\alpha}$. □

**Theorem 25.** *The angle between a tangent to a circle and any chord through the point of contact is equal to the angle in the alternate segment.*

*Hypothesis:*$AT$ is a tangent and $AB$ is a chord, and $X$ is any point in the segment $ACB$.

*Conclusion:*
$\hat{\alpha}_1 = \hat{\alpha}$.

*Construction:*Draw the diameter $AC$; and find $M$, the centre of the arc $ACX$.

*Proof:*
$\widehat{CXA}$ is a right angle (angle of a semicircle), and so is $\widehat{CAT}$ (between diameter and the tangent.)
∴ $\widehat{CXA}$=$\widehat{CAT}$.
When we rotate through the angle $\widehat{XMA}$ about $M$, $X$ is placed on $A$, and $XC$ is laid along $AC$ (Theorem 21).
And since $\widehat{CXA}$=$\widehat{CAT}$, it follows that $XA$. is laid along the tangent $AT$.
But $XB$ is laid along the line $AB$ (Theorem 21).
.i. $\hat{\alpha}$ is laid exactly on $\hat{\alpha}_1$. ∴ $\hat{\alpha}_1 = \hat{\alpha}$. □

## Atora

*Tá $\widehat{PAB}$ (fóirlíon $\hat{\alpha}_1$) = uille an mhionteascáin $AB$.*

## Cleachtaithe

1. Cruthuigh gur dronuilleóg é gach parallélogram cóimhchiorcalach.
2. Má síntear slios ceathairshleasáin chóimhchiorcalaigh, teaspáin gurb ionann an uille amuigh agus an uillinn ós a cóir istigh.

**Corollary.** *The angle $\widehat{PAB}$ (the complement of the angle $\hat{\alpha}_1$) = the angle of the minor segment $AB$.*

## Exercises

1. Prove that every cyclic parallelogram is a rectangle
2. If a side of a cyclic quadrilateral is extended, show that the external angle is equal to the opposite inside angle.

# Cleachtaithe



1) Tadhlann dhá chiorcal a chéile ag T, agus ar dhá chórda TX agus TY i gciorcal ~~acu~~ acu ~~'siad~~ is iad TL, TM na córdaí a ghearras an ⊙ eile. Cruthuigh (i) go bhfuil LM // le XY, agus (ii) go bhfuil na tadhlaithe ag L agus X parailéileach.

2) Triantán i gciorcal is ea ABC, agus is i bpointe P a theagmhaíos an tadhlaí ag A leis an mbonn BC. Cruthuigh go bhfuil uilleacha an triantáin PAB cóimhionann le h-uilleacha an triantáin PCA.

3) 'Siad P agus Q pointí teagmhála dhá chiorcal ar láir dóibh O agus $O_1$, agus córda dúbailte ar bith is ea LPM.
Cruthuigh go bhfuil uilleacha an △ LMQ cóimhionann le h-uilleacha an △ $OO_1P$.

4) Ceathairshleasán cóimhchiorcalach is ea ABCD, agus tagann na sleasa AB is DC le chéile in X. Faigh scáth na líne BC i gcómhroinnteóir na h-uilleann BXC, agus cruthuigh go bhfuil sé // le AD.

5) Pointí is ea X, Y, Z ar na sleasa BC, CA, AB sa triantán ABC. Tagann an ⊙ AYZ agus an ⊙ BZX le chéile arís in D.
Cruthuigh gur pointí cóimhchiorcalacha iad C, X, Y, D, ionnus go dtéigheann an ⊙ CXY tré D freisin.

Tosach leathanach 80 (atá i ndiaidh 79+1 sa LSS) sa LSS.

## Cleachtaithe

1. Tadhlann dhá chiorcai a chéile ag $T$, agus ar dhá chórda $TX$ agus $TY$ i gciorcal acu is iad $TL, TM$ na córdaí a ghearras a $\odot$ eile. Cruthuigh (i) go bhfuil $LM\|$ le $XY$, agus (ii) go bhfuil na tadhlaithe ag $L$ agus $X$ parallélach.

2. Triantán i gciorcal is ea $ABC$, agus is i bpointe $P$ a teagmhaíos an tadhlaí ag $A$ leis an mbonn $BC$. Cruthuigh go bhfuil uilleacha an triantáin $PAB$ cóimhionann le h-uilleacha an triantáin $PCA$.

3. 'Siad $P$ agus $Q$ pointí teagmhála dhá chiorcal ar láir dóibh $O$ agus $O_1$, agus córda dúbailte ar bith is ea $LPM$.

   Cruthuigh go bhfuil uilleacha an $\Delta LMQ$ cóimgionann le h-uilleacha an $\Delta OO_1P$.

4. Ceathairshleasán cóimhchioralach is ea $ABCD$, agus tagann na sleasa $AB$ is $DC$ le chéile in $X$. Faigh scáth na líne $BC$ i gcómhroinnteoir na h-uilleann $BXC$, agus cruthuigh go bhfuil sé $\|$ le $AD$.

5. Pointí is ea $X, Y, Z$ ar na sleasa $BC, CA, AB$ sa triantán $ABC$. Tagann an $\odot AYZ$ agus an $\odot BZX$ le chéile arís in $D$.

   Cruthuigh gur pointí cóimhchiorcalacha iad $C, X, Y, D$, ionas go dtéigheann an $\odot CXY$ tré $D$ freisin.

(b) I gceist 5 má thárlaíonn go bhfuil X, Y, Z in aon dronlíne amháin, cruthuigh go bhfuil D ar iomchiorcal an Δ ABC freisin.

7) Triantán i gciorcal iúa ABC agus 'sé M lár an stuaigh BC a ghabhas an uille A. Tarraingítear ingir MX agus MY ar na sleasa AB agus AC. Tré chasadh timpeall M, cruthuigh:-

(a) go bhfuil an dá thriantán BXM agus CYM congruach;

(b) má's ar $A_1$ a cuirtear A, go bhfuil $AY = YA_1$; $AX = AY = \frac{1}{2}(AB + AC)$; $BX = CY$ = leath na difríochta idir AB is AC:

(c) go bhfuil $B\hat{M}X = C\hat{M}Y$ = leath na difríochta idir na h-uilleacha B agus C.

8) I gceist 7 má 'siad NP agus NQ na h-ingir ar na sleasa AB, AC ó lár an stuaigh BAC, cruthuigh (i) go bhfuil $BP = CQ = \frac{1}{2}(AB + AC)$; (ii) go bhfuil $AP = AQ$ = leath na difríochta idir AB is AC.

## Lóci

Má ghluaiseann pointe sóinseálach i gcaoi go bhfuil coinníollacha geoméitreacha á chóimhlíonadh aige ar feadh na gluaiseachta tugtar rian nó lócus an phointe ar an lúib a ghineas sé.

Iarrtar ar an léitheoir na somplaí seo a thabhairt fá deara.

| An Coinníoll | An Rian |
|---|---|
| (a) Tá pointe sóinseálach an fhad chéanna ó dhá phointe shuite A, B | Ais shuiméitreachta na dronlíne AB. (teoirim IV) |
| (b) Tá pointe sóinseálach an fhad chéanna ó dhá dhronlíne shuite l, m. | Dhá ais shuiméitreachta na línte l, m, má's línte trasgabhlacha iad, ach dronlíne atá // leo má's línte // iad. |
| (c) Pointe sóinseálach iúa P ionnus go bhfuil buan-fhairsinge sa Δ PAB | Dronlíne atá // le AB (teoirim XVI) |
| (d) Pointe sóinseálach iúa P ionnus gur buan-uille í APB. Má's dronuille í APB | Stuagh ciorcail dírithe tré A, B. (teoirim XXI) Ciorcal ar AB mar lárlíne. |

Tosach leathanach 79+1, gan uimhir sa LSS.

6. I gceist 5 má thárlaíonn go bhfuil $X, Y, Z$ in aon dronlíne amháin, cruthuigh go bhfuil $D$ ar iomchiorcal an $\Delta ABC$ freisin.

7. Triantán i gciorcal is ea $ABC$ agus 'sé $M$ lár an stua $BC$ a ghabhas an uille $A$. Tarraingítear ingir $MX$ agus $MY$ ar na sleasa $AB$ agus $AC$. Tré chasadh timpeall $M$, cruthuigh:—
(a) go bhfuil an dá thriantán $BXM$ agus $CYM$ congrúach;
(b) má's ar $A_1$ a cuirtear $A$, go bhfuil $AY = YA_1$; $AX = AY = \frac{1}{2}(AB+AC)$; $BX = CY =$ leath na difríochta idir $AB$ is $AC$;
(c) go bhfuil $\widehat{BMX} = \widehat{CMY} =$ leath na difríochta idir na h-uilleacha $B$ agus $C$.

8. I gceist 7 má'siad $NP$ agus $NQ$ na h-ingir ar na sleasa $AB, AC$ ó lár an stua $BAC$, cruthuigh (i) go bhfuil $BP = CQ = \frac{1}{2}(AB + AC)$; (ii) go bhfuil $AP = AQ =$ leath na difríochta idir $AB$ is $AC$.

## Exersises

1. Two circles are tangent to one another at $T$, and on two chords $TX$ and $TY$ in one of the circle $TL, TM$ are the chords that cut the other ⊙. Prove (i) that $LM \|$ to $XY$, and (ii) that the tangents at $L$ and $X$ are parallel.

2. $ABC$ is a triangle inscribed in a circle, and $P$ is the point where the tangent at $A$ meets the base $BC$. Prove that the angles in the triangle $PAB$ are equal to the angles in the triangle $PCA$.

3. $P$ and $Q$ are the common points of two circles having centres $O$ and $O_1$, and $LPM$ is a double chord.

   Prove that the angles of the $\Delta LMQ$ are equal to the angles of the $\Delta OO_1P$.

4. $ABCD$ is a cyclic quadrilateral , and the sides $AB$ and $DC$ meet at $X$. Find the reflection of the line $BC$ in the bisector of the angle $BXC$, and prove that it is $\|$ to $AD$.

5. $X, Y, Z$ are points on the sides $BC, CA, AB$ in the triangle $ABC$. The $\odot AYZ$ and the $\odot BZX$ meet again at $D$.

   Prove that the points $C, X, Y, D$ are concyclic, so that the $\odot CXY$ passes through $D$ as well.

6. In Exercise 5 if it happens that $X, Y, Z$ are in a single straight line, prove that $D$ is also on the circumcircle of the $\Delta ABC$.

7. $ABC$ is a triangle inscribed in a circle, and $M$ is the centre of the arc $BC$ that subtends the angle $A$. Perpendiculars $MX$ and $MY$ are drawn to the sides $AB$ and $AC$. By rotating around $M$, prove:—

(a) that the two triangles $BXM$ and $CYM$ are congruent;
(b) if $A_1$ is where $A$ is placed, that $AY = YA_1$; $AX = AY = \frac{1}{2}(AB + AC)$; $BX = CY =$ half the difference between $AB$ and $AC$;
(c) that $\widehat{BMX} = \widehat{CMY} =$ half the difference between the angles $B$ and $C$.

8. In Exercise 7 if $NP$ and $NQ$ are the perpendiculars on the sides $AB$, $AC$ from the centre of the arc $BAC$, prove (i) that $BP = CQ = \frac{1}{2}(AB + AC)$; (ii) that $AP = AQ =$ half the difference between $AB$ and $AC$.

## Loci

Má ghluaiseann pointe sóinseálach i gcaoi go bhfuil coinníolacha geométreacha á chóimhlíonadh aige ar feadh na gluaiseachta tugtar *rian* nó *locus* an phointe ar an lúb a ghineas sé .

Iarrtar ar an léitheoir na samplaí seo a thabhairt fá deara:

| **An Coinníoll** | **An Rian** |
|---|---|
| (a) Tá pointe sóinseálach an fhad chéanna ó dhá phointe shuite $A, B$. | Ais shuiméitreachta na dronlíne $AB$ (teoirim IV) |
| (b) Tá pointe sóinseálach an fhad chéanna ó dhá dhronlíne shuite $\ell, m$. | Dhá ais shuiméitrachta na línte $\ell, m$ má's línte teagmhálacha iad, ach dronlíne ∥ leo má's línte ∥ iad. |
| (c) Pointe sóinseálach is ea $P$ ionas go fhuil buan-fhairsinge sa $\Delta PAB$. | Dronlíne atá ∥ le $AB$ (Teoirim XVI) |
| (d) Pointe sóinseálach is ea $P$ ionas gur buan-uille í $APB$. | Stua ciorcail áirithe tré $A, B$ (Teoirim XII) |
| Má's dronuille í $APB$ | Ciorcal ar $AB$ mar lárlíne. |

| | |
|---|---|
| (e) Lár ciorcail ise P a thadhlas ⊙ áirithe (nó dronlíne áirithe) ag pointe shuite A | Dronlíne áirithe tré P [sonrú an taghlai] |
| Lár córda sóinseálaigh i gciorcal ise P, a bhfuil buan-fhad sa gcórda | Ciorcal cō-lárach a ghabhas tré lár chórda ar bith acu. |

Cleachtaithe

1) Faigh rian lár na gciorcal a thadhlas dhá dronlíne.

2) Faigh rian lár na gciorcal go bhfuil ga seasmhach ionntu agus a thadhlas ciorcal áirithe.

3) Taispáin cé'n chaoi a línítear dhá chiorcal a bhfuil ga áirithe ionntu agus a thadhlas amuigh dá chiorcal a tugtar.

4) Tugtar dhá phointe A, B, agus dronlíne áirithe $\underline{l}$. Aimsigh (i) pointe in $\underline{l}$ atá cómhfhada ó A agus B; (ii) pointe $\underline{P}$ in $\underline{l}$, ionnus go mbeidh fairsinge áirithe sa Δ PAB.

5) Aimsigh lár an chiorcail a ghabhas tré phointe shuite A, agus a thadhlas dronlíne áirithe $\underline{l}$ ag pointe B a tugtar.

6) Tarraing ⊙ a thadhlas ⊙ áirithe ag pointe A a tugtar, agus a ghabhas tré phointe áirithe eile B.

7) Faigh rian an phointe shóinseálaigh P, má's eol go luigheann lár OP ar dhronlíne áirithe, áit gur pointe shuite é $\underline{O}$ a tugtar.

8) Ceangailtear pointe sóinseálach Q ar ⊙ áirithe, gurb é $\underline{O}$ a lár, le pointe shuite A. 'Sé P lár na dronlíne AQ. Cruthuigh go bhfuil fad sheasmhach idir P agus lár AO, agus d'á chionn sin faigh rian an phointe P.

9) Tré phointe áirithe tarraing córda ciorcail a mbeidh fad áirithe ann.

10) Gearrann dhá chiorcal a chéile in P agus Q agus B'iad O is $O_1$, láir na gciorcal. Córda dúbailte tré P ise LPM. Ó'n bpointe $C$, lár-phointe na líne $OO_1$, tarraingítear an t-ingear CX ar LM.

Cruthuigh CP = CX áit gurb é X lár an chórda dhúbailte LPM agus d'á chionn sin faigh rian an phointe X.

Tosach leathanach 81 sa LSS.

| | |
|---|---|
| (e) Lár ciorcail is ea $P$ a thadhlas $\odot$ áirithe (nó dronlíne áirithe) ag pointe shuite $A$ | Dronlíne áirithe tré $P$ [ sonnraí an tadhlaí ] |
| (f) Lár córda sóinseálaigh i gciorcal is ae $P$, a bhfuil buab-fhad sa gcórda | Ciorcal cómhlárach a ghabhas tré lár chórda ar bith acu |

## Loci

If a variable point moves in such a way that some geometrical conditions are satisfied while it moves, then the curve that it traces out is called a *locus*.

The reader is asked to consider the following examples:

| **The Condition** | **The Locus** |
|---|---|
| (a) The variable point is the same distance from two fixed points $A, B$. | The axis of symmetry of the straight line $AB$ (Theorem 4) |
| (b) The variable point is the same distance from two fixed straight lines $\ell, m$. | The two axes of symmetry of the lines $\ell, m$ if they are lines that meet, but a straight line $\parallel$ to them if they are $\parallel$ lines. |
| (c) $P$ is a point such that the $\Delta PAB$ has a fixed area. | A straight line $\parallel$ to $AB$ (Theorem 16) |
| (d) $P$ is a variable point such that the angle $APB$ is constant. | An arc of some circle through $A, B$ (Theorem 12) |
| $APB$ is a right angle | the circle having $AB$ as a diameter. |

| | |
|---|---|
| (e) $P$ is the centre of some circle that is tangent to a particular $\odot$ (or a particular straight line) at a fixed point $A$ | A particular straight line through $P$ [ definition of the tangent] |
| (f) $P$ is the centre of a variable chord in a circle, where the chord has a constant length. | A concentric circle that passes through any the centre of any one of the chords. |

## Cleachtaithe

1. Faigh rian lár na gciorcal a thadhlas dhá dronlíne.

2. Faigh rian lár na gciorcal go bhfuil ga seasmhach ionntu agus a thadhlas ciorcal áirithe.

3. Teaspáin cé'n chaoi a línítear dhá chiorcal a bhfuil ga áirithe ionntu agus a thadhlas amuigh dá chiorcal a tugtar.

4. Tugtar dhá phointe $A, B$, agus dronlíne áirithe $\ell$. Aimsigh
(i) pointe in $\ell$ atá cómhfhada ó $A$ agus $B$;
(ii) pointe $P$ in $\ell$, ionas go mbeidh fairsinge áirithe sa $\Delta PAB$.

5. Aimsigh lár an chiorcail a ghabhas tré phointe shuite $A$, agus a thadhlas dronlíne áirithe $\ell$ ag pointe $B$ a tugtar.

6. Tarraing $\odot$ a thadhlas $\odot$ áirithe ag pointe $A$ a tugtar, agus a ghabhas tré phointe áirithe eile $B$.

7. Faigh rian an phointe shóinseálaigh $P$, má's eol go luigheann lár $OP$ ar dhronlíne áirithe, áit gur pointe shuite é $O$ a tugtar.

8. Ceangailtear pointí sóinseálach $Q$ ar $\odot$ áirithe gurb é $O$ a lár, le pointe shuite $A$. 'Sé $P$ lár na dronlíne $AQ$. Cruthuigh go bhfuil fad sheasmhach idir $P$ agus lár $AO$, agus d'á chionn sin faigh rian an phointe $P$.

9. Tré phointe áirithe tarraing córda ciorcail a mbeidh fad áirithe ann.

10. Gearrann dhá chiorcal a chéile in $P$ agus $Q$ agus is iad $O$ is $O_1$, láir na gciorcail. Córda dúbailte tré $P$ is ea $LPM$. Ó bpointe $C$, lár-phointe na líne $OO_1$, tarraingítear an t-ingear $CZ$ ar $LM$.

    Cruthuigh $CP = CX$ áit gurb é $X$ lár an chórda dhúbailte $LPM$ agus d'á chionn sin faigh rian an phointe $X$.

## Cleachtaithe

1. Find the locus of the centres of the circles tangent to two straight lines .

2. Find the locus of the circles of some fixed radius that are tangent to some particular circle.

3. Show how to draw two circles that have a particular radius and are externally tangent to two given circles.

4. You are given two points $A, B$, and a particular straight line $\ell$. Find (i) a point in $\ell$ that is equidistant from $A$ and $B$; (ii) a point $P$ in $\ell$, such that the $\Delta PAB$ will have a specified area.

5. Find the centre of the circle that passes through a fixed point $A$, and that is tangent to a a particular straight line $\ell$ at a given point $B$.

6. Draw a ⊙ that is tangent to a particul ⊙ at a given point $A$, and that passes through another particular point $B$.

7. Find the locus of the variable point $P$, if it is known that $OP$ lies on a particular straight line , where $O$ is a given fixed point.

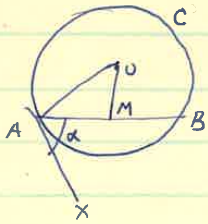
8. Variable points $Q$ on a particular ⊙ with centre $O$ are joined to a fixed point $A$. $P$ is the centre of the straight line $AQ$. Prove that the distance from $P$ to the centre of $AO$ is constant, and hence find the locus of the point $P$.

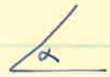
9. Through a particular point draw a chord of a circle having a specified length.

10. Two circles cut one another at $P$ and $Q$ and $O$ and $O_1$, are the centres of the circles. $LPM$is a double chord. Fromthe point $C$, the midpoint of the line $OO_1$, the perpendicular $CZ$ is drawn on $LM$.

    prove that $CP = CX$ where $X$ is the centre of the double chord $LPM$ and hence find the locus of the point $X$.

Ceist 1 Teascán ciorcail a thógáil ar bhonn áirithe, agus uille an teascáin a bheith cóimhionann le h-uillinn áirithe.




Abair gurb-é AB an bonn agus gurb í $\hat{\alpha}$ an uille a tugtar.

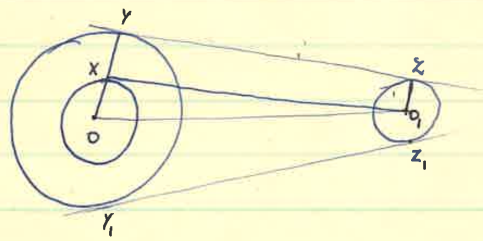
Réiteach

Tarraing an dronlíne AX a ghníos an uille $\hat{\alpha}$ le AB.
Tóg ingear ar AX ag A a ghearras ais shuimeitreachta AB in O.
Línigh an ⊙ fá'n ga OA ar lár dó O. 'Sé ACB an stuaf a fheileas.

Cruthúnas De bhrí go bhfuil O ar a.s. na líne AB, tá OA = OB, agus gabhann an ⊙ tré B freisin.

Ó thárla $O\hat{A}X = 90°$, tadhlaí ia AX, ionnas go bhfuil uille an teascáin ACB cóimhionann le $\hat{\alpha}$ (teoirim XXIV)

Ceist 2 Cóimhthadhlaí a tharraint go dtí dhá chioreal.




Abair gurb iad $O, O_1$, láir na gciorcal ar gatha dóibh $R, r$ $(R > r)$.

Réiteach

Línigh an ⊙ dárb ga $R-r$ gurb é O a lár, agus tarraing tadhlaí $O_1X$ ón bpointe $O_1$ go dtí é.

Sín OX go mbuailidh sé an imlíne in Y, agus tarraing $O_1Z$ // le OY.

Cóimhthadhlaí ia YZ.

Cruthúnas

Rinneadh $OX = OY - O_1Z$. ∴ Tá XY = agus // le $O_1Z$ de réir tógála.
Fágann sin gur ▭ é $XYZO_1$ (teoirim XIII), agus de bhrí gur dronuille í $O\hat{X}O_1$, dronuilleóg ia é.
∴ Dronuilleacha ia $\hat{Y}$ agus $\hat{Z}$, ionnas go dtadhlann YZ an dá chioreal.

Tosach leathanach 82 sa LSS.

## Ceist 1

Teascán ciorcail a thógáil ar bhonn áirithe agus uille an teascáin a bheith cóimhionann le h-uillinn áirithe.

Tá Fíoghair anseo sa LSS, leathanach 82.

Abair gurb é $AB$ an bonn agus gurb é $\hat{\alpha}$ an uille a tugtar.

*Réiteach:*

Tarraing an dronlíne $AX$ a ghníos an uill $\hat{\alpha}$ le $AB$.
Tóg ingear ar $AX$ ag $A$ a ghearas ais shuiméitreachta $AB$ in $O$.
Línigh an ⊙ fá'n nga $OA$ ar lár dó $O$. 'Sé $ACB$ an stua a fheileas.

*Cruthúnas:*

De bhrí go bhfuil $O$ ar a.s. na líne $AB$, tá $OA = OB$, agus gabhann an ⊙ tré $B$ freisin.

Ó thárla $\widehat{OAX} = 90°$, tadhlaí is ea $AX$, ionas go bhfuil uille an teascáin $ACB$ cóimhionann le $\hat{\alpha}$ (Teoirim XXIV).

## Ceist 2

Cómhthadhlaí a tharraingt go dtí dhá chiorcal.

Tá Fíoghair anseo sa LSS, leathanach 82.

Abair gurb iad $O, O_1$ láir na gciorcal ar gatha dóibh $R, r$ $(R > r)$.

*Réiteach:*

Línigh an ⊙ dárb ga $R - r$ gurb é $O$ a lár, agus tarraing tadhlaí $O_1X$ ó'n bpointe $O_1$ go dtí é.

Sín $OX$ go mbuailidh sí an imlíne in $Y$, agus tarraing $O_1Z \parallel$ le $OY$.

Cómhthadhlaí is ea $YZ$.

*Cruthúnas:*

Rinneadh $OX = OY = O_1Z$. ∴ Tá $XY$ = agus ∥ le $O_1Z$ de réir tógála. Fágann sin gur ▱ é $XYZO_1$ (Teoirim XIII), agus de bhrí gur dronuille í $\widehat{OXO_1}$, dronuillleóg is ea é .

∴ Dronuilleacha is ea $\hat{Y}$ agus $\hat{Z}$, ionas go dtadhlann $YZ$ an dá chiorcal.

## Question 1

To construct a segment of a circle on a given base so that the angle of the segment will be the same as a particular angle.

Suppose that $AB$ is the base and that $\hat{\alpha}$ is the given angle.

*Solution:*

Draw the straight line $AX$ making the angle $\hat{\alpha}$ with $AB$.
Erect a perpendicular to $AX$ at $A$ cutting the axis of symmetry of $AB$ at $O$.
Draw the ⊙ with radius $OA$ and centre $O$.
$ACB$ is the arc that suits.

*Proof:*
Since $O$ is on the a.s. of the line $AB$, we have $OA = OB$, and the circle passes through $B$ as well.
Since $\widehat{OAX} = 90°$, $AX$ is a tangent, so that the angle of the segment $ACB$ is equal to $\hat{\alpha}$ (Theorem 24).

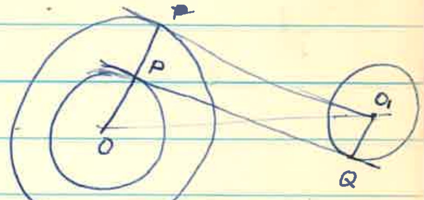
## Question 2

To draw a common tangent to two circles.
Suppose that $O, O_1$ are the centres of the circles, and the radii are $R, r$ $(R > r)$.

*Solution:*
Draw the ⊙ of radius $R - r$ with centre $O$, and draw the tangent $O_1X$ from the point $O_1$ to it.
Extend $OX$ to meet the perimeter at $Y$, and draw $O_1Z \|$ to $OY$.
Then $YZ$ is a common tangent.

*Proof:*
Draw $OX = OY = O_1Z$. ∴ $XY$ = and ∥ to $O_1Z$ by construction. It follows that $XYZO_1$ is a ▱ (Theorem 13), and since $\widehat{OXO_1}$ is a right angle, it is a rectangle.
∴ $\hat{Y}$ and $\hat{Z}$ are right angles, so that $YZ$ is tangent to the two circles.

Atora 1. Is léir gur cómhtadhlaí eile é $Y_1Z_1$, scáth na líne YZ in $OO_1$. Cómhthadhlaithe díreacha a tugtar ar YZ, agus $Y_1Z_1$.




Atora 2. Má 'sé R+r (in ionad R-r) ga an ⊙ a tuilinitear gur lár dó O, agus má leantar don réiteach thuas focal ar fhocal ina dhiaidh sin gheofar cómhthadhlaí eile PQ, d'a ngoirtear cómhthadhlaí trasnánach. Cómhthadhlaí trasnánach eile isea scáth na líne PQ in $OO_1$.

Nóta. Ní bhíonn ceithre tadhlaithe ann i gcómhnaí. Má theagmhaíonn na cioreail le chéile i ndá phointe dhifriúla, níl ach dhá chómhthadhlaí acu, de bhrí go bhfuil $O_1$ taobh istigh den ⊙ ar thagramar dó in atora 2.

Má thadhlann an dá ⊙ a chéile, tá trí cómhthadhlaithe acu má's tadhall amuigh é, ach níl thar cheann amháin má's tadhall istigh é.

Inchioreal agus Eischioreail Thriantáin.

I dteoirim B (caib.III) is cómhfhada na h-ingir ó I ar shleasa an triantáin ABC, de bhrí gur scátha a chéile iad i gcómhroinntebíri na n-uilleann.

Fágann sin go dtadhlann trí sleasa an triantáin an ⊙ ar lár do I go bhfuil ga an ⊙ sin cothrom leis an ingear ó I ar shlios ar bith. Inchioreal an triantáin a tugtar ar an gcioreal sin. 'Sé I an t-inlár.

Mar an gcéanna tá ⊙ eile ann, ar lár dó $I_1$, a thadhlas an slios BC istigh agus a thadhlas an dá shlios eile amuigh. Eischioreal de thrí eischioreail an triantáin isea é, agus 'sé $I_1$ an t-eislár atá ós cóir na h-uilleann A.

## Cleachtaithe

1) Deimhnigh gur ar líne cheangail na lár a ghearras an dá chómhthadlaí dhíreacha a chéile, agus gurb amhlaí don dá cheann eile é má's ann dóibh.

2) Tadhlann dhá ⊙ amuigh ag A, agus cómhthadhlaí díreach isea XY. Má's in B a ghearras XY an cómhthadlaí ag A, cruthuigh (i) go bhfuil BA = BX = BY, agus (ii) gur dronuille í $X\hat{A}Y$

Tosach leathanach 83 sa LSS.

### Atora 1

*Is léir gur cómhtadhlaí eile é $Y_1Z_1$, scáth na líne $YZ$ in $OO_1$.*

Cómhthadhlaithe *díreacha* a tugtar ar $YZ$, agus $Y_1Z_1$.

### Atora 2

*Má 'sé $R+r$ (in ionad $R-r$) ga an ⊙ a línítear gur lár dó $O$, agus má leantar den réiteach thuasfocal ar fhocal ina dhiaidh sin gheofar cómhthadhlaí eile $PQ$, d'a ngoirtear cómhthadhlaí* treasnánach. *Cómhthadhlaí treasnánach eile is ea scáth na líne $PQ$ in $OO_1$.*

Tá Fíoghair anseo sa LSS, leathanach 83.

### Nóta

Ní bhíonn ceithre tadhlaithe ann i cgómhnaí . Má theagmhaíonn na ciorcail le chéile i ndá phointe dhifriúla, níl ach dhá chómhthadhlaí acu, de bhrí go bhfuil$O_1$ taobh istigh den ⊙ ar thagramar dó in atora 2.

Má thadhlann an dá ⊙ le chéile, tá trí cómhthadhlaithe ann má's tadhall amuigh é ach níl thar cheann amháin má's tadhall istigh é .

### Inchiorcal agus Eischiorcail Thriantáin

I dteoirim B (Caib III) is cómhfhada na hingir ó $I$ ar shleasa an triantáin $ABC$, de bhrí gur scátha a chéile iad gcómhroinnteoirí na n-uilleann.

Fágann sin go dtadhlann trí sleasa an triantáin an ⊙ ar lár dó $I$ go bhfuil ga an ⊙ sin cothrom leis an ingear ó $I$ ar shlios ar bith. *Inchiorcal* an triantáin a tugtar ar an gciorcal sin. 'Sé $I$ an *t-imlár*.

Mar an gcéanna tá ⊙ eile ann, ar lár dó $I_1$, thadhlas an slios $BC$ istigh agus a thadhlas an dá shlios eile amuigh. *Eisciorcal* de thrí eischiorcail an triantáin is ea é , agus 'sé $I_1$ an t-eislár atá ós cóir na h-uilleann $A$.

**Corollary 1.** *It is clear that $Y_1Z_1$, the reflection of the line $YZ$ in $OO_1$ is another common tangent.*

$YZ$, and $Y_1Z_1$ are called *direct common tangents.*

**Corollary 2.** *If $R+r$ (instead of $R-r$) were the radius of the ⊙ drawn with centre $O$, and if one followed the above solution word-for-word from then on, one would get another common tangent $PQ$, which is known as a* transverse common tangent. *The reflection of $PQ$ in $OO_1$ is another transverse common tangent.*

## Note

There are not always four tangents. If the circles meets in two different points, there are only two common tangents, because $O_1$ is inside the ⊙ we referred to in Corollary 2.

If the two circles are tangent to one another, there are three common tangents if it is external tangency, but there is no more than one in the case of internal tangency.

## The Incircle and Excircles of a Triangle

In Theorem B (Chapter III) the perpendiculars from $I$ on the sides of the triangle $ABC$ are the same length, because they are reflections of one another in the bisectors of the angles.

It follows that the three sides of the triangle are tangent to the ⊙ with centre $I$ and radius equal to the perpendicular from $I$ on any side at all. That circle is called the *incircle* of the triangle. $I$ is the *incentre*.

Similarly, there is another ⊙, with centre $I_1$, tangent to the side $BC$ inside and tangent to the other two sides outside. It is one of the three *excircles* of the triangle, and $I_1$ is *the excentre opposite the angle A*.

3) Trí phointe A tarraing an dronlíne go bhfuil fad an ingir uirthi ó phointe áirithe B comhfhada le líne a tugtar.

4) Tarraing tadhlaí do chiorcal áirithe a bheas // le dronlíne áirithe.

5) Tarraingítear dhá thadhlaí ag foirchinn láirlíne, agus gearrann siad mír AB ar thadhlaí ar bith eile. Cruthuigh go ngabhann AB dronuille ag lár an chiorcail.

6) Ceathairshleasán isea ABCD gur féidir ⊙ a inscríobhadh ann (i.e. tá ⊙ ann a thadhlas na sleasa uilig istigh). Cruthuigh $AB + CD = BC + AD$

7) ~~I gceathairshleasán~~ cruthuigh fá, cheathairshleasán chómhchiorcalach gur ar imlíne an chiorcail a bhuaileas cómhroinnteóir uilleann ar bith le cómhroinnteóir na h-~~i~~uilleann ós a cóir amuigh.

8) Tarraing tadhlaí go dtí ⊙ ionnus go mbeidh fad áirithe sa gcórda a ghearras ⊙ áirithe eile air.

9) Tadhlann dhá chiorcal istigh ag A, agus córda ciorcail acu isea XY a thadhlas ~~i gciorcal acu~~ an ciorcal eile ag B. Cruthuigh gurb í AB cómhroinnteóir na h-uilleann $X\hat{A}Y$.

10) Má 'sé I inlár an △ ABC, cruthuigh $B\hat{I}C = 90° + \frac{1}{2}A$.

Faigh rian inlár thriantáin gurb eol a bhonn agus a stuacuille.

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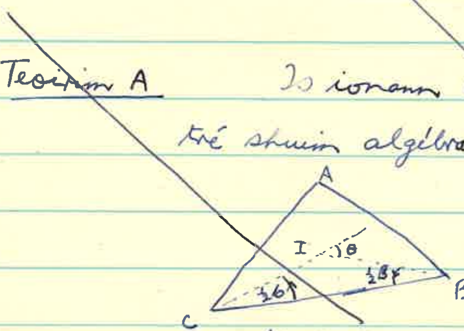
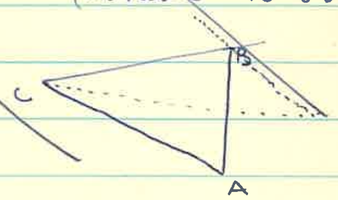
## Teoirmí Breise

**Teoirim A** Is ionann dhá chasadh as éadan agus casadh singil trí shuim algébrach na n-uilleann (marab é zéro nó 360° an t-suim)




Fioghair I



## Cleachtaithe

1. Deimhnigh gur ar líne cheangail na lár a ghearras an dá chómhthadhlí dhíreacha a chéile, agus gurb amhlaí don dá cheann eile é má's ann dóibh.

2. Tadhlann dhá ⊙ amuigh ag $A$, agus cómhthadhlaí díreach is ea $XY$. Má's in $B$ a ghearras $XY$ an cómhthadhlaí ag $A$, cruthuigh (i) go bhfuil $BA = BX = BY$, agus (ii) gur dronuille é $\widehat{XAY}$.

   Tosach leathanach 84 sa LSS.

3. Tré phointe $A$ tarraing an dronlíne go bhfuil fad an ingir uirthi ó phointe áirithe $B$ cómhfhada le líne a tugtar.

4. Tarraing tadhlaí do chiorcal áirithe a bhéas ∥ le dronlíne áirithe.

5. Tarraingítear dhá thadhlaí ag foirchinn láirlíne agus gearrann siad mír $AB$ ar thadhlaí ar bith eile. Cruthuigh go ngabhann $AB$ dronuille ag lár an chiorcail.

6. Ceathairshleasán is ea $ABCD$ gur féidir ⊙ a inscríobhadh ann (i.e. tá ⊙ ann a thadhlas na sleasa uilig istigh). Cruthuigh

$$AB + CD = BC + BD.$$

7. Cruithigh fá cheathairshleasán cóimhchiorcalach gur ar imlíne an chiorcail a bhuaileas cómhroinnteóir uilleann ar bith le cómhroinnteóir na huilleann ós a cóir amuigh.

8. Tarraing tadhlaí go dtí ⊙ ionas go mbeidh fad áirithe sa gcórda a ghearras ⊙ áirithe eile air.

9. Tadhlann dhá chiorcal istigh ag $A$, agus córda ciorcail acu is ea $XY$ a thadhlas an ciorcal eile ag $B$. Cruthuigh gurb í $AB$ cómhroinnteoir na h-uilleann $\widehat{XAY}$.

10. Má 'sé $I$ inlár an $\Delta\ ABC$, cruthuigh $\widehat{BIC} = 90° + \frac{1}{2}A$.
    Faigh rian imlár thriantáin gurb eol a bhonn agus a stuacuille.

## Exercises

1. Verify that the two direct common tangents meet on the line that joins the centres, and that the same is the case for the other two if they exist.

2. Two circles are externally-tangent at $A$, and $XY$ is a direct common tangent. If $B$ is where $XY$ cuts the common tangent at $A$, prove (i) that $BA = BX = BY$, and (ii) that $\widehat{XAY}$ is a right angle.

3. Through a point $A$ draw the straight line such that the length of the perpendicular to it from a particular point $B$ is the same as that of a given line.

4. Draw a tangent to a particular circle that will be $\parallel$ to a particular straight line .

5. Two tangents are drawn at the ends of a diameter and they cut a segment $AB$ on any other tangent. Prove that $AB$ subtends a right angle at the centre of the circle.

6. $ABCD$ is a quadrilateral in which a $\odot$ can be inscribed (i.e. there is a $\odot$ that is tangent to all the sides inside). Prove

$$AB + CD = BC + BD.$$

7. Prove that for any cyclic quadrilateral that the bisectors of opposite angles meet on the circle through the vertices.

8. Draw a tangent to a $\odot$ such that the tangent cuts another given circle in a chord of specified length.

9. Two circles are internally tangent at $A$, and $XY$ is a chord of one circle and is tangent to the other circle at $B$. Prove that $AB$ is the bisector of the angle $\widehat{XAY}$.

10. If $I$ is the incentre of the $\Delta\ ABC$, prove $\widehat{BIC} = 90^\circ + \frac{1}{2}A$.
    Find the locus of the incentre of the triangles having a given base and apex angle.

# Ceisteanna Ilgnéitheacha

1) Sa bparallélogram ABCD, téasnán ~~ísea~~ BD. Teasgmhaíonn comhroinnteóirí na n-uilleann A agus C leis sna pointí X agus Y. Cruthuigh AX = ~~CY~~ agus // le CY.

2) Tóg rhombus a bheás comhfhairsing le parallélogram a tugtar.

3) Má ghearann ⊙ córdaí comhfhada ar thrí sleasa an triantáin, cá bhfuil an lár? Tarraing ciorcal ionas go mbeidh na trí córdaí a ghearras sé ar shleasa an triantáin comhfhada le míolíne a tugtar duit.

4) I dtriantán ar bith ABC tarraingitear dhá chiorcail ar láirlínte dhóibh AB agus AC. Teaspáin go dteasgmhaíonn siad le chéile arís ar an slios BC.

5) I dtriantán dronuilleach, cruthuigh go dtagann aist simeitreachta na slios le chéile ag lár an hipotenúis.

6) Triantán i gciorcal ísea ABC. Tarraingitear dronlíne parallélach leis an tadhlaí ag A, agus gearrann sé AB, AC sna pointí X, Y. Cruthuigh go bhfuil na pointí X, Y, B, C comhchiorcalach.

7) Tagann dhá ○ le chéile ag P agus Q. Ó phointe ar bith X ar chiorcal acu tarraingitear na línte XPA agus XQB a ghearras an ○ eile sna pointí A agus B. Cruthuigh go bhfuil AB // leis an tadhlaí ag X.

8) Pointe ísea X ar an slios AB sa ▱ ABCD. Tá dronlíne // le AB a ghearras na línte AD, XD, XC, BC sna pointí L, M, N, P. Cruthuigh go bhfuil fairsinge an ▱ ABPL = 2 △XDN.

9) Tadhlann dhá ⊙ a chéile ag X agus córda dúbailte ísea PXQ. Cruthuigh go bhfuil uille an teascáin ar XP i gciorcal acu cothrom leis an uillinn ar an teascán XQ sa gceann eile.

10) Tarraingitear tadhlaithe ó phointe shuite A go dtí fuireann de chiorcail chomhláracha. Faigh réan na bpointí tadhaill.

11. I gceathairshleasán ar bith ABCD, má comhroinntear dhá uillinn chómhgharach C, D cruthuigh go bhfuil an uille idir na comhroinnteoirí cothrom le leath-shuim A agus B.

Má comhroinntear na ceithre uilleacha, cruthuigh go ngeintear ceathairshleasán comhchiorcalach leo.

12. Gearrann dhá line ingearacha a chéile ag O. Pointí orthu isea X, Y. Má shleamhnaíonn X, Y orthu i gcaoi go bhfanann fad XY buan, faigh rian lár-phointe XY.

13. Tagann dhá chórda ciorcail le chéile ag pointe istigh. Cruthuigh gurb ionann an uille eatorra agus an uille ~~atá gabhtha~~ a ghabhas leath-shuim a stuaga ag an imlíne.

Má's amuigh a ghearras siad a chéile, cruthuigh nach mór leath na difríochta a rá in áit leath na suime.

14. Pointe isea P ar an trasnán BD sa □ ABCD. Cruthuigh Δ ABP = Δ BCP i bhfairsinge.

15. Gluaiseann pointe P i gcaoi ~~go~~ gur buan don uillinn idir an dá thadhlaí ó P go dtí ciorcal áirithe. Faigh rian P.

16. Sé O iomlár an Δ ABC agus sé X lár an tsleasa AB. Cruthuigh $A\hat{O}X = \hat{C}$.

Má 'sé AD an t-ingear ar BC, cruthuigh go bhfuil $O\hat{A}D$ = ~~leath na difríochta~~ an difríocht idir $\hat{B}$ agus $\hat{C}$.

17. Sa gceathairshleasán ABCD, 'sé O lár an trasnáin BD agus pointe in BC isea X ionnas go bhfuil OX ∥ AC. Cruthuigh go gcomhroinneann AX an 4-shleasán.

18. Tagann dhá O le chéile in X, Y agus corda dúbailte isea AXB. Gearrann na tadhlaithe ag A agus B a chéile ag C. Cruthuigh CAYB comhchiorcalach.

19. Sa Δ ABC, 'siad X, Y, Z lár na slios BC, CA, AB agus D é AD an ⊥-ingear ó A ar BC. Cruthuigh (i) gur scátha a chéile in YZ iad na Δ YZA agus YZD; (ii) go bhfuil $Y\hat{X}Z = Y\hat{D}Z$; (iii) go ngabhann an ⊙ XYZ trí D.

20. ✓ Ceathairshleasán é ABCD go dtagann AB agus DC le chéile in X, agus go dtagann DA, CB le chéile in Y. Teagmhaíonn na ⊙ XBC agus YAB le chéile in B agus i bpointe eile Z.
Cruthuigh (i) $Y\hat{Z}B = \hat{A}$, $B\hat{Z}C = A\hat{X}D$ (ii) go ngabhann an ⊙ YDC trí Z; (iii) mar an gcéanna go ngabhann an ⊙ ADX trí Z.
Sé sin le rá, maidir leis na ceithre triantáin a gintear le ceithre dronlíne ar bith, tá pointe áirithe ar cheithre imchiorcail na dtriantán sin.

21. ✓ Má tharlaíonn i gceist 20 gur ceathairshleasán comhchiorclach é ABCD, taispeáin go bhfuil X, Z, Y in aon dronlíne amháin.

Tosach leathanach 85 sa LSS.

## 7.8 Ceisteanna Ilgnéitheacha

1. Sa bparallélogram $ABCD$, treasnán is ea $BD$. Teagmhaíonn cómhroinnteóirí na n-uilleann $A$ agus $C$ leis sna pointí $X$ agus $Y$. Cruthaigh $AX$ = agus ∥ le $CY$.

2. Tóg rhombus a bhéas cómhfhairsing le parallélogram a tugtar.

3. Má ghearrann ⊙ córdaí cómhfhada ar thrí sleasa an triantáin, cá bhfuil an lár? Tarraing ciorcal ionas go mbeidh na trí córdaí a ghearras sé ar shleasa an triantáin cómhfhada le mírlíne a tugtar duit.

4. I dtriantán ar bith $ABC$ tarraingítear dhá chiorcal ar lárlínte dóibh $AB$ agus $AC$. Teaspáin go dteagmhaíonn siad le chéile arís ar an shlios $BC$.

5. I dtriantán dronuilleach, cruthuigh go dtagann aisí suiméitreachta na slios le chéile ag lár an hipoteús.

6. Triantán i gciorcal is ea $ABC$. Tarraingítear dronlíne parllélach leis an tadhlaí ag $A$, agus gearrann sí $AB, AC$ sna pointí $X, Y$. Cruthuigh go bhfuil na pointí $X, Y, B, C$ cóimhchiorcalach.

7. Tagann dhá ⊙ le chéile ag $P$ agus $Q$. Tré phointe ar bith $X$ ar chiorcal acu tarraingítear na línte $XPA$ agus $XQB$ a gharras an ⊙ eile sna pointí $A$ agus $B$. Cruthuigh go bhfuil $AB$ ∥ leis an tadhlaí ag $X$.

8. Pointe is ea $X$ ar an slios $AB$ sa ▱ $ABCD$. Tá dronlíne ∥ le $AB$ a ghearras na línte $AD, XD, XC, BC$ sna pointí $L, M, N, P$. Cruthuigh go bhfuil fairsinge an ▱ $ABPL = 2AXDN$.

9. Tadhlann dhá chiorcal a chéile ag $X$ agus córda dúbailte is ea $PXQ$. Cruthuigh go bhfuil uille an teascáin ar $XP$ is gcioral acu cothrom leis an uillinn ar an teascán $XQ$ sa gceann eile.

10. Tarraingítear tadhlaithe ó phointe shuite $A$ go dtí foirceann de ciorcail cómhláracha. Faigh rian na bpointí tadhaill.

    Tosach leathanach 86 sa LSS.

11. I gceathairshleasáin ar bith $ABCD$, má cómhroinntear dhá uillinn chómhgharach $C, D$ cruthuigh go bhfuil an uille idir na cómhroinnteóirí cothrom le leath shuim $A$ agus $B$.

    Má cómhroinntear na ceithre uilleacha, cruthuigh go ngintear ceathairshleasán cómhciorcalach leo.

12. Gearrann dhá líne ingearacha a chéile ag $O$. Pointí orthu is ea $X, Y$. Má shleamhníonn $X, Y$ orthu i gcaoin go bhfanann fad $XY$ buan, faigh rian lár-phointe $XY$.

13. Tagann dhá córda chiorcail le chéile ag pointe istigh. Cruthuigh gurb ionann an uille eatorru agus an uille a ghabhas leath-shuim a stua ag an imlíne.

    Má's amuigh a ghearras siad a chéile cruthuigh nachmór leath na difríochta a rá in áit leath na suime.

14. Pointe is ea $P$ ar treasnán$BD$ san $\varparallelogram$ $ABCD$. Cruthuigh $\Delta ABD = \Delta BCP$ i bhfairsinge.

15. Gluaiseann pointe $P$ i gcaoin gur buan don uillinn idir an dá thadhlaí ó $P$ go dtí ciorcal áirithe. Faigh rian $P$.

16. Sé $O$ iomlár an $\Delta ABC$ agus sé $X$ lár an tsleasa $AB$. Cruthuigh $\widehat{AOX} = \hat{C}$.

    Má 'sé $AD$ an t-ingear ar $BC$, cruthuigh go bhfuil $\widehat{OAD}$ = an difríocht idir $\hat{B}$ agus $\hat{C}$.

17. Sa gceathairshleasán $ABCD$, 'sé $O$ lár an treasnáin $BD$ agus pointe in $BC$ is ea $X$ ionas go bhfuil $OX \parallel AC$. Cruthuigh go gcómhroinneann $AX$ an 4-shleasán.

18. Tagann dhá $\odot$ le chéile in $X, Y$ agus córda dúbailte is ea $AXB$. Gearrann na tadhlaithe ag $A$ agus $B$ le chéile ag $C$. Cruthuig $CAYB$ cóimhchiorcalach.

    Tosach leathanach 87 sa LSS.

19. Sa $\Delta$ $ABC$ 'siad $X, Y, Z$ láir na slios $BC, CA, AB$ agus 'sé $AD$ an t-ingear ó $A$ ar $BC$. Cruthuigh (i) gur scátha a chéile in $YZ$ iad na $\Delta$ $YZA$ agus $YZD$; (ii) go bhfuil $\widehat{YXZ} = \widehat{YDZ}$; (iii) go bgabhann an $\odot$ $XYZ$ tré $D$.

20. Ceathairshleasán é $ABCD$ go dtagann $AB$ agus $DC$ le chéile in $X$, agus go dtagann $DA, CB$ le chéile in $Y$. Teagmhaíonn na $\odot$ $XBC$ agus $YAB$ le chéile in $B$ agus i bpointe eile $Z$. Cruthuigh (i) $\widehat{YZB} = \hat{A}$, $\widehat{BXC} = \widehat{AXD}$, (ii) go ngabhann an $\odot$ $YDC$ tré $Z$, (iii) mar an gcéanna go ngabhann an $\odot$ $ADX$ tré $Z$.

    Sé sin le rá , maidir leis na ceithre triantáin a gintear le ceithre dronlínte ar bith, tá pointe áirithe ar cheithre iomchiorcail na dtriantán sin.

21. Má thárlaíonn i gceist 20 gur ceathaircshleasán cóimhchiorcalach é $ABCD$, teaspáin go bhfuil $X, Y, Z$ is aon dronlíne amháin.

## 7.9 Miscellaneous Exercises

1. In the parallelogram $ABCD$, $BD$ is a diagonal. The bisectors of the angles $A$ and $C$ meet it in the points $X$ and $Y$. Prove that $AX$ = and $\parallel$ to $CY$.

2. Construct a rhombus having the same area as a given parallelogram.

3. If a ⊙ cuts chords of equal length on the three sides of a triangle, where is the centre?

   Draw a circle with the property that the three chords it cuts on the sides of a triangle have equal length, equal to some given line segment.

4. In any triangle $ABC$ two circles are drawn having diameters $AB$ and $AC$. Show that they meet one another again on the side $BC$.

5. In a right-angle triangle, prove that the axes of symmetry of the sides meet at the centre of the hypotenuse.

6. $ABC$ is a triangle inscribed in a circle. A straight line is drawn parallel to the tangent at $A$, and it cuts $AB, AC$ at the points $X, Y$. Prove that the points $X, Y, B, C$ are concyclic.

7. Two circles meet one another at $P$ and $Q$. Through any point $X$ one one of the circles the lines $XPA$ and $XQB$ are drawn, cutting the other circle at the points $A$ and $B$. Prove that $AB \parallel$ to the tangent at $X$.

8. $X$ is a point on the side $AB$ in the ▱ $ABCD$. There is a straight line ∥ to $AB$ that cuts the lines $AD, XD, XC, BC$ at the points $L, M, N, P$. Prove that the area of the ▱ $ABPL = 2AXDN$.

9. Two circles are tangent to one another at $X$ and $PXQ$ is a double chord. Prove that the angle of the segment on $XP$ in one of the circles is equal to the angle of the segment $XQ$ in the other one.

10. Tangents are drawn from a fixed point $A$ to the edges of concentric circles. Find the locus of the points of tangency.

11. In any quadrilateral $ABCD$, if the two adjacent angles $C, D$ are bisected, prove that the angle between the bisectors is equal to half the sum of $A$ and $B$.

    If the four angles are bisected, prove that generates a cyclic quadrilateral .

12. Two perpendicular lines meet at $O$. $X, Y$ are points on them. If $X, Y$ slide on them in such a way that the length of $XY$ stays constant, find the locus of the midpoint of $XY$.

13. Two chords of a circle meet at a point inside it. Prove that the angle between them is equal to that subtended by half the sum of their arcs at the perimeter.

    If they meet outside prove that you have to say half the difference instead of half the sum.

14. $P$ is a point on the diagonal $BD$ in the ▱ $ABCD$. Prove $\Delta ABD = \Delta BCP$ in area.

15. A point $P$ moves so that the angle between the two tangents from $P$ to a particular circle is constant. Find the locus of $P$.

16. $O$ is the circumcentre of the $\Delta ABC$ and $X$ is the centre of the side $AB$. Prove that $\widehat{AOX} = \hat{C}$.

    If $AD$ is the perpendicular on $BC$, prove that $\widehat{OAD}$ = the difference between $\hat{B}$ and $\hat{C}$.

17. In the quadrilateral $ABCD$, the centre of the diagonal $BD$ is $O$ and $X$ is a point $BC$ such that $OX \| AC$. Prove that $AX$ bisects the quadrilateral .

18. Two $\odot$ meet at $X, Y$ and $AXB$ is a double chord. The tangents at $A$ and $B$ meet at $C$. Prove that $CAYB$ is concyclic.

19. In the $\Delta$ $ABC$ the centres of the sides $BC, CA, AB$ are $X, Y, Z$ and $AD$ is the perpendicular from $A$ on $BC$. Prove (i) that $\Delta$ $YZA$ and $YZD$ are reflections of one another in $YZ$. (ii) that $\widehat{YXZ} = \widehat{YDZ}$; (iii) that the $\odot$ $XYZ$ passes through $D$.

20. $ABCD$ is a quadrilateral for which $AB$ and $DC$ meet at $X$, and $DA, CB$ meet at $Y$. The $\odot XBC$ and $YAB$ meet one another at $B$ and in another point $Z$. Prove (i) $\widehat{YZB} = \hat{A}$, $\widehat{BXC} = \widehat{AXD}$, (ii) that the $\odot YDC$ passes through $Z$, (iii) and similarly that the $\odot ADX$ passes through $Z$.

    That is to say, with regard to the four triangles generated by any four straight lines at all, there is a special point on the four circumcircles of those triangles.

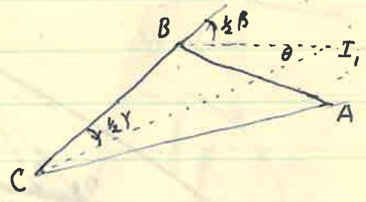
21. If it happens in Exercise 20 that the quadrilateral $ABCD$ is cyclic, show that $X, Y, Z$ lie on a single straight line.

# Teoirmí Breise

## Teoirim A

Is ionann dhá chasadh as éadan agus casadh singil tré shuim algébrach na n-uilleann (marab é zéro nó 360° an t-suim)


Fíogh I




Fíogh II

## Hipotesis

Castar an plána timpeall B agus C as éadan tré na h-uilleacha $\hat{\beta}$ is $\hat{\gamma}$.
[I bhFíogh I tá $\hat{\beta}$ agus $\hat{\gamma}$ deimhneach; i bhFíogh II tá $\beta$ deimhneach ach tá $\gamma$ diúltach]

**Tógáil** Tarraing an líne BA a leagtar fan BC de bharr an chasta timpeall B, agus an líne BA go leagtar bB uirthi sa gcasadh eile.

Abair gurb iad BI, bI comhroinnteóirí na n-uilleann $\hat{\beta}$ is $\hat{\gamma}$.

Is ionann an dá chasadh agus casadh singil timpeall I.

## Cruthúnas

Is ionann an casadh timpeall B agus dhá scáthú sna línte BI agus BC as a chéile [Teoirim A, caib III]

Mar an gcéanna is ionann an casadh timpeall b agus dhá scáthú sna línte bB agus bI as ~~a chéile~~ éadan.

~~Má chuirtear le chéile ga~~ Ag cur na scáthuithe sin le chéile dúinn, ní miste neamh-shuim a dheanamh den dá scáthú in BC as éadan, agus fágtar dhá scáthú in BI agus bI gurb ionann le chéile iad agus casadh timpeall I tré n uillinn $2\theta$

Ach, de thairbhe teoirme , tá $2\theta = \hat{\beta} + \hat{\gamma}$ (Fíogh I), nó $2\theta = \hat{\beta} - \hat{\gamma}$ (Fíogh II)

Q.E.D

1) Má's pointí iad X, Y ar dhá dhronlíne // $l$ agus $m$, ionnus go bhfuil XY ⊥ le $l$, teaspáin gurb ionann dhá scáthú in $l$ agus $m$ agus aistriú fan XY tré 2XY.

2) Má's é zéro (nó 360°) suim na n-uilleann $\hat{\beta}$ is $\hat{\gamma}$ sa teoirim, cruthuigh gurb ionann agus aistriú áirithe an dá scáthú.

3) Síntear sleasa triantáin ABC, agus castar an plána timpeall A, B, C as éadan tré na h-uilleacha deimhneacha amuigh.
Teaspáin gurb ionann é sin agus aistriú fan na líne bA ina gcuirleann gach pointe an fhad AB+BC+CA de.

Tosach leathanach 88 sa LSS.

## 7.10 Teoirmí Breise

# Extra Theorems

## 7.11 Teoirim A

*Is ionann dhá chasadh as éadan agus casadh singil tré shuim algébrach na n-uilleann (morab í zéro nó* 360° *an tsuim).*

Tá Fíoghair anseo sa LSS, leathanach 88.

*Hipotéis:*

Castar an plána timpeall $B$ agus $C$ as éadan tré na h-uilleacha $\hat{\beta}$ is $\hat{\gamma}$.
[ I bhFiog I tá $\hat{\beta}$ agus $\hat{\gamma}$ deimhneach; in bhFiog II tá $\beta$ deimhneach ach tá $\gamma$ diúltach.]

*Tógáil:*

Tarraing an líne $BA$ a leagtar fan $BC$ de bharr an chasta timpeall $B$, agus an líne $CA$ go leagtar $CB$ uirthi sa gcasadh eile.

Abair gurb iad $BI, CI$ cómhroinnteoirí na n-uilleann $\hat{\beta}$ is $\hat{\gamma}$.

Is ionann an dá chasadh agus casadh singil timpeall $I$.

*Cruthúnas:*

Is ionann an casadh timpeall $B$ agus dhá scáthú sna línte $BI$ agus $BC$ as a chéile [Teoirim A, Caib III].

Mar an gcéanna is ionann an casadh timpeall $C$ agus dhá scáthaú sna línte $CB$ agus $CI$ as éadan.

Ag cur na scáthuithe sin le chéile dúinn, ní miste neamhs-shuim a déanamh den dá scáthú in $BC$ as éadan, agus fágtar dhá scáthú in $BI$ agus $CI$ gur ionann le chéile iad agus casadh timpeall $I$ tré uillinn $2\theta$.

Ach, de thairbhe teoirme, tá $2\theta = \hat{\beta} + \hat{\gamma}$ (Fiogh I), nó $2\theta = \hat{\beta} - \hat{\gamma}$ (Fiogh II). □

**Theorem A.** *Two successive rotations are equivalent to single rotation through the algebraic sum of the angles (except when the sum is zero or* 360°*).*

*Hypothesis:*

The plane is rotated around $B$ and $C$ in sequence through the angles $\hat{\beta}$ and $\hat{\gamma}$.
[ In Fig I, $\hat{\beta}$ and $\hat{\gamma}$ iare positive; in Fig II, $\beta$ is positive but $\gamma$ is negative.]

*Construction:*

Draw the line $BA$ which is laid along $BC$ by the rotation about $B$, and the line $CA$ on which $CB$ iis laid by the other rotation.

Suppose that $BI, CI$ are the bisectors of the angles $\hat{\beta}$ and $\hat{\gamma}$.

The two rotations are equivalent to a single rotation about $I$.

*Proof:*

The rotation about $B$ is equivalent to two reflections in the lines $BI$ and $BC$ one after the other [Theorem A, Chapter III].

Similarly the rotation about $C$ is equivalent to the two reflections in the lines $CB$ and $CI$ iin order.

Putting those reflections together, we can ignore the two reflections $BC$ after one another, and we are left with two reflections in $BI$ and $CI$ which together are the same as a rotation about $I$ through the angle $2\theta$.

But by a theorem, $2\theta = \hat{\beta} + \hat{\gamma}$ (Fig I), or $2\theta = \hat{\beta} - \hat{\gamma}$ (Fig II). □

1. Má's pointí iad $X, Y$ ar dhá dhronlíne ∥ le $\ell$ agus $m$ ionas go bhfuil $XY$ ⊥ le $\ell$, teaspáin gurb ionann dhá scáthú in $\ell$ agus $m$ agus aistriú fan $XY$ tré $2XY$.

2. má 'sé zéro (nó 360°) suimna h-uilleann $\beta$ is $\gamma$ sa teoirim, cruthuigh gurb ionann agus aistriú áirithe an dá scáthú .

3. Síntear sleasa triantáin $ABC$, agus castar an plána timpeall $A, B, C$ as éadan tré na huilleacha deimhneacha amuigh.

   Teaspáin gurb ionann é sin agus aistriú fan na líne $CA$ ina gcuireann gach pointe an fhad $AB + BC + CA$ de.

1. If $X, Y$ are points on two straight lines ∥ to $\ell$ and $m$ so that $XY$ ⊥ le $\ell$, show that two reflections in $\ell$ and $m$ are the same as the translation along $XY$ through $2XY$.

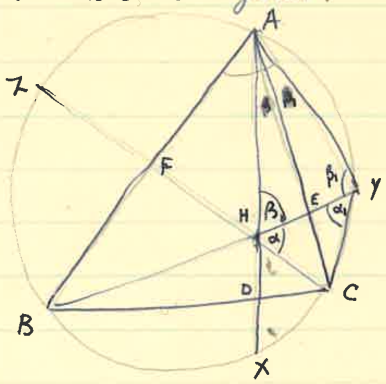
2. If the sum of the angles $\beta$ and $\gamma$ in the theorem is zero (or 360°), prove that the two reflections are cthe same as some translation.

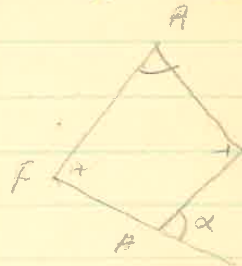
3. The sides of a triangle $ABC$ are extended, and the plane is rotated about $A, B, C$ in turn through the positive external angles.

   Show that that is equivalent to the translation along the line $CA$ in which each point moves the distance $AB + BC + CA$.

## Teoirim B

I dtriantán ar bith línte comhreathacha is ea na h-ingir ó na reanna ar na sleasa ós a gcóir.




if H outside ABC ?

### Hipotesis

Is í bpointe H a thagann na h-ingir BE is CF ar na sleasa AC, AB le chéile.

### Tátall

'Sé AH an t-ingear ó A ar BC. go dtí X, Y, Z.

### Tógáil

Línigh iomchiorcal an Δ, agus sín AH, BH, CH, ~~ceangail~~ AY, CY.

### Cruthúnas

Ó's dronuilleacha iad $\hat{E}$ agus $\hat{F}$, pointí comhchiorcalacha is ea A, F, H, E, ionnus go bhfuil $\hat{\alpha} = B\hat{A}C$ (teoirim XXI)

Ach tá $B\hat{A}C = \hat{\alpha}_1$ atá ar aon stuagh leis. $\therefore$ Tá $\hat{\alpha} = \hat{\alpha}_1$.

$\therefore$ Triantán comhchosach is ea CHY, agus ó tharla CE ⊥ le HY, scátha a chéile in AC is ea H agus Y.

Fágann sin $\hat{\beta} = \hat{\beta}_1$ atá ar aon stuagh amháin le $\hat{C}$.

De tháirbhe $\hat{\beta} = \hat{C}$, 4-shleasán comhchiorcalach is ea HECD, agus ó's dronuille $\hat{E}$ (hipotesis), dronuille eile is ea $\hat{D}$ (teoirim XXI)

Q.E.D.

**Teárma** Ingearlár an Δ a tugtar ar H; 'sé DEF triantán bhun na n-ingear.

**Atora 1** 'Siad X, Y, Z scátha an phointe H sna sleasa BC, CA, AB.

**Atora 2** 'Siad A, B, C láir na stuagh YZ, ZX agus XY
mar tá $Z\hat{C}A = Y\hat{C}A$ (scátha a chéile in AC), etc.

**Atora 3** Ó tharla HD = DX, HE = EY, HF = FZ, tá sleasa an Δ DEF // le sleasa an Δ XYZ.

'Sé H inlár an Δ DEF, agus 'siad A, B, C na h-eislár.

Tosach leathanach 89 sa LSS.

## 7.12 Teoirim B

*I dtriantán ar bith línte cóimhreathacha is ea na hingir ó na reanna ar na sleasa ós a gcóir.*

Tá Fíoghair anseo sa LSS, leathanach 89.

*Hipotéis:*
Is i bpointe $H$ a thagann na hingir $BE$ is $CF$ ar na sleasa $AC, AB$ le chéile.

*Tátall:*
'Sé $AH$ an tingear ó $A$ ar $BC$.

*Tógáil:*
Línigh ionchiorcal an $\Delta$, agus sín $AH, BH, CH$ go dtí $X, Y, Z$. Ceangail $AY, CY$.

*Cruthúnas:*
'Os dronuilleacha iad $\hat{E}$ agus $\hat{F}$, pointí cóimhchiorcalacha is ea $A, F, H, E$, ionas go bhfuil $\hat{\alpha} = \widehat{BAC}$ (Teoirim XXI).

Ach tá $\widehat{BAC} = \hat{\alpha}_1$ atá ar aon stua leis. $\therefore$ Tá $\hat{\alpha} = \hat{\alpha}_1$.

$\therefore$ triantán cómhchosach is ea $CHY$, agus ó thárla $CE \perp$ le $HY$, scátha a chéile in $AC$ is ea $H$ agus $Y$.

Fágann sin $\hat{\beta} = \hat{\beta}_1$, atá ar aon stua amháin le $\hat{C}$.

De thairbhe $\hat{\beta} = \hat{C}$, 4-shleasán cóimhchiorcalach is ea $HECD$, agus ó's dronuille $\hat{E}$ (hipotesis), dronuille eile is ea $\hat{D}$ (Teoirim XXI). □

**Theorem B.** *In any triangle at all the perpendiculars from the vertices on the opposite sides are concurrent.*

*Hypothesis:*
$H$ is the point where the perpendiculars $BE$ and $CF$ on the sides $AC, AB$ meet one another.

*Conclusion:*
$AH$ is the perpendicular from $A$ on $BC$.

*Construction:*
Draw the circumcircle of the $\Delta$, and extend $AH, BH, CH$ to $X, Y, Z$. Join $AY, CY$.

*Proof:*
Since $\hat{E}$ and $\hat{F}$ are right angles, the points $A, F, H, E$ are concyclic, so that $\hat{\alpha} = \widehat{BAC}$ (Theorem 21).

But $\widehat{BAC} = \hat{\alpha}_1$ which is on the same arc. $\therefore$ Tá $\hat{\alpha} = \hat{\alpha}_1$.

$\therefore$ $CHY$ is an isosceles triangle, and since $CE \perp$ to $HY$, the points $H$ and $Y$ are reflections of one another in $AC$.

It follows that $\hat{\beta} = \hat{\beta}_1$, which is on the same arc as $\hat{C}$.

Since $\hat{\beta} = \hat{C}$, the quadrilateral $HECD$ is cyclic, and since $\hat{E}$ is a right angle (hypothesis), $\hat{D}$ is also a right angle (Theorem 21). □

## Téarma

*Ingearlár* a tugtar ar $H$; 'sé $DEF$ triantán bun na n-ingear.

## Atora 1

*'Siad $X, Y, Z$ scátha an phointe $H$ sna sleasa $BC, CA, AB$.*

## Atora 2

*'Siad $A, B, C$ láir na stua $YZ, ZX$ agus $XY$*

Mar tá $\widehat{ZCA} = \widehat{YCA}$ (scátha a chéile in $AC$), etc.

## Atora 3

*Ó thárla $HD = DX$, $HE = EY$, $HF = FX$, tá sleasa an $\Delta DEF \parallel$ le sleasa an $\Delta XYZ$.*

'Sé $H$ inlár an $\Delta DEF$, agus 'siad $A, B, C$ ha h-eisláir.

## Definition

$H$ is called the *orthocentre*; $DEF$ is called the *orthic triangle.*

**Corollary 1.** *$X, Y, Z$ are the reflections of the point $H$ in the sides $BC, CA, AB$.*

**Corollary 2.** *$A, B, C$ are the centres of the arcs $YZ, ZX$ and $XY$*

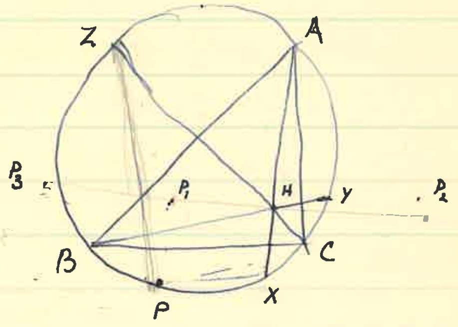
for $\widehat{ZCA} = \widehat{YCA}$ (reflections of one another in $AC$), etc.

**Corollary 3.** *Since $HD = DX$, $HE = EY$, $HF = FX$, the sides of the $\Delta DEF \parallel$ to the sides of the $\Delta XYZ$.*

*$H$ s the incentre of the $\Delta DEF$, and $A, B, C$ are the excentres.*

## Teoirim C

Trí pointí in aon dronlíne amháin is ea scátha pointe ar imlíne chiorcail i sleasa thriantáin inscríobhtha.



### Hipotesis

'Siad $P_1, P_2, P_3$ scátha $P$ sna sleasa $BC$, $CA$, $AB$.

### Tátall

Luigheann $P_3$ (agus $P_2$) ar líne cheangail $H$ agus $P_1$.

### Cruthúnas

'Sé $B$ lár an stuaigh $ZX$, agus ó tharla $X\hat{B}C = C\hat{B}H$, $Z\hat{B}A = A\hat{B}H$, (teoirim A), tá $X\hat{B}Z = 2\hat{B}$. $= X\hat{P}Z$ $\quad X \to Z$

∴ buirtear $XP$ fan $ZP$ de bharr chasta tré'n uillinn $2\hat{B}$ timpeall $B$, gurb ionann é agus dhá scathú in $BC$ is $BA$ as a chéile (Caib III).

'Sé $HP_1$ scáth na líne $XP$ in $BC$, agus fágann sin gur scátha a chéile iad na línte $HP_1$ agus $ZP$ sa slios $AB$.

∴ Tá $P_3$, scáth $P$ in $AB$ ar an líne $HP_1$, agus mar an gcéanna tá $P_2$ uirthi freisin. $[P \in ZP \Leftrightarrow P_3 \in HP_1]$

Q.E.D.

**Atora 1** Pointí comhlíneacha is ea bonn na n-ingear a tarraingítear ar shleasa thriantáin ó phointe ar bith ar an iomchiorcal.

Mar is iad láir na línte $PP_1$, $PP_2$, $PP_3$ na trí bonn sin, conas go luigheann siad ar an dhronlíne tré lár $PH$ atá // le $P_1P_2P_3H$.

Troighlíne $P$ a tugtar ar an líne úd.

**Atora 2** Má scáithtear i sleasa thriantáin dronlíne ar bith tré'n ingearlár, is trí dronlínte comhreathacha a gheintear, agus tagann siad le chéile ar an iomchiorcal.

Tosach leathanach 90 sa LSS.

## 7.13 Teoirim C

*Trí pointí in aon dronlíne amháin is ea scátha pointe ar imlíne chiorcail i sleasa thriantáin inscríobhthe.*

Tá Fíoghair anseo sa LSS, leathanach 90.

*Hipotéis:*

'Siad $P_1, P_2, P_3$ scátha $P$ sna sleasa $BC, CA, AB$.

*Tátall:*

Luigheann $P_3$ (agus $P_2$) ar líne cheangail $H$ agus $P_1$.

*Cruthúnas:*

'Sé $B$ lár an stua $ZX$, agus ó thárla $\widehat{XBC}=\widehat{CBH}$, $\widehat{ZBA}=\widehat{ABH}$, (Teoirim A), tá $\widehat{XBZ}=2\hat{\beta}$. $=\widehat{XPZ}$ $X \to Z$

∴ Cuirtear $XP$ fan $ZP$ de bharr chasta tré'n uillinn $2\hat{\beta}$ timpeall $B$, gurb ionann é agus dhá scáthú in $BC$ is $BA$ as a chéile (Caib III).

'Sé $HP_1$ scáth na líne $XP$ in $BC$, agus fágann sin gur scátha a chéile iad na línte $HP_1$ agus $ZP$ sa slios $AB$.

∴ Tá $P_3$, scáth $P$ in $AB$ ar an líne $HP_1$, agus mar an gcéanna tá $P_2$ uirthi freisin.
[$P \in ZP \iff P_3 \in HP_1$ ] □

**Theorem C.** *The reflections of a point on the perimeter of a circle in the sides of any inscribed triangle are three collinear points.*

*Hypothesis:*

$P_1, P_2, P_3$ are the reflections of $P$ in the sides $BC, CA, AB$.

*Conclusion:*

$P_3$ (and $P_2$) lie on the line joining $H$ and $P_1$.

*Proof:*

$B$ is the centre of the arc $ZX$, and since $\widehat{XBC}=\widehat{CBH}$, $\widehat{ZBA}=\widehat{ABH}$, (Theorem A), we have $\widehat{XBZ}=2\hat{\beta}$. $=\widehat{XPZ}$ $X \to Z$

∴ $XP$ s placed along $ZP$ by the rotation through the angle $2\hat{\beta}$ about $B$, which is the same as two reflections in $BC$ and $BA$ in turn (Chapter III).

$HP_1$ is the reflection of the line $XP$ in $BC$, and iit follows that the lines $HP_1$ and $ZP$ are reflections of one another in the side $AB$.

∴ $P_3$, the reflection of $P$ in $AB$ is on the line $HP_1$, and in the same way $P_2$ s also on it.
[$P \in ZP \iff P_3 \in HP_1$ ] □

## Atora 1

*Pointí cóimhlíneacha is ea bun na n-ingear a tarraingítear ar shleasa thriantáin ó phointe ar bith ar an iomchiorcal.*

Mar siad láir na línte $PP_1, PP_2, PP_3$ na trí bun sin, ionas go luigheann sian aaaaar an dhronlíne tré lár $PH$ atá ∥ le $P_1P_2P_3H$.

*Troighlíne P* a tugtar ar an líne úd.

## Atora 2

*Má scáithtear i sleasa thriantáin dronlíne ar bith tré'n ingearlár, is trí dronlínte cóimhreathacha a gintear, agus tagann siad le chéile ar an iomchiorcal.*

**Corollary 1.** *The feet of the perpendiculars on the sides of a triangle from any point on the circumcircle are collinear.*

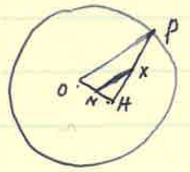
For those three feet are the centres of the line $PP_1, PP_2, PP_3$ so that they lie on the straight line through the centre of $PH$ that is ∥ to $P_1P_2P_3H$.

That line is called a *Simson line.*

**Corollary 2.** *If any straight line through the orthocentre of a triangle is reflected in the sides of the triangle, then the three straight lines that result are concurrent, and the point where they meet is on the circumcircle.*

## Teoirim D   Ciorcal Naoi bPointe an Triantáin

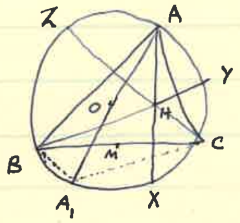
**Lemma** Má ceangailtear pointe socair H le pointe sóinseálach P ar imlíne chiorcail, luíonn lár na líne ceangail ar chiorcal áirithe.



Má 'sé X lár HP agus má 'sé N lár HO, is léir go bhfuil $NX = \frac{1}{2} OP$.
Athraíonn X dá réir i gcaoi go bhfuil fad áirithe idir é agus pointe socair N.

Fágann sin gur ciorcal é (ar lár dó N) lorg an phointe X.

**Teoirim D** I dtriantán ar bith, gabhann ciorcal tré na naoi bpointí seo; lár na slios, bun na n-ingear ó na reanna ar na sleasa, agus lár-phointí na línte a cheanglaíos an t-ingearlár agus reanna an triantáin.



**Tógáil** Tarraing $AOA_1$, an láirlíne trí A. Ceangail $A_1B$, $A_1C$.

**Cruthúnas**

Ó's láirlíne í $AA_1$, dronuillé is ea $A_1BA$, agus fágann sin go bhfuil $A_1B \parallel$ le CH, mar táid $\perp$ le AB.

Mar an gcéanna tá $A_1C \parallel$ le BH, ionas gur ▱ é $BHCA_1$.

$\therefore$ Tá lár an tsleasa BC leathbhealaigh idir H agus $A_1$.

Is léir anois go gcomhroinneann na naoi bpointí a luaitear sa teoirim naoi línte áirithe ó H go dtí an imlíne, ionas go luíonn siad féin ar chiorcal dárb ga $\frac{1}{2} OA$, go bhfuil a lár leath-bhealaigh idir H agus O.

Ciorcal naoi bpointe an triantáin a tugtar ar an gciorcal sin.

**Atora** Ó tharla $A_1O = OA$ agus $A_1M = MH$, tá $OM = \frac{1}{2} AH$.

Tosach leathanach 91 sa LSS.

## 7.14 Teoirim D (Ciorcal Naoi bPointe an Triantáin)

### Lemma

Má ceangailtear pointe socair $H$ le pointe sóinseálach $P$ ar imlíne chiorcail, luíonn lár na líne ceangail ar chiorcal áirithe.

Tá Fíoghair anseo sa LSS, leathanach 91.

Má 'sé $X$ lár $HP$ agus má 'sé $N$ lár $HO$, is léir go bhfuil $NX = \frac{1}{2}OP$. Athríonn $X$ d'áréir i gcaoi go bhfuil fad áirithe idir é agus pointe socair $N$.

Fágann sin gur ciorcal é (ar lár dó $N$) lorg an phointe $X$.

### Teoirim D

*I dtriantán ar bith, gabhann ciorcal tré na naoi bpointí seo; lár na slios, bun na n-ingear ó na reanna ar na sleasa, agus lár-phointí na línte a cheanglaíos an t-ingearlár agus reanna an triantáin.*

Tá Fíoghair anseo sa LSS, leathanach 91.

*Tógáil:*

Tarraing $AOA_1$ an lárlíne tré $A$. Ceangail $A_1B, A_1C$.

*Cruthúnas:*

Ó's láirlíne í $AA_1$, dronuille is ea $A_1BA$, agus fágann sin go bhfuil $A_1B\|$ le $CH$, mar táid $\perp$ le $AB$.

Mar an gcéanna tá $A_1C\|$ le $BH$, ionas gur $\varparallelogram$ é $BHCA_1$.

$\therefore$ Tá lár an tsleasa $BC$ leath-bealaigh idir $H$ agus $A_1$.

Is léir anois go gcómhroinneann na naoi bpointí a luaitear sa teoirim naoi línte áirithe ó $H$ go dtí an imlíne, ionas go luíonn siad féin ar chiorcal dárb ga $\frac{1}{2}OA$, go bhfuil a lár leath-bealaigh idir $H$ agus $O$.

Ciorcal naoi bpointe an triantáin a tugtar ar an gciorcal sin.

### Atora

*Ó thárla $A_1O = OA$ agus $A_1M - MH$, tá $OM = \frac{1}{2}AH$.*

## Theorem D (The Nine-point Circle of the Triangle)

**Lemma.** *If a fixed point $H$ is joined to a variable point $P$ on the perimeter of a circle, then the centres of the join-lines lie on a certain circle*

If $X$ is the centre of $HP$ and if $N$ is the centre of $HO$, it is clear that $NX = \frac{1}{2}OP$. So it follows that $X$ moves is such a way as to keep a fixed distance between it an the fixed point $N$.

It follows that the locus of the point $X$ is a circle (with centre $N$).

**Theorem D.** *In any triangle at all a circle passes through these nine points: the centres of the sides, the feet of the perpendiculars from the vertices on the sides, and the centres of the lines that join the orthocentre to the vertices of the triangle.*

*Construction:*

Draw $AOA_1$ the diameter through $A$. Join $A_1B$, $A_1C$.

*Proof:*

Since $AA_1$ is a diameter, $A_1BA$ is a right angle, and it follows that $A_1B\|$ to $CH$, for they are $\perp$ to $AB$.

In the same way, $A_1C\|$ to $BH$, so that $BHCA_1$ is a $\varnothing$.

$\therefore$ The centre of the side $BC$ is halfway between $H$ and $A_1$.

It is now clear that the nine points mentioned in the theorem bisect nine particular lines from $H$ to the perimeter, so that they theemselves lie on a circle with radius $\frac{1}{2}OA$ and with centre halfway between $H$ and $O$.

That circle is called the nine-point circle of the triangle.

**Corollary.** *Since $A_1O = OA$ and $A_1M - MH$, we have $OM = \frac{1}{2}AH$.*

# Cleachtaithe

1) Má 'sé H ingearlár an $\Delta ABC$, deimhnigh gur ingearlár é gach pointe de na ceithre cinn $A, B, C, H$, sa triantán ar reanna dhó na trí pointe eile.

Teaspáin gurb é an t-aon chioreal amháin é cioreail naoi bpointe na dtriantán $ABC$, $HAB$, $HBC$, $HAB$.

2) Sa $\Delta ABC$ 'sé $I$ an t-inlár agus 'siad $I_1, I_2, I_3$ na h-eisláir. Cruthuigh gur triantán agus a ingearlár a dhealbhíonn na pointí $I, I_1, I_2, I_3$.

3) Má 'sé $X$ lár na líne $II_1$, teaspáin go bhfuil $X$ ar an gcioreal $ABC$, agus cruthuigh $XI = XB = XC$.

Deimhnigh freisin gurb é $X$ lár an stuaigh $BC$ thíos, agus gurb é lár-phointe $I_2 I_3$ lár an stuaigh $BC$ thuas.

4) Cruthuigh gurb iad $180^\circ - 2A$, $180^\circ - 2B$, $180^\circ - 2C$ uilleacha thriantáin bhun na n-ingear.

5) Faigh an uille a ghabhas an slios $BC$ ag an ingearlár $H$.

Aimsigh rian $H$ nuair is eol bonn agus stuacuille an triantáin. Sa gcaoi céanna faigh rian lár chioreail na naoi bpointe.

6) Tadhlann an slios $BC$ inchioreal an triantáin ag $P$, agus is ag $Q$ a thadhlas sé an t-eischioreal atá os cóir $A$.

Cruthuigh (i) $2BP = AB + BC - CA$; (ii) $2BQ = BC + CA - AB$.

7) Tarraingitear na tadhlaithe ag $A, B, C$ d'ionchioreal an $\Delta ABC$. Teaspáin go ngintear $\Delta$ leo gurb ionann a uilleacha agus uilleacha thriantáin bhun na n-ingear.

8) Triantán a thógáil gurb eol ionaid na dtrí n-eisláir ann.

9) Pointe ar iomchioreal an $\Delta ABC$ isea $P$, agus is i bpointe $Q$ a ghearras an t-ingear ó $P$ ar $BC$ an chioreal $ABC$ arís. 'Sé $X$ scáth an ingearlair in $BC$.

Cruthuigh (i) gur scátha a chéile iad $AQ$ agus $XP$ in ais shuimeatreachta $AX$: (ii) go bhfuil $AQ \parallel$ le $HP_1$, agus le troighlíne an phointe $P$: (iii) go bhfuil lár $AP_1$ ar chioreal na naoi bpointe.

10) Má's foirchinn láirlíne iad $P, R$ sa gcioreal $ABC$ cruthuigh go bhfuil troighlínte na bpointe sin de réir an $\Delta ABC$ ingearach le chéile, agus go ngearrann siad ar chioreal na naoi bpointe

[Ride: na línte trí A atá $\parallel$ leo a bhreithniú i dtosach]

11) Maidir leis na ceithre triantáin a gintear le ceithre dronlínte ar bith, agus an pointe O ina dtagann a n-iomchioreail le chéile (feic leath. — ) cruthuigh:—

(1) gur pointe é O go bhfuil a scátha sna ceithre dronlínte comhlíneach;
(2) go luigheann ingearláir na gceithre dtriantán san dronlíne sin;
(3) gur dronlíne í an líne cheangail go bhfuil a scátha sna ceithre dronlínte comhreathach.

Tosach leathanach 92 sa LSS.

## Cleachtaithe

1. Má 'sé $H$ ingearlár an $\Delta ABC$, deimhnigh gur ingearlár é gach pointe de na ceithre cinn $A, B, C, H$, sa triantán ar reanna dhó na trí pointí eile.

   Teaspáin gurb é an t-aon chiocal amháin é ciorcail naoi bpointe na dtriantán $ABC$, $HAB$, $HBC$, $HCA$.

2. Sa $\Delta ABC$ 'sé $I$ an t-inlár agus 'siad $I_1, I_2, I_3$ na heisláir. Cruthuigh gur triantán agus a ingearlár a dhealbhíonn na pointí $I, I_1, I_2, I_3$.

3. Má 'sé $X$ lár na líne $II_1$, teaspáin go bhfuil $X$ ar an gciorcal $ABC$ agus cruthuigh $XI = XB = XC$.

   Deimhnigh freisin gurb é $X$ lár an stua $BC$ thíos, agus gurb é lárphointe $I_2I_3$ lár an stua $BC$ thuas.

4. Crutuigh gurb iad $180° - 2A$, $180° - 2B$, $180° - 2C$ uilleacha thriantáin bun na n-ingear.

5. Faigh an uille a ghabhas an slios $BC$ ag an ingearlár $H$.

   Aimsigh rian $H$ nuair is eol bonn agus staucuille an triantáin.

   Sa gcás céanna faigh rian lár chiorcail na naoi bpointe.

6. Tadhlann an slios $BC$ inchiorcal an triantáin ag $P$, agus is ag $Q$ a thadhlas sé an t-eischiorcal atá óscóir $A$.

   Cruthuigh (i) $2BP = AB + BC - CA$; (ii) $2BP = BC + CA - AB$.

7. Tarraingítear na tadhlaithe ag $A, B, C$ d'iomchiorcal an $\Delta ABC$.
   Teaspáin go ngintear $\Delta$ leo gurb ionann a uilleacha agus uilleach thriantáin bhunna n-ingear.

8. Triantán a thógáil gurb eol ionaid na dtrí n-eisláir ann.

9. Pointe ar iomchiorcal an $\Delta ABC$ is ea $P$, agus is i bpointe $Q$ a ghearras an t-ingear ó $P$ ar $BC$ an chiorcal $ABC$ arís. 'Sé $X$ scáth an ingearláir in $BC$.

   Cruthuigh (i) gur scátha a chéile iad $AQ$ agus $XP$ in ais shuiméitreachta $AX$; (ii) go bhfuil $AQ \|$ le $HP_1$, agus le le troighlíne an phointe $P$; (iii) go bhfuil lár $AP_1$ ar chiorcal na naoi bpointe.

10. Má's foirchinn lárlíne iad $PR$ sa gciorcal $ABC$ cruthuigh go bhfuil troighllíte na bpointí sin de réir an $\Delta ABC$ ingearach le chéile, agus go ngearrann siad ar chiorcal na naoi bpointe.

    [Lide: na línte tré $A$ atá $\|$ leo a bhreithiú i dtosach.]

11. Maidir leis na ceithre triantáin a gintear le ceithre dronlíte ar bith, agus an pointe *O* ina dtagann a n-iomchiorcail le chéile (feic leathanach 255) cruthuigh:—

    (1) gur pointe é *O* go bhfuil a scátha sna ceithre dronlínte cóimlíneach;

    (3)i go luigheann ingearláir na gceithre dtriantáin san dronlíne sin;

    (3) gur dronlíne í an líne cheangail go bhfuil a scátha sna ceithre dronlínte cóimhreathach.

12) Pointe taobh istigh a dtriantáin ABC is ea P. Séad L, M, N lár na slios BC, CA, AB, agus séad X, Y, Z lár na línte PA, PB, PC. Tagann na ⊙ NXY agus LYZ le chéile in Y agus i bpointe eile K, abair.

Cruthuigh (i) $N\hat{K}Y = A\hat{B}P$, $Y\hat{K}L = P\hat{B}C$; (ii) go ngabhann an ⊙ NLM tré K freisin (iii) mar an gcéanna go ngabhann an ⊙ MZX tríd.

'Sé sin, maidir leis na ceithre triantáin a gfintear ó cheithre phointe ar bith, tá pointe amháin ar chiorcail naoi bpointe na gceithre triantán sin, san am chéanna.

Tosach leathanach 93 sa LSS.

12. Pointe taobh istigh an dtriantán $ABC$ is ea $P$. Siad $L, M, N$ láir na slios $BC, CA, AB$, agus siad $X, Y, Z$ láir na línte $PA, PB, PC$. Tagann na $\odot NXY$ agus $LYZ$ le chéile in $Y$ agus i bpointe eile $K$, abair.

    Cruthuigh (i) $\widehat{NKY} = \widehat{ABP}$, $\widehat{YKL} = \widehat{PBC}$; (ii) go ngabhann an $\odot NLM$ tré $K$ freisin; (iii) mar an gcéanna go ngabhann an $\odot MZX$ tríd.

    'Sé sin, maidir leis na ceithre triantáin a gintear ó cheithre phointí ar bith, tá pointe amháin ar chiorcail naoi bpointe na gceithre triantán sin, san am chéanna.

## Exercises

1. If $H$ is the orthocentre of the $\Delta ABC$, verify that each of the four points $A, B, C, H$ is the orthocentre of the triangle having the other three as vertices.

   Show that all the triangles $ABC, HAB, HBC, HCA$ have the same nine-point circle.

2. In the $\Delta ABC$ the incentre is $I$ and the excentres are $I_1, I_2, I_3$. Prove that the points $I, I_1, I_2, I_3$ determine a triangle and its orthocentre.

3. If $X$ is the centre of the line $II_1$, show that $X$ iis on the circle $ABC$ and prove $XI = XB = XC$.

   Verify as well that $X$ is the centre of the lower arc $BC$, and that the centre of $I_2I_3$ is the centre of the upper arc $BC$.

4. Prove that $180° - 2A$, $180° - 2B$, $180° - 2C$ are the angles in the orthic triangle.

5. Find the angle that the side $BC$ subtends at the orthocentre $H$.

   Find the locus of $H$ when the base and apex angle of the triangle are given.

   In the same case find the locus of the centre of the nine-point circle.

6. The side $BC$ is tangent to the incircle at $P$, and it is tangent to the excircle opposite $A$ at $Q$.

   Prove (i) $2BP = AB + BC - CA$; (ii) $2BP = BC + CA - AB$.

7. The tangents at $A, B, C$ to the circumcircle of the $\Delta ABC$ are drawn.
   Show that a $\Delta$ is generated by them that has the same angles as the orthic triangle.

8. To construct a triangle when the positions of the three excentres are known.

9. $P$ is a point on the circumcircle of the $\Delta ABC$, and $Q$ is the point where the perpendicular from $P$ on $BC$ cuts the circle $ABC$ again. $X$ is the reflection of the orthocentre in $BC$.

Prove (i) that $AQ$ and $XP$ are reflections of one another in the axis of symmetry of $AX$; (ii) that $AQ\|$ to $HP_1$, and to the Simson line of the point $P$; (iii) that the centre of $AP_1$ is on the nine-point circle.

10. If $P$ and $R$ are the extremeties of a diameter of the circle $ABC$, prove that the Simson line of those points with respect to the triangle $\Delta ABC$ are perpendicular to one another, and that they meet on the nine-point circle.

    [Hint: consider first the lines through $A$ that are $\|$ to them.]

11. With regard to the four triangles that can be made from four arbitrary straight lines, and the point $O$ in which their circumcircles meet (see page 257) prove:—
    (1) that the reflections of the point $O$ in the four straight lines are collinear;
    (2) That the orthocentres of the four triangles lie on that line;
    (3) that the reflections of that line in the four lines concurrent.

12. $P$ is a point inside the triangle $ABC$. The centres of vthe sides $BC, CA, AB$ are $L, M, N$, and $X, Y, Z$ are the centres of the lines $PA, PB, PC$. The $\odot NXY$ and $LYZ$ meet at $Y$ and in another point $K$, say.

    Prove (i) $\widehat{NKY} = \widehat{ABP}$, $\widehat{YKL} = \widehat{PBC}$; (ii) that the $\odot NLM$ also passes through $K$; (iii) similarly, that the $\odot MZX$ passes through it.

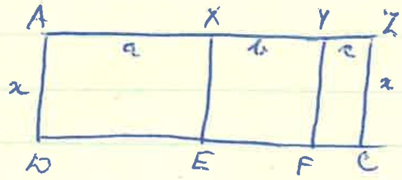
    That is, with regard to the four triangles generated by four arbitrary points, there is a single point that lies on ithe nine-point circles of those four triangles.

## Caibidil VIII

## Toradh Dhá Dhronlíne

Má tá fad $a$ agus $b$ in dhá dhronlíne áirithe, is follusach cé'n chaoi a n-aimsítear dronlíne eile a mbeidh fad $a+b$ inte (nó fad $a-b$ nuair atá $a > b$).

Má's mian linn toradh dhá dhronlíne a threacadh ar bhealach geométrach, níl aon chaoi is túisce a chuirfeadh an léitheoir uirthi ná fairsinge na dronuilleoige a gheintear ón dá líne sin a tharraint chuige. Níor mhór a bheith cinnte áfach go dtagann an "toradh" seo le rialacha an mhéaduithe de réir Algébair; e.g. go mbeadh $x(a+b+c) = xa+xb+xc$. Is furusta a thaispeáint go bhfuil sé sin amhlaidh.




Tarraing ar dhronlíne ar bith na míreanna AX, XY, YZ atá $a$, $b$, $c$ ar fad. Tarraing ingear ag A a bhfuil fad $x$ ann, agus slánuigh na dronuilleoga AZCD etc.

De bhrí go bhfuil an dron. AZCD cothrom leis na trí dronuilleoga AXED, XYFE, YZCF le chéile, tá

$$x(a+b+c) = xa+xb+xc.$$

Nóta 1 'Sé AP.PB a scríobhtar chun fairsinge na dronuilleoige ar sleasa dhi AP agus PB a chomhartha. Ciallaíonn $AP^2$ an chearnóg AP.AP.

Nóta 2 Is léir ó'n dronuilleoig ar sleasa dhi $a+b$ agus $c+d$, go bhfuil $(a+b)(c+d) = ac+ad+bc+bd$.

e.g. nuair $a=c$, $b=d$, tá $(a+b)^2 = a^2+2ab+b^2$.

∴ Sa bhFíog. thuas tá $AY^2 = (AX+XY)^2 = AX^2+2AX.XY+XY^2$.

De bhrí go dtagann toradh dhá dhronlíne le gnáth-rialacha Algébair, is féidir teoirmí geométracha a ghnóthú le modh algébrach

# Caibidil 8

# Toradh Dhá Dhronlíne

Tosach leathanach 94 sa LSS.

## The Product of Two Straight Lines

Má tá fad $a$ agus $b$ in dhá dhronlíne áirithe is follasach cé'n chaoi a n-aimsítear dronlíne eile a mbeidh fad $a+b$ innti (nó fad $a-b$ nuair atá $a>b$).

Má's mian linn *toradh* dhá dhronlíne a bhreacadh ar bhealach geométrach, níl aon chaoi a chuimhneóis an léitheoir uirthi ná fairsinge na dronuilleoige a gintear óndá líne sin a tharraingt chuige. Níor mhór a beith cinnte áfach go dtagann an "toradh" seo le riallacha an mhéaduithe de réir Algébair: e.g. go mbeadh $x(a+b+c) = xa+xb+xc$. Is furasta a theaspáint go bhfuil sí sin amhlaidh.

Tá Fíoghair anseo sa LSS, leathanach 94.

Tarraing ar dhronlíne ar bith na míreanna $AX, XY, YZ$ atá $a, b, c$ ar fhad. Tarraing ingear ag $A$ a bhfuil fad $x$ ann, agus slánuigh na dronuilleoga $AZCD$ etc.

De bhrí go bhfuil an dron. $AZCD$ cothrom leis na trí dronuilleoga $AXED, XYFE, YZCF$ le chéile, tá

$$x(a+b+c) = xa+xb+xc.$$

If $a$ and $b$ are the lengths of two given straight lines , then it is straightforward to find another straight line having length $a+b$ (or length $a-b$, when $a>b$).

If we want to get the *product* of two straight lines in a geometric way, the only way that will come to the reader's mind is to use the area of the rectangle generated by those two lines. It is necessary, however, to check that this 'product' respects the rules of multiplication according to Algebra: e.g. that $x(a+b+c) = xa+xb+xc$. It is easy to show that this is the case.

Draw on any straight line segments $AX, XY, YZ$ of length $a, b, c$. Draw a perpendicular at $A$ with length $x$, and complete the rectangles $AZCD$ etc.

Since the rectangle $AZCD$ is equal to the three rectangles $AXED, XYFE, YZCF$ together, we have

$$x(a+b+c) = xa+xb+xc.$$

## Nóta 1

'Sé $AP \cdot PB$ a scríobhtar chun fairsinge na dronuilleoige ar sleasa dhi $AP$ agus $PB$ a chomharthú . Ciallaíonn $AP^2$ an chearnóg $Ap \cdot AP$.

## Nóta 2

Is léir ó'n dronuilleog, ar sleasa dhi $a+b$ agus $c+d$, go bhfuil $(a+b)(c+d) = ac+ad+bc+bd$.

e.g. nuair $a = c, b = d$, tá $(a+b)^2 = a^2+2ab+b^2$.

∴ Sa bhFiog. thuas tá
$AY^2 = (AX+XY)^2 = AX^2+2AX \cdot XY + XY^2$.

De bhrí go dtagann tora dhá dhronlíne le gnáth-rialacha Algébair, is féidir teoirmí geométracha a gnothú le modh algébrach.

## Note 1

We write $AP \cdot PB$ to represent the area of the rectangle with sides $AP$ and $PB$. The meaning of $AP^2$ is the square $AP \cdot AP$.

## Note 2

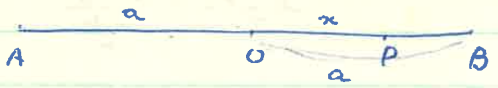
It is clear from the rectangle with sides $a+b$ and $c+d$, that $(a+b)(c+d) = ac+ad+bc+bd$.

e.g. when $a = c, b = d$, we have $(a+b)^2 = a^2+2ab+b^2$.

∴ In the Fig above, we have
$AY^2 = (AX+XY)^2 = AX^2+2AX \cdot XY + XY^2$.

Since the product of two straight lines agrees with the ordinary rules of Algebra, it is possible to prove geometric theorems by algebraic methods.

E.G.(a) Má's istigh a roinneann P an dronlíne AB ar lár di O, agus má tá $AO = a$, $OP = x$, tá ~~is soiléir go bhfuil~~




$AP = a + x$ ; $PB = a - x$.

Is soiléir anois de réir algébair go bhfuil,

I $AP + PB = 2OP$

II $AP.PB = AO^2 - OP^2$

III $AP^2 + PB^2 = 2AO^2 + 2OP^2$

Is minic a bainfear feidhm as na teoirmí sin.

(b) Má's amuigh a roinneann P an líne AB, agus


má cuirtear $AO = a$, $OP = x$ arís, faightear $AP = x + a$, $PB = x - a$.

∴ I′ $AP + PB = 2OP$

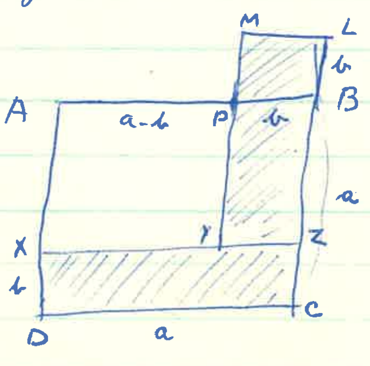
II′ $AP.PB = OP^2 - OA^2$

III′ $AP^2 + PB^2 = 2OP^2 + 2OA^2$.

Ar an taobh eile áfach is féidir teoirmí algébracha áirithe a léiriú de réir geométreachta.

Sompla 1.

Breacadh geométreach na teoirme $(a-b)^2 = a^2 - 2ab + b^2$ a thabhairt.




Tógáil

Bíodh $AB = a$, $PB = b$, ionnus go bhfuil $AP = a - b$. Tóg cearnóga ABCD, PBLM ar AB agus PB. Déan $XD = PB = b$, agus tarraing $XZ \parallel AB$.

Cruthúnas

Ó tharla $BL = CZ = b$, tá $ZL = BC = a$.

∴ Is ionann leabh gach dronuilleog MLZY, agus XZCD.

Tosach leathanach 95 sa LSS.

## E.G.

(a) Má's istigh a roinneann $P$ an dronlíne $AB$ ar lár di $O$, agus má tá $AO = a, OP = x$, tá

$$AP = a + x; \quad PB = a - x.$$

Tá Fíoghair anseo sa LSS, leathanach 95.

Is soiléir anois de réir algébar go bhfuil,

$\therefore$

$$\begin{array}{rlcl} I' & AP + PB & = & 2OP \\ II' & AP \cdot PB & = & AO^2 - OP^2 \\ III' & AP^2 + PB^2 & = & 2AO^2 + 2OP^2 \end{array}$$

Is minic a bainfear feidhm as na teoirmí sin.
(b) Má's amuigh a roinneann $P$ an líne $AB$, agus

Tá Fíoghair anseo sa LSS, leathanach 95.

má cuirtear $AO = a, OP = x$, arís, faightear $AP = x + a$, $PB = x - a$.

$\therefore$

$$\begin{array}{rlcl} I' & AP + PB & = & 2OP \\ II' & AP \cdot PB & = & OP^2 - OA^2 \\ III' & AP^2 + PB^2 & = & 2OP^2 + 2OA^2 \end{array}$$

Ar an taobh eile dhe is féidir teoirmí albébracha áirithe a léiriú de réir geométrachta.

## E.G.

(a) If $P$ divides the straight line $AB$ with centre $O$ internally, and if $AO = a, OP = x$, we have

$$AP = a + x; \quad PB = a - x.$$

It is now clear by algebra that

$\therefore$

$$\begin{array}{rlcl} I' & AP + PB & = & 2OP \\ II' & AP \cdot PB & = & AO^2 - OP^2 \\ III' & AP^2 + PB^2 & = & 2AO^2 + 2OP^2 \end{array}$$

This theorem is much used.

(b) If $P$ divides the line $AB$ externally, and if we put $AO = a, OP = x$, again, we get $AP = x + a$, $PB = x - a$.
$\therefore$

$$
\begin{array}{lll}
I' & AP + PB & = 2OP \\
II' & AP \cdot PB & = OP^2 - OA^2 \\
III' & AP^2 + PB^2 & = 2OP^2 + 2OA^2
\end{array}
$$

On the other hand it is possible to illustrate algebraic theorems geometrically.

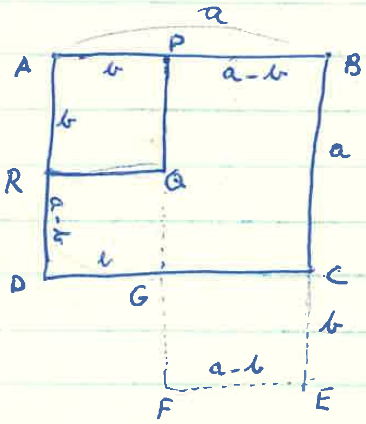
De bhrí go bhfuil $AX = a - b = AP$, cearnóg í sea $APYX$ gurb ionann é agus $(a-b)^2$

Ach tá an chearnóg $AB$ + an chearnóg $MB$

= an chearnóg $AY + 2ab$.

$\therefore\ a^2 + b^2 = (a-b)^2 + 2ab$, nó $(a-b)^2 = a^2 - 2ab + b^2$.

Sampla 2 Breacadh geoméitreach na teoirme $(a+b)(a-b) = a^2 - b^2$ a thabhairt.




Tógáil

Tóg na cearnóga AC, AQ ar $AB = a$, $AP = b$. Déan $CE = b$ agus slánuigh an dronuilleog PBEF.

Cruthúnas

De bhrí go bhfuil $RD = a - b = GC$, agus go bhfuil $DG = b = CE$, is ionann an dá dron. RG agus GE.

~~Ach~~ Fágann sin go bhfuil an dron. PE = an dron. PC + an dron. RG

Ach tá an dron. PC + an dron. PG = $AB^2 - AP^2$

$\therefore$ Tá an dron. PE = $AB^2 - AP^2$, nó $(a-b)(a+b) = a^2 - b^2$.

~~Ceisteanna~~

(1) Teaspáin le fioghair go bhfuil an chearnóg ar dhronlíne níos mó faoi cheathair ná an chearnóg ar leath na líne.

(2) Tabhair breacadh geoméitreach i gcóir:—

(i) $a(a-b) = a^2 - ab$

(ii) $(a+b)^2 = a^2 + 2ab + b^2$.

## Sampla 1

*Breacadh geométreach na teoirme* $(a-b)^2 = a^2 - 2ab + b^2$ *a thabhairt.*

Tá Fíoghair anseo sa LSS, leathanach 95.

*Tógáil:*

Bíodh $AB = a, PB = b$, ionas go bhfuil $AP = a - b$. Tóg cearnóga $ABCD, PALM$, ar $AB$ agus $PB$. Déan $XD = PB = b$, agus tarraing $XZ \| AB$.

*Cruthúnas:*

Ó thárla $BL = CZ = b$, tá $ZL = BC = a$.

$\therefore$ Is ionann le $ab$ gach dronuilleóg $MLZY$ agus $XZCD$.

Tosach leathanach 96 sa LSS.

De bhrí go bhfuil $AX = a - b = AP$, cearnóg is ea $APYX$ gurb ionann é agus $(a-c)^2$.

Ach tá an chearnóg $AC$ + an chearnóg $MB$

= an chearnóg $AY + 2ab$

.i. $a + 2 + b^2 = (a-b)^2 + 2ab$, nó $(a-b)^2 = a^2 - 2ab + b^2$.

## Sample 1

*To give a geometric version of the theorem* $(a-b)^2 = a^2 - 2ab + b^2$.

*Construction:*

Let $AB = a, PB = b$, so that $AP = a - b$. Construct squares $ABCD, PALM$, on $AB$ and $PB$. Make $XD = PB = b$, and draw $XZ \| AB$.

*Proof:*

Since $BL = CZ = b$, we have $ZL = BC = a$.

$\therefore$ $ab$ is equal to each of the rectangles $MLZY$ and $XZCD$.

Since $AX = a - b = AP$, $APYX$ is a square iwhich is equal to $(a-c)^2$.

But the square $AC$ + the square $MB$

= the square $AY + 2ab$

.i. $a + 2 + b^2 = (a-b)^2 + 2ab$, or $(a-b)^2 = a^2 - 2ab + b^2$. □

## Sampla 2

*Breacadh geométrach na teoirme* $(a+b)(a-b) = a^2 - b^2$ *a thabhairt.*

Tá Fíoghair anseo sa LSS, leathanach 96.

*Tógáil:*

Tóg na cearnóga $AC, AQ$ ar $AB = a, AP = b$. Déan $CE = b$ agus slánuigh an dronuilleóg $PBEF$.

*Cruthúnas:*

De bhrí go bhfuil $RD = a - b = GC$, agus go bhfuil $DG = b = CE$, is ionann an dá dhron. $RG$ agus $GE$.

Fágann sin go bhfuil an dron. $PE$ = an dron. $PC$ + an dron. $RG$.
Ach tá an dron. $PC$+ an dron. $PG = AB^2 - AP^2$.
∴ Tá an dron. $PE = AB^2 - AD^2$, nó $(a-b)(a+b) = a^2 - b^2$.

## Sample 2

*To give a geometric illustration of the theorem* $(a+b)(a-b) = a^2 - b^2$.

*Construction:*

Construct squares $AC, AQ$ on $AB = a, AP = b$. Make $CE = b$ and complete the rectangle $PBEF$.

*Proof:*

Since $RD = a - b = GC$, and $DG = b = CE$, the two rectangles $RG$ and $GE$ are equal.
It follows that the rect. $PE$ = the rect. $PC$ + the rect. $RG$.
But the rect. $PC$+ the rect. $PG = AB^2 - AP^2$.
∴ the rect. $PE = AB^2 - AD^2$, or $(a-b)(a+b) = a^2 - b^2$. □

## Ceisteanna

1. Teaspáin le fioghair go bhfuil an chearnóg ar dhronlíne níos mó faoi cheathair ná an chearnóg ar leath na líne.

2. Tabhair breacadh geométrach i gcóir:—

   (i) $a(a-b) = a^2 - ab$,

   (ii) $(a+b)^2 = a^2 + 2ab + b^2$.

## Questions

1. Show with a figure that the square on a straight line is four times greater than the square on half the line.

2. Give geometric illustrations for:—

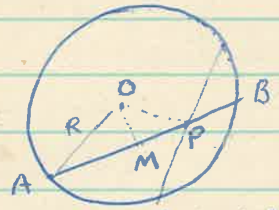
   (i) $a(a-b) = a^2 - ab$,

   (ii) $(a+b)^2 = a^2 + 2ab + b^2$.

## Teoirim I

Má ghabhann córdaí ciorcail tré'n bpointe céanna, is buan-fhairsinge an dronuilleog fá mhíreanna gach córda.

### Cás I. Nuair is istigh a ghearras na córdaí.




b'fhearr "r" ná "a" "ná" "g"

Abair gur córda ar bith é AB tré'n bpointe socair P, na ciorcal ar lár dó O agus ar ga dó R.

Tarraing an t-ingear OM ar AB, a chomhroinneas é.

Teaspáinfear go bhfuil $AP.PB = R^2 - OP^2$.

**Cruthúnas** $= AM^2 - MP^2$

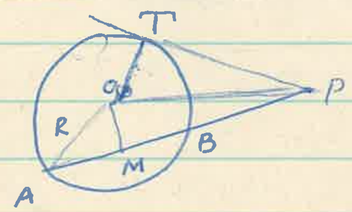
Ó tharla $AM = MB$, tá $AP.PB = (AM+MP)(AM-MP)$

Is ionann é sin agus $(AM^2+MO^2)-(MP^2+MO^2) = R^2 - OP^2$.

∴ $AP.PB = R^2 - OP^2$.

Beireann $R^2 - OP^2$ leis an bpointe P, ach tá sé neamhspleách den chórda ar leith a tarraingtear tré P, ionnus go bhfuil an fhairsinge bhuan $R^2 - OP^2$ san dronuilleog fá mhíreanna gach córda.

### Cás II Nuair a ghabhas na córdaí tré phointe P amuigh.




Teaspáinfear go bhfuil $PA.PB = OP^2 - R^2$.

**Cruthúnas**

Tá $PA.PB = (PM+MA)(PM-MA) = PM^2 - MA^2$.

Is ionann é sin agus $(PM^2+OM^2)-(MA^2+OM^2) = OP^2 - R^2$.

∴ Tá an dronuilleog fá mhíreanna gach córda $= OP^2 - R^2$

**Atoradh 1** I gcás II, (má's é PT an tadhlaí ag T) is soiléir ó'n triantán dronuilleach OTP go bhfuil

$$PT^2 = OP^2 - R^2 = PA.PB.$$

Tosach leathanach 97 sa LSS.

## 8.1 Teoirim I

*Má ghabhann córdaí ciorcail tré'n bpointe céanna, is buan fhairsinge an dronuilleog fá mhíreanna gach córda.*

### Cás I

*Nuair is istigh a ghearra na córdaí*

Tá Fíoghair anseo sa LSS, leathanach 97.

Abair gur códa ar bith é $AB$ tré'n bpointe socair $P$ sa ciorcal ar lár dó $O$ agus ar ga dó $R$.
Tarraing an t-ingear $OM$ ar $AB$, a chomhroinneas é .
Teaspáinfear go bhfuil $AP \cdot PB = R^2 - OP^2$.

*Cruthúnas:*
Ó thárla $AM = MB$, tá $AP \cdot PB = (AM + MP)(AM - MP)$.
Is ionann é sin agus $(AM^2 + MO^2) - (MP^2 + MO^2) = R^2 - OP^2$.

$$AP \cdot PB = R^2 - OP^2.$$

Baineann $R^2 - OP^2$ leis an bpointe $P$, ach tá sé neamhspeách den chórda ar leith a tarraingítear tré $P$, iona gi bhfuil an fhairsinge bhuan $R^2 - OP^2$ san dronuilleoig fá mhíreanna fach córda.

### Cás II

*Nuair a ghabhas na córdaí tré phointe $P$ amuigh.*

Tá Fíoghair anseo sa LSS, leathanach 97.

Teaspáinfear go bhfuil $PA \cdot PB = OP^2 - R^2$.

*Cruthúnas:*
Tá $PA \cdot PB = (PM + MA)(PM - MA) = PM^2 - MA^2$.
Is ionann é sin agus $(PM^2 + OM^2) - (MA^2 + OM^2) = OP^2 - R^2$.
∴ Tá an dronuilleog fá mhíreanna gach córda $= OP^2 - R^2$.

**Theorem** (8-1)**.** *If chords of a circle pass through the same point, then the area of the rectangles on the two pieces of each chord is constant.*

## Case I

*When the chords meet inside*

Suppose $AB$ is any chord through the fixed point $P$ in the circle with centre $O$ and radius $R$.

Draw the perpendicular $OM$ on $AB$, bisecting it.

We shall show that $AP \cdot PB = R^2 - OP^2$.

*Proof:*

Since $AM = MB$, we have $AP \cdot PB = (AM + MP)(AM - MP)$.
That is the same as $(AM^2 + MO^2) - (MP^2 + MO^2) = R^2 - OP^2$.

$$AP \cdot PB = R^2 - OP^2.$$

The quantity $R^2 - OP^2$ involves the point $P$t is independent of which particular chord is drawn through $P$, so that the rectangle on the pieces of the chord has the constant area $R^2 - OP^2$ .

## Cás II

*When the chords pass through an outside point $P$.*

We shall show that $PA \cdot PB = OP^2 - R^2$.

*Proof:*

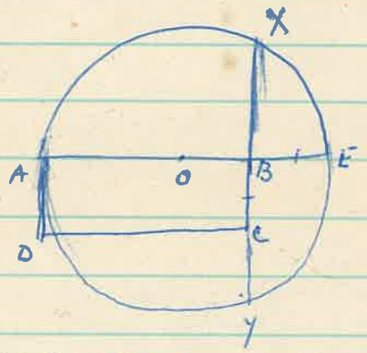
We have $PA \cdot PB = (PM + MA)(PM - MA) = PM^2 - MA^2$.
That is the same as $(PM^2 + OM^2) - (MA^2 + OM^2) = OP^2 - R^2$.
∴ The rectangle on the pieces of each chord is $= OP^2 - R^2$. □

**Atora 2** Má's dronlíne í PT a tarraingítear go dtí ⊙ ó phointe P amuigh, iomḟás go bhfuil $PT^2 = PA.PB$ áit gur corda ar bith é tré P é AB, tadhlann PT an ⊙ sin ag T.

Mar sa △ OTP tugtar $PT^2 = OP^2 - OT^2$ ∴ $OT^2 + PT^2 = OP^2$, iomḟás gur dronuillé í $O\hat{T}P$, agus d'á bhrí sin tadhlaí isea TP.

**Nóta** De chairbhe teoirme I is féidir an cheist seo a réiteach.

**Ceist 1** Cearnóg a tharraint a bheas comhfhairsing le dronuilleoig áirithe.




Abair gurb é ABCD an dronuilleog a tugtar.

**Réiteach**

Sín AB go dtí E chum go mbeidh BE = BC. Tarraing ⊙ ar an láirlíne AE, agus sín BC go dtéigh go dteagmhaíonn sé leis an gciorcal in X agus Y. 'Sé an chearnóg ar BX an chearnóg a fheileas.

**Cruthúnas**

De chairbhe teoirme I tá $AB.BE = BX.BY = BX^2$, mar tá BX = BY de bhrí go bhfuil XY ⊥ leis an láirlíne AE.

Ach rinneadh BE = BC.

∴ Tá an dron. ABCD $= BX^2$.

**Ceisteanna**

1) Is dronuillé í A sa △ ABC agus 'sé AD an t-ingear ar BC. Cruthuigh (a) $BD.DC = AD^2$ (b) go dtadhlann AC an ciorcal ADB ag A agus d'á chionn sin go bhfuil $BC.CD = AC^2$.

2) I dtriantán ar bith ABC pointe in BC isea X iomḟás go bhfuil $X\hat{A}C = ABC$. Cruthuigh $BC.CX = AC^2$.

3) Tarraing cearnóg a bheas comhfhairsing le triantán áirithe.

### Atora 1

*I gcás II, má's é $PT$ an tadhlaí ag $P$, is soiléir ó'n triantán dronuilleach $OTP$ go bhfuil*

$$PT^2 = OP^2 - R^2 = PA \cdot PB.$$

Tosach leathanach 98 sa LSS.

### Atora 2

*Má's dronlíne é $PT$ a tarraingítear go dtí ⊙ ó phointe $P$ amuigh, ionas go bhfuil $PT^2 = PA \cdot PB$ áit gur córda ar bith tré $P$ é $AB$, tadhlann $PT$ at ⊙ sin ag $T$.*

Mar sa $\Delta OTP$ tugtar $PT^2 = OP^2 - OT^2$ .i. $OT^2 + PT^2 = OP^2$, ionas gur dronlíne é $\widehat{OTP}$, agus d'á bhrí sin tadhlaí is ea $TP$.

**Corollary 1.** *In case II, if $PT$ is the tangent at $P$, it is clear from the right-angle triangle $OTP$ that*

$$PT^2 = OP^2 - R^2 = PA \cdot PB.$$

**Corollary 2.** *If $PT$ is a straight line drawn to a ⊙ from an outside point $P$, so that $PT^2 = PA \cdot PB$ where $AB$ is any chord through $P$, then $PT$ is tangent to that ⊙ at $T$.*

For in the $\Delta OTP$ we have $PT^2 = OP^2 - OT^2$ .i. $OT^2 + PT^2 = OP^2$, so that the angle $\widehat{OTP}$ is a straight line, and hence $TP$ is a tangent. □

### Nóta

De thairbhe Teoirme I is féidir an cheist seo a réiteach.

### Ceist 1

*Cearnóg a tharraint a bheas comhfhairsing le dronuilleoig áirithe.*

Tá Fíoghair anseo sa LSS, leathanach 98.

Abair gurb é $ABCD$ an dronuilleóg a tugtar.

*Réiteach:*

Sín $AB$ go dtí $E$ chun go mbeidh $BE = BC$. Tarraing ⊙ ar an láillíne $AE$, agus sín $BC$ go dteagmhaíonn sé leis an gciorcal in $X$ agus $Y$. 'Sí an chearnóg ar $BX$ an chearnóg a fheileas.

*Cruthúnas:*

De thairbhe Teoirme I tá $AB \cdot BE = BX \cdot BY = BX^2$, mar tá $BX = BY$ de bhrí go bhfuil $XY \perp$ leis an láirlíne $AE$.

Ach rinneadh $BE = BC$.

∴ Tá an dron. $ABCD = BX^2$.

## Note

Theorem 8-1 allows us to solve the following question:

## Question 1

*To draw a square having the same area as a given rectangle.*

Suppose that $ABCD$ is the given rectangle.

*Solution:*

Extend $AB$ to $E$ to make $BE = BC$. Draw a $\odot$ on the diameter $AE$, and extend $BC$ to meet the circle at $X$ and $Y$. The square on $BX$ is a square that suits.

*Proof:*

By Theorem 8-1we have $AB \cdot BE = BX \cdot BY = BX^2$, because we have $BX = BY$ because $XY \perp$ to the diameter $AE$.

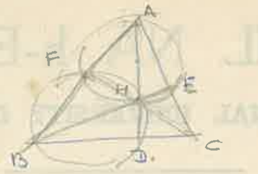
But we made $BE = BC$.

$\therefore$ The rectangle $ABCD = BX^2$. □

## Ceisteanna

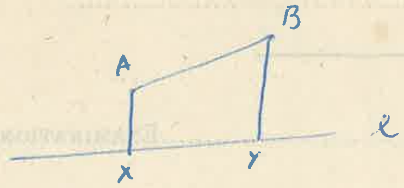
1. Is dronuille í $A$ sa $\Delta ABC$ agus 'sé $AD$ an t-ingear ar $BC$. Cruthuigh
   (a) $BD \cdot DC = AD^2$.
   (b) go dtadhlann $AC$ an ciorcal $ADB$ ag $A$ agus dá chionn sin go bhfuil
$$BC \cdot CD = AC^2.$$
2. I dtriantán ar bith $ABC$ pointe in $BC$ is ea $X$ ionas go bhfuil $\widehat{XAC} = \widehat{ABC}$. Cruthuigh
$$BC \cdot CX = AC^2.$$
3. Tarraing cearnóg a bheas comhfhairsing le triantán áirithe.

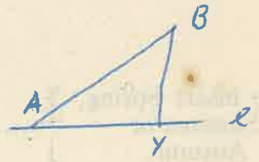
4) I dtriantán ar bith ABC ~~[illegible]~~ 'sé AD an t-ingear ó A ar BC, agus 'sé BE an t-ingear ó B ar AC. Cruthuigh BC.CD = AC.CE.




CD.CB = CH.CF = CE.CA

## Téarma



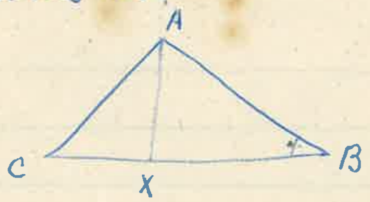




Má's iad X, Y bunanna na n-ingear ó A agus B ar dhronlíne l, 'sé XY leagan na dronlíne AB ar l. Má tharlaíonn go bhfuil A ar l, 'sé AY leagan AB ar l.

## Teoirim II

~~Má's géaruillinn [illegible] a ghabhas slios i dtriantán, is ionann an chearnóg ar an slios sin agus suim na gcearnóg ar an dá shlios eile minus dúbailt na dronuilleoige.~~

Má's géaruillinn a ghabhas slios i dtriantán, is ionann an chearnóg ar an slios sin agus suim na gcearnóg ar an dá shlios a chrioslaíonn an ghéaruillinn minus dúbailt na dronuilleoige fá shlios acu agus leagan an tsleasa eile air.




Abair gur géaruillinn í $\hat{B}$ san triantán ABC, agus go bhfuil $AX \perp BC$. Tá le cruthú go bhfuil $AC^2 = AB^2 + BC^2 - 2CB.BX$.

## Cruthúnas

Ó'n dtriantán dronuilleach CXA faightear $AX^2 = AC^2 - CX^2$, agus mar an gcéanna tá $AX^2 = AB^2 - BX^2$. Scríobh $CB - BX$ in áit $CX$.

$$\therefore \quad AC^2 - (CB - BX)^2 = AB^2 - BX^2.$$

$$\therefore \quad AC^2 - CB^2 + 2CB.BX - BX^2 = AB^2 - BX^2$$

Fágann sin $AC^2 - CB^2 + 2CB.BX = AB^2$, nó $AC^2 = AB^2 + BC^2 - 2CB.BX$

Q.E.D.

Nóta [Féach Teoirim ~~XVII~~ Caib. V nóta]

Tosach leathanach 99 sa LSS.

4. I dtriantán ar bith $ABC$ 'sé $AD$ an t-ingear ó $A$ ar $BC$, agus 'sé $BE$ an t-ingear ó $B$ ar $AC$. Cruthuigh $BC \cdot CD = AC \cdot CE$.

   Tá Fíoghair anseo sa LSS, leathanach 99.

## Questions

1. $A$ is a right angle in the $\Delta ABC$ and $AD$ is the perpendicular on $BC$. Prove
   (a) $BD \cdot DC = AD^2$.
   (b) that $AC$ is tangent to the circle $ADB$ at $A$ and as a result that
   $$BC \cdot CD = AC^2.$$
2. In any triangle $ABC$ the point $X$ in $BC$ is such that $\widehat{XAC} = \widehat{ABC}$. Prove that
   $$BC \cdot CX = AC^2.$$
3. Draw a square that has the same area as a given triangle.
4. In any triangle $ABC$ the perpendicular from $A$ on $BC$, is $AD$, and the perpedicular from $B$ on $AC$. is $BE$. Prove $BC \cdot CD = AC \cdot CE$.

### Téarma

Tá Fíoghair anseo sa LSS, leathanach 99.

Má's iad $X, Y$ bun na n-ingear ó $A$ agus $B$ are dhronlíne $\ell$, 'sé $XY$ *leagan na dronlíne AB ar* $\ell$. Má thárlaíonn go bhfuil $A$ ar $\ell$, 'sé $AY$ leagan $AB$ ar $\ell$.

## 8.2 Teoirim II

*Má's géaruille a ghabhas slios i dtriantán, is ionann an chearnóg ar an slios sin agus suim na gcearnóg ar an dá shlios a chrioslaíonn an ghéaruille minus dúbailt na dronuilleoige fá shlios acu agus leagan an tslios eile air.*

Tá Fíoghair anseo sa LSS, leathanach 99.

Abair gur géaruille é $\hat{B}$ san triantán $ABC$, agus go bhfuil $AX \perp BC$. Tá le crutú go bhfuil $AC^2 = AB^2 + BC^2 - 2CB \cdot BX$.

*Cruthúnas:*

Ó'n dtriantán dronuilleach $CXA$ faightear $AX^2 = AC^2 - CX^2$, agus mar an gcéanna tá $AX^2 = AB^2 - BX^2$. Scriobh $CB - BX$ in áit $CX$.

$$\therefore\ AC^2 - (CB - BX)^2 = AB^2 - BX^2.$$

$$.i. AC^2 - CB^2 + 2CB \cdot BX - BX^2 = AB^2 - BX^2.$$

Fágann sin $AC^2 - CB^2 + 2CB \cdot BX = AB^2$, nó $AC^2 = AB^2 + BC^2 - 2CB \cdot BX$. □

## Nóta

[Féach Teoirim XXII Caib. V nóta.]

## Definition

If $X, Y$ are the feet of the perpendiculars from $A$ and $B$ on a straight line $\ell$, then $XY$ is the *projection of the straight line $AB$ on $\ell$*. If it happens that $A$ is on $\ell$, then $AY$ is the projection of $AB$ on $\ell$.

**Theorem** (8-2)**.** *If a side of a triangle subtends an acute angle, then the square on that side is equal to the sum of the squares on the two sides that embrace the acute angle minus twice the rectangle on one of those sides and the projection of the other side on it.*

Suppose that $\hat{B}$ is an acute angle in the triangle $ABC$, and that $AX \perp BC$. We have to prove that $AC^2 = AB^2 + BC^2 - 2CB \cdot BX$.

*Proof:*
From the right-angle triangleh $CXA$ we get $AX^2 = AC^2 - CX^2$, and similarly we have $AX^2 = AB^2 - BX^2$. Write $CB - BX$ in place of $CX$.

$$\therefore\ AC^2 - (CB - BX)^2 = AB^2 - BX^2.$$

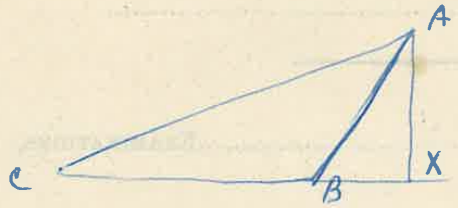
$$.i. AC^2 - CB^2 + 2CB \cdot BX - BX^2 = AB^2 - BX^2.$$

It follows that $AC^2 - CB^2 + 2CB \cdot BX = AB^2$, or $AC^2 = AB^2 + BC^2 - 2CB \cdot BX$. □

## Note

[See the note after Theorem 22 in Chapter V.]

Teoirim III  Má's maoluille a ghabhas slios i dtriantán, is ionann an chearnóg ar an slios sin agus suim na gcearnóg ar an dá shlios a chrioslaíonn an mhaoluille plus dúbailt na dronuilleoige fá shlios acu agus leagan an t-sleasa eile air.




Abair gur maoluille í $\hat{B}$ sa triantán $ABC$, agus go bhfuil $AX \perp BC$

Tá le cruthú go bhfuil $AC^2 = AB^2 + BC^2 + 2\,BC.\,BX$.

Cruthúnas

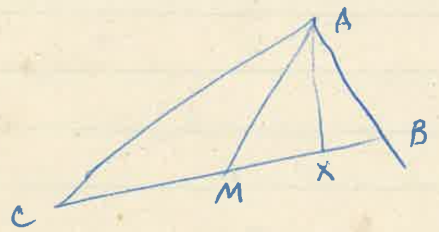
Ó'n dtriantán dronuilleach $AXC$ tá $AX^2 = AC^2 - CX^2$, agus mar an gcéanna tá $AX^2 = AB^2 - BX^2$. Scríobh $BC + BX$ in áit $CX$.

$\therefore \quad AC^2 - (BC + BX)^2 = AB^2 - BX^2$.

$\therefore \quad AC^2 - BC^2 - 2BC.BX - BX^2 = AB^2 - BX^2$.

Fágann sin $AC^2 - BC^2 - 2BC.BX = AB^2$, nó $AC^2 = AB^2 + BC^2 + 2BC.BX$.

Teoirim IV  Is ionann suim na gcearnóg ar dhá shlios triantáin agus dúbailt na cearnóige ar leath an tríú sleasa maille le dúbailt na cearnóige ar an meánlíne a chomhroinneas an tríú slios.




Sa $\triangle ABC$ abair gur meánlíne í $AM$ (i.e. $BM = MC$), agus go bhfuil $AX \perp BC$.

Tá le cruthú go bhfuil $AB^2 + AC^2 = 2MC^2 + 2AM^2$.

Cruthúnas

Maran $\triangle$ comhchosach é $ABC$ géaruille is ea uille amháin den dá uillinn fhóirlíontacha $A\hat{M}C$, $A\hat{M}B$, agus maoluille is ea an uille eile.

Sa $\triangle$ maoluilleach $AMC$, tá $AC^2 = AM^2 + MC^2 + 2CM.MX$ (Teoirim III)

Sa $\triangle$ géaruilleach $AMB$, tá $AB^2 = AM^2 + MB^2 - 2BM.MX$ (Teoirim II)

Ach tugtar $MC = MB$.

$\therefore$ Le suimiú faightear $AB^2 + AC^2 = 2AM^2 + 2MC^2$.

Tosach leathanach 100 sa LSS.

## 8.3 Teoirim III

*Má's maoluille a ghabhas slios i dtriantán, is ionann an chearnóg ar an slios sin agus suim na gcearnóg ar an dá shlios a chrioslaíonn an mhaoluille plus dúbailt na dronuilleoige fá shlios acu agus leagan an tsleasa eile air.*

Tá Fíoghair anseo sa LSS, leathanach 100.

Abair gur maoluille é $\hat{B}$ sa triantán $ABC$, agus go bhfuil $AX \perp BC$.
Tá le cruthú go bhfuil $AC^2 = AB^2 + BC^2 + 2BC \cdot BX$.

*Cruthúnas:*

Ó'n triantán dronuilleach $AXC$ tá $AX^2 = AC^2 - CX^2$, agus mar an gcéanna tá $AX^2 = AB^2 - BX^2$. Scríobh $BC + BX$ in áit $CX$.
$\therefore AC^2 - (BC + BX)^2 = AB^2 - BX^2$.
.i. $AC^2 - BC^2 - 2BC \cdot BX - BX^2 = AB^2 - BX^2$.
Fágann sin $AC^2 - BC^2 - BC \cdot BX = AB^2$, nó $AC^2 = AB^2 + BC^2 + 2BC \cdot BX$.

**Theorem** (8-3)**.** *If a side in a triangle subtends an obtuse angle, then the square on that side is equal to the sum of the squares on the two sides that embrace the obtuse angle plus twice the rectangle on one of those sides and the projection of the other side on it.*

Suppose that $\hat{B}$ is an obtuse angle in the triangle $ABC$, and that $AX \perp BC$.
We have to prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BX$.

*Proof:*

From the right-angle triangle $AXC$ we have $AX^2 = AC^2 - CX^2$, and similarly $AX^2 = AB^2 - BX^2$. Write $BC + BX$ in place of $CX$.
$\therefore AC^2 - (BC + BX)^2 = AB^2 - BX^2$.
.i. $AC^2 - BC^2 - 2BC \cdot BX - BX^2 = AB^2 - BX^2$.
This gives $AC^2 - BC^2 - BC \cdot BX = AB^2$, nó $AC^2 = AB^2 + BC^2 + 2BC \cdot BX$. □

## 8.4 Teoirim IV

*Is ionann suim na gcearnóg ar dhá shlios triantáin agus dúbailt na cearnóige ar leath an tríú shleasa maille le dúbailt na cearnóige ar an meánlíne a chomhroinneas an tríú slios.*

Tá Fíoghair anseo sa LSS, leathanach 100.

Sa $\Delta ABC$ abair gurmeánlíneé $AM$ (i.e. $BM = MC$), agus go bhfuil $AX \perp BC$.
Tá le crutú go bhfuil $AB^2 + AC^2 = 2MC^2 + 2AM^2$.

*Cruthúnas:*

Moran $\Delta$ comhchosach é $ABC$ géaruille is ea uille amháin den dá uillinn fhóirlíontacha $\widehat{AMC}, \widehat{AMB}$, agus maoluille is ea an uille eile.

Sa $\Delta$ maoluilleach $AMC$, tá $AC^2 = AM^2 + MC^2 + 2CM \cdot MX$ (Teoirim III).

Sa $\Delta$ géaruilleach $AMB$, tá $AC^2 = AM^2 + MB^2 - 2BM \cdot MX$ (Teoirim II).

Ach tugtar $MC = MB$.

$\therefore$ Le suimiú faightear $AB^2 + AC^2 = 2MC^2 + 2AM^2$.

**Theorem** (8-4)**.** *The sum of the squares on two sides of a triangle is equal to twice the square on half of the third side plus twice the square on the median that bisects the third side.*

In the $\Delta ABC$ suppose $AM$ is a median (i.e. $BM = MC$), and that $AX \perp BC$.
We have to prove that $AB^2 + AC^2 = 2MC^2 + 2AM^2$.

*Proof:*

Unless the $\Delta$ $ABC$ is isosceles, one or other of the complementary angles $\widehat{AMC}, \widehat{AMB}$ is acute, and the other is obtuse.

In the obtuse-angled $\Delta$ $AMC$, we have $AC^2 = AM^2 + MC^2 + 2CM \cdot MX$ (Theorem 8-3).

In the acute-angled triangle $\Delta$ $AMB$, we have $AC^2 = AM^2 + MB^2 - 2BM \cdot MX$ (Teoirim 8-2).

But we are given that $MC = MB$.

$\therefore$ By adding we get $AB^2 + AC^2 = 2MC^2 + 2AM^2$. □

## Ceisteanna

1) Sa triantán ABC, tá $AC^2 = AB^2 + BC^2 + AB.BC$; cruthuigh go bhfuil $\hat{B} = 120°$.

2) I dtriantán ar bith ABC deimhnigh gurb ionann na dronuilleoga AB fá leagan BC ar AB, agus BC fá leagan AB ar BC.

3) ~~I gceathairshleasán~~ Taispáin gurb ionann suim na gcearnóg ar shleasa pharalléilograim agus suim na gcearnóg ar an dá thrasnán.

4) I dtriantán ar bith is ionann suim na gcearnóg ar na sleasa fá thrí agus suim na gcearnóg ar na meánlínte fá cheathair.

5) I gceathairshleasán ar bith ABCD 'siad X, Y lár na dtrasnán AC agus BD. Cruthuigh $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4XY^2$.

6) De bhrí gur córda é CX sa gciorcal ar an lárlíne AC (Teoirim IV), cruthuigh $CM.MX = \frac{1}{4}(AC^2 - CB^2)$

7) Sa triantán ABC tá AB = AC agus síntear AB go dtí D i gcaoi go bhfuil AB = BD. Cruthuigh $CD^2 = AB^2 + 2BC^2$.

8) Teagmhaíonn dhá chiorcal le chéile in A agus B, agus pointe ar bith ar shíneadh AB isea P. Taispáin gur cómhfhada na tadhlaithe a tarraingítear ó P go dtí an dá chiorcal.

9) Faigh rian an phointe P a ghluaiseas i gcaoi go bhfuil $AP^2 + PB^2$ buan, áit gur pointí socra iad A agus B.

10) Sa triantán ABC pointe in BC isea K i gcaoi go bhfuil $mKC = nBK$ áit gur uimhreacha iad m, n.
Cruthuigh $mAB^2 + nAC^2 = (m+n)AK^2 + mBK^2 + nKC^2$.

Tosach leathanach 101 sa LSS.

## Ceisteanna

1. Sa triantán $ABC$, tá $AC^2 = AB^2 + BC^2 + AB \cdot BC$; cruthuigh go bhfuil $\hat{B} = 120°$.

2. I dtriantán ar bith $ABC$ deimhnigh gurb ionann na dronuilleoga $AB$ fá leagan $BC$ ar $AB$, agus $BC$ fá leagan $AB$ ar $BC$.

3. Teaspáin gurb ionann suim na dgearnóg ar shleasa pharallélogram agus suim na gcearnóg ar an dá threasnán.

4. I dtriantán ar bith is ionann suim na gcearnóg ar na sleasa fá thrí agus suim na gcearnóg ar na meánlínte fá cheathair.

5. I gceathairshleasán ar bith $ABCD$ 'siad $X, Y$ láir na dtreasnán $AC$ agus $BD$. Cruthuigh
$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4XY^2.$$

6. De bhrí gur córda é $CX$ sa gciorcal ar an láirlíne $AC$ (Teoirim IV), cruthuigh
$$CM \cdot MX = \frac{1}{4}(AC^2 - CB^2).$$

7. Sa triantán $ABC$ tá $AB = AC$ agus síntear $AB$ go dtí $D$ ionas go bhfuil $AB = BD$. Cruthuigh $CD^2 = AB^2 + 2BC^2$.

8. Teagmhaíonn dhá chiorcal le chéile in $A$ agus $B$, agus pointe ar bith ar síneadh $AB$ is ea $P$. Teaspáin gur cómhfhada na tadhlaithe a tarraingítear ó $P$ go dtí an dá chiorcail.

9. Faigh rian an phointe $P$ a ghluaiseas i gcaoi go bhfuil $AP^2 + PB^2$ buan, áit gur pointí socra iad $A$ agus $B$.

10. Sa triantán $ABC$ pointe in $BC$ is ea $K$ ionas go bhfuil $mKC = nBK$ áit gur uimhreacha iad $m, n$.

    Cruthuigh
$$mAB^2 + nAC^2 = (m+n)AK^2 + mBK^2 + nKC^2.$$

## Questions

1. In a triangle $ABC$, we have $AC^2 = AB^2 + BC^2 + AB \cdot BC$; Prove that $\hat{B} = 120°$.

2. In any triangle at all $ABC$ verify that the rectangles $AB$ times the projection of $BC$ on $AB$, and $BC$ times the projection of $AB$ on $BC$ are equal (in area).

3. Show that the sum of the squares on the sides of a parallelogram is equal to the sum of the squares on the two diagonals.

4. In any triangle at all the sum of the squares on the sides times three is equal to the sum of the squares on the medians times four.

5. In any quadrilateral $ABCD$ let $X, Y$ be the centres of the diagonals $AC$ and $BD$. Prove
$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4XY^2.$$

6. Using the fact that $CX$ is a chord in the circle with diameter $AC$ (Theorem 4), prove that $CM \cdot MX = \frac{1}{4}(AC^2 - CB^2)$.

7. In the triangle $ABC$ with $AB = AC$ the side $AB$ is extended to $D$ so that $AB = BD$. Prove that $CD^2 = AB^2 + 2BC^2$.

8. Two circles meet one another at $A$ and $B$, and $P$ is any point on the extension of $AB$. Show that the tangents from $P$ to the two circles are the same length.

9. Find the locus of the point $P$ that moves so that $AP^2 + PB^2$ is constant, where the points $A$ and $B$ are fixed.

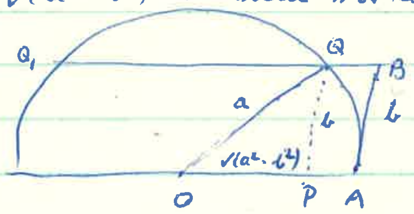
10. In the triangle $ABC$ a point $K$ in $BC$ is such that $mKC = nBK$ where $m, n$ are numbers.

    Prove that
$$mAB^2 + nAC^2 = (m+n)AK^2 + mBK^2 + nKC^2.$$

## An Chudromóid Chearnach san Geoméatreacht

Nuair tugtar dhá mhírlíne atá $a$ agus $b$ ar fad, is furasta an líne $\sqrt{(a^2+b^2)}$ a thógáil de thairbhe theoirme ~~Pythagoras~~ Phítagorais.

Is féidir $\sqrt{(a^2-b^2)}$ a aimsiú mar seo a leanas, nuair $a > b$.




Tarraing O ar gá dó $OA = a$, agus ar an ingear ag A déan $AB = b$. Abair go ngearrann an pharailéil tré B an Θ in Q, agus gurb é QP an k-ingear ar OA.

De bhrí go bhfuil $OQ = a$, $PQ = b$, $O\hat{P}Q = 90°$, tá $OP^2 = a^2 - b^2$.

<u>Nóta</u> Má déantar leithchiorcal ar OA agus má's córda é AX san gciorcal go bhfuil fad $b$ ann, feicfear go bhfuil $OX^2 = a^2 - b^2$.

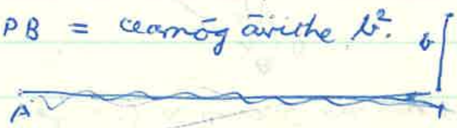
## An Chudromóid Chearnach

Is ionann le chéile $x^2 + 2ax - b^2 = 0$ agus $(x+a)^2 = a^2 + b^2$, ionnas go gcinntear dhá réiteach na cudromóide de réir $x + a = \pm\sqrt{(a^2+b^2)}$.

Mar an gcéanna, is ionann $x^2 - 2ax + b^2 = 0$ agus $(x-a)^2 = a^2 - b^2$, agus gurb iad $x = a \pm \sqrt{(a^2-b^2)}$ na réiteacha, áit a bhfuil $a > b$.

Is iomdha ceist san geoméatreacht gurb ionann í agus cudromóid chearnach a réiteach, agus cé gur gnáthach an obair a choiriú i gcomharthaíocht geoméatreach 'sí an obair chéanna í agus réiteach algébrach na cudromóide. Is maith ann an léide nach féidir an cheist a réiteach d'uireasa na líne $\sqrt{(a^2+b^2)}$ nó $\sqrt{(a^2-b^2)}$ de réir na cudromóide a bhfuil gnotha againn leithi.

<u>Sompla 1</u> Dronlíne AB a roinnt i bpointe P istigh, ionnas go mbeidh $AP.PB$ = cearnóg áirithe $b^2$.





<u>Réiteach</u> Faigh O lár AB. Má's é P an pointe atá á lorg bíodh $OA = OB = a$, $OP = x$. Fágann sin $AP = a + x$; $PB = a - x$.

$\therefore$ caithfidh $(a+x)(a-x) = b^2$ $\therefore x^2 = a^2 - b^2$ nó $x = \pm\sqrt{(a^2-b^2)}$

Mínítear thuas cé'n chaoi a n-aimsítear an líne $\sqrt{(a^2-b^2)}$ agus cinntear ionad P dá réir.

Is léir gur réiteach eile é P, (scáth P in O); is don fhréamh eile $x = -\sqrt{(a^2-b^2)}$ a fhreagraíos seisean.

Tosach leathanach 102 sa LSS.

## 8.5 An Chudromóid Chearnach san Geométracht

Nuair a tugtar dhá mhírlíne atá$a$ agus $b$ ar fad, is furasta an líne $\sqrt{(a^2+b^2)}$ a thógáil de thairbhe Theoirme Phutagorais.

Is féidir $\sqrt{(a^2-b^2)}$ a aimsiú mar seo a leanas, nuair $a > b$.

Tá Fíoghair anseo sa LSS, leathanach 102.

Tarraing ⊙ ar ga dó $OA = a$, agus ar an ingear ag $A$ déan $AB = b$. Abair go ngearrann an pharallél tré $B$ an ⊙ in $Q$, agus gurb é $QP$ an t-ingear ar $OA$.

De bhrí go bhfuil $OA = a, PQ = b, \widehat{OPQ} = 90°$, tá $OP^2 = a^2 - b^2$.

### Nóta

Má déantar leith-chiorcal ar $OA$ agus má's córda é $AX$ san gciorcal go bhfuil fad $b$ ann, feicfear go bhfuil $OX^2 = a^2 - b^2$.

## 8.6 The Quadratic Equation in Geometry

When two line segments that are of lengths $a$ and $b$, it is easy to construct the line $\sqrt{(a^2+b^2)}$ with the aid of Pythagoras' Theorem.

It is possible to find $\sqrt{(a^2-b^2)}$ as follows, when $a > b$.

Draw a ⊙ with radius $OA = a$, and on the perpendicular at $A$ make $AB = b$. Suppose the parallel through $B$ cuts the circle at $Q$, and that $QP$ is the perpendicular on $OA$.

Since $OA = a, PQ = b, \widehat{OPQ} = 90°$, we have $OP^2 = a^2 - b^2$.

### Note

If a semicircle is made on $OA$ and if $AX$ is a chord in the circle of length $b$, it will be seen that $OX^2 = a^2 - b^2$.

### An Chudromoid Chearnach

Is ionann le chéile $x^2-2ax-b^2 = 0$ agus $(x+a)^2 = a^2+b^2$, ionas go gcinntear dhá réiteach na chudromóide de réir $x + a = \pm\sqrt{(a^2+b^2)}$.

Mar an gcéanna, is ionann $x^2 - 2ax + b^2 = 0$ agus $(x-a)^2 = a^2 - b^2$, agus siad $x = a \pm \sqrt{(a^2-b^2)}$ na réiteacha, áit a bhfuil $a > b$.

Is iomdha ceist san geométreacht gurb ionann í agus cudromóid chearnach a réiteach, agus cé gur gnáthach an obair a chóiriú i gcomharthaíocht geométreac 'sí an obair chéanna í agus réiteach algébrach na cudromóide. Is maith ann an leide nach féidir an cheist a réiteach d'uireasa na líne $\sqrt{(a^2+b^2)}$ nó $\sqrt{(a^2-b^2)}$ de réir na cudromóide a bhfuil gnotha againn leithu.

## The Quadratic Equation

The equations $x^2 - 2ax - b^2 = 0$ and $(x+a)^2 = a^2 + b^2$ are equivalent to one another, so two solutions to the equation are determined according to $x + a = \pm\sqrt{(a^2+b^2)}$.

In the same way, the equations $x^2 - 2ax + b^2 = 0$ and $(x-a)^2 = a^2 - b^2$ are equivalent, and the solutions are $x = a \pm \sqrt{(a^2-b^2)}$, when $a > b$.

Many a question in geometry is equivalent to solving a quadratic equation, and even though it is usual to organise the work in geometrical symbolism, it is the same work as finding the algebraic solution of the equation. It is a good hint that the question cannot be solved without using the line $\sqrt{(a^2+b^2)}$ or $\sqrt{(a^2-b^2)}$, depending on the equation with which we are dealing.

## Sampla 1

*Dronlíne a roinnt i bpointe $P$ istigh, ionas go mbeidh $AP \cdot PB$ = cearnóg áirithe $b^2$.*

Tá Fíoghair anseo sa LSS, leathanach 102.

*Réiteach:*

Faigh $O$ lár $AB$. Má's é $P$ an pointe atá á lorg bíodh $OA = OB = a$, $OP = x$. Fágann sin $AP = a + x$; $PB = a - x$.

Caithfidh $(a+x)(a-x) = b^2$, .i. $x^2 = a^2 - b^2$ nó $x = \pm\sqrt{(a^a - b^2)}$.

Mínítear thuas cé'n chaoi a n-aimsítear an líne $\sqrt{(a^2-b^2)}$ agus cinntear ionad $P$ d'á réir.

Is léir gur réiteach eile é $P_1$ (scáth $P$ in $O$); is don phréamh eile $x = -\sqrt{(a^2-b^2)}$ a fhreagraíos seisean.

## Sample 1

*To divide a straight line at an inside point $P$, so that we will have $AP \cdot PB$ = a given square $b^2$.*

*Solution:*

Find $O$, the centre of $AB$. If $P$ is the point we seek, let $OA = OB = a$, $OP = x$. It follows that $AP = a + x$; $PB = a - x$.

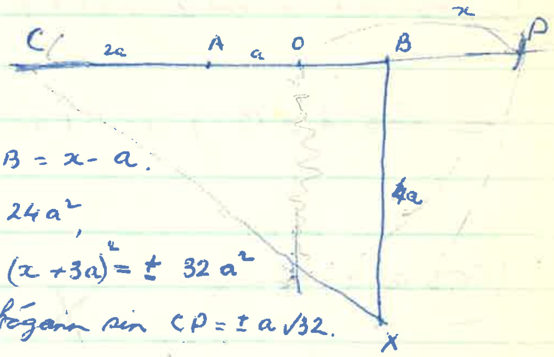
We must have $(a+x)(a-x) = b^2$, .i. $x^2 = a^2 - b^2$ or $x = \pm\sqrt{(a^a - b^2)}$.

It is explained above how to find the line $\sqrt{(a^2-b^2)}$ and accordingly the position of $P$ is determined.

Clearly another solution is $P_1$ (the reflection of $P$ in $O$); that corresponds to the other root $x = -\sqrt{(a^2-b^2)}$.

**Sompla 2** Roinn an dronlíne AB i bpointe P amuigh i gcaoi go mbeidh $2AP^2 - PB^2 = 6AB^2$

**Réiteach**




Bíodh $AO = OB = a$, $OP = x$

ionnas go bhfuil $AP = x + a$, $PB = x - a$.

Tugtar $2(x+a)^2 - (x-a)^2 = 24a^2$,

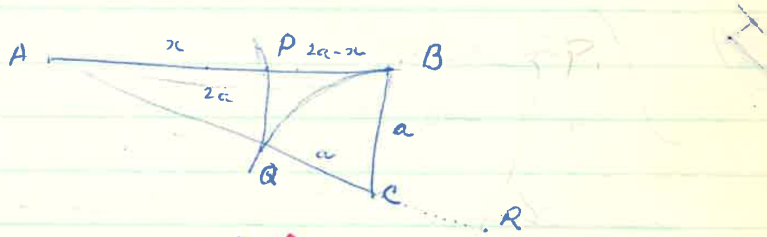
∴ $x^2 + 6ax = 23a^2$ nó $(x+3a)^2 = \pm 32a^2$

Má tá $CA = AB = 2a$, fágann sin $CP = \pm a\sqrt{32}$.

Má geartar $BX = CB = 4a$ ar ingear ag B is léir go bhfuil $CX = a\sqrt{32}$, agus má's í P a gearras an ⊙ gur lár dó C agus gur ga dó CX an líne AB comhlíonann P na coinníollacha. Gheobhfar pointe eile a fhileas ar an taobh eile de C.

**Sompla 3** ~~Dronlíne~~ Meán-teascadh a dhéanamh ar dhronlíne AB.

∴ pointe P in AB a aimsiú ionnas go mbeidh $AB.BP = AP^2$.




Bíodh $AB = 2a$, $AP = x$, ionnas go bhfuil $PB = 2a - x$.

Tugtar $2a(2a - x) = x^2$ ∴ $x^2 + 2ax = 4a^2$ nó $(x+a)^2 = 5a^2$.

Fágann sin go bhfuil $AP = a(\sqrt{5} - 1)$.

Chun teacht ar ionad an phointe P ní mór an líne $a\sqrt{5}$ a thógáil i dtosach, agus má geartar $BC = a$ ar ingear ag B is léir go bhfuil $AC^2 = AB^2 + BC^2 = 4a^2 + a^2$. ∴ Tá $AC = a\sqrt{5}$.

Nuair baintear $CQ = CB = a$ den líne sin faightear $AQ = a(\sqrt{5}-1)$.

∴ Nuair geartar $AP = AQ$ ar an líne AB, 'sé P an pointe a bhí á lorg.

**Nota** Réiteach eile is ea $x = -(\sqrt{5}+1)a$, agus má cuirtear $CR = CB = a$ le AC, tá $AR = a(\sqrt{5}+1)$. Má's é Q an pointe ar shíneadh BA i gcaoi go bhfuil $AQ = AR = a(\sqrt{5}+1)$, gheobhfar amach go bhfuil $AB.BQ = AQ^2$,

Deirtear gurb é Q pointe an mheánteascaidh amuigh.

Tosach leathanach 103 sa LSS.

## Sampla 2

*Roinn an dronlíne AB ag bpointe P amuigh i gcaoi go mbeidh* $2AP^2 - PB^2 = 6AB^2$.

Tá Fíoghair anseo sa LSS, leathanach 103.

*Réiteach:*

Bíodh $AO = OB = a$, $OP = x$ ionas go bhfuil $AP = x + a, PB = x - a$.

Tugtar $2(x+a)^2 - (x-a)^2 = 24a^2$,

$\therefore x^2 + 6ax = 23a^2$ nó $(x+3a)^2 = \pm 32a^2$.

Má tá $CA = AB = 2a$, fágann sin $CP = \pm a\sqrt{32}$.

Má gearrtar $BX = CB = 4a$ ar ingear ag $B$ is léir go bhfuil $CX = a\sqrt{32}$, agus má's in $P$ a gearras an ⊙ gur láir dó $C$ agus gur ga dó $CX$ an líne $AB$ cóimhlíonann $P$ na coinníollacha.

Gheofar pointe eile a fheileas ar an taobh eile de $C$.

## Sample 2

*Divide the straight line AB at an outside point P so that we will have* $2AP^2 - PB^2 = 6AB^2$.

*Solution:*

Let $AO = OB = a$, $OP = x$ so that $AP = x + a, PB = x - a$.

We are given that $2(x+a)^2 - (x-a)^2 = 24a^2$,

$\therefore x^2 + 6ax = 23a^2$ or $(x+3a)^2 = \pm 32a^2$.

If $CA = AB = 2a$, it follows that $CP = \pm a\sqrt{32}$.

If we cut $BX = CB = 4a$ on a perpendicular at $B$ it is clear that $CX = a\sqrt{32}$, and if $P$ is where the ⊙ with centre $C$ and radius $CX$ cuts the line $AB$, then $P$ satisfies the conditions.

One finds another suitable point on the other side of $C$.

## Sampla 3

*Meán teascach a dhéanamh ar dhronlíne AB.*
.i. pointe $P$ in $AB$ a aimsiú ionas go mbeidh $AB \cdot BP = AP^2$.

Tá Fíoghair anseo sa LSS, leathanach 103.

Bíodh $AB = 2a$, $AP = x$, ionas go bhfuil $PB = 2a - x$.
Tugtar $2a(2a - x) = x^2$ .i. $x^2 + 2ax = 4a^2$ nó $(x+a)^2 = 5a^2$.

Fágann sin go bhfuil $AP = a(\sqrt{5} - 1)$.

Chun teacht ar ionad an phointe $P$ ní mór an líne $a\sqrt{5}$ a thógáil is dtosach, agus má gearrtar $BC = a$ ar ingear ag $B$ is léir go bhfuil $AC^2 = AB^2 + BC^2 = 4a^2 + a^2$. $\therefore$ Tá $AC = a\sqrt{5}$.

Nuair baintear $CQ = CB = a$ den líne sin faightear $AB = a(\sqrt{5} - 1)$.
$\therefore$ Nuair gearrtar $AD = AQ$ ar an líne $AB$, 'sé $P$ an pointe a bhí á lorg.

## Sample 3

*To make the mean proportional cut of a straight line AB.*
.i. to find the point $P$ in $AB$ such that

$$AB \cdot BP = AP^2.$$

Let $AB = 2a$, $AP = x$, so that $PB = 2a - x$.
We are given $2a(2a - x) = x^2$ .i. $x^2 + 2ax = 4a^2$ or $(x + a)^2 = 5a^2$.

It follows that $AP = a(\sqrt{5} - 1)$.

To locate the position of the point $P$ we have to construct the line $a\sqrt{5}$ first, and if we cut $BC = a$ on a perpendicular at $B$ it is clear that $AC^2 = AB^2 + BC^2 = 4a^2 + a^2$. ∴ we have $AC = a\sqrt{5}$.

Taking away $CQ = CB = a$ from that line leaves $AB = a(\sqrt{5} - 1)$.
∴ When $AD = AQ$ is cut on the line $AB$, then $P$ is the point we were seeking.

The point $P$ is that which cuts the interval $AB$ in two intervals such that $AP$ is the geometric mean of $AB$ and $BP$, i.e.

$$\frac{AP}{AB} = \frac{PB}{AP}.$$

I'm not sure what name is used for it. Perhaps it should be the '(internal) mean proportional cut point' of the interval, or 'the point which divides the interval in mean proportions'. Another possibility is the simple 'mean cut' or the 'mean proportion point'.

### Nóta

Réiteach eile is ea $x = -(\sqrt{5}+1)a$, agus má cuirtear $CR = CB = a$ le $AC$, tá $AR = a(\sqrt{5}+1)$. Má's é $Q$ an pointe ar síneadh $BA$ i gcaoi go bhfuil $AQ = AR = a(\sqrt{5} + 1)$, gheobhfar amach go bhfuil $AB \cdot BQ = AQ^2$.

Deirtear gurb é $Q$ pointe mheábteascaidh *amuigh*.

### Note

Another solution is $x = -(\sqrt{5} + 1)a$, and if we add $CR = CB = a$ to $AC$, we have $AR = a(\sqrt{5} + 1)$. If $Q$ is the point on the extension of $BA$ such that $AQ = AR = a(\sqrt{5} + 1)$, it will be found that $AB \cdot BQ = AQ^2$.

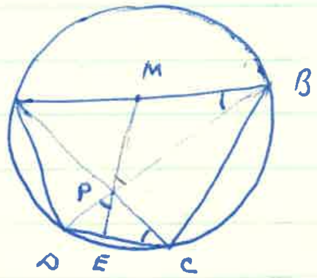
One says that $Q$ is the *external* mean proportional cut point.

## Anailís Geoméatreach

Nuair atá teoirim le ~~cruthú ag~~ cruthú (nó ceist le réiteach) againn, nach léir dúinn an cruthúnas roimh ré, is féidir teacht ar an réiteach go minic ar an gcuma seo a leanas.

Glac le fírinne na teoirme (nó ~~an~~ cuir i gcás go gcomhlíonann figiúr áirithe coinníollacha na ceiste) gan cruthúnas ar bith i dtosach. De thairbhe teoirmí eile atá ar eolas againn tig linn oibriú siar uaidh agus tátaill eile a ghnóthú as. Má's féidir sa gcaoi sin tátal áirithe a fháil gurb eol a chruthúnas cheana agus gur leor fírinne an tátaill chun fírinne na teoirme a dheimhniú, tig linn cruthúnas na teoirme a bhunú ar chruthúnas an tátaill.

e.g. (1) Sa gceathairshleasán comhchiorcalach ABCD tágann na trasnáin le chéile go h-ingearach ag P. Cruthuigh go bhfuil líne cheangail P le lár AB ingearach le CD.




Tuigtear (1) gur ceathairshleasán i gciorcal é ABCD: (2) go bhfuil $A\hat{P}B = 9$[0°] (3) go bhfuil $AM = MB$.

Tá le cruthú go bhfuil $PED = 90°$.

### Anailís

De bhrí gur dronuillé é DPC, chun go mbeadh $P\hat{E}D = 90°$, ba leor a chruthú go bhfuil $D\hat{P}E = 90 - D = P\hat{C}E$.

.i. go bhfuil $M\hat{P}B = M\hat{B}P$ (mar uillinneacha sa teascán cheanna ise~~a~~ $P\hat{C}E$ agus $M\hat{B}D$).

Ach ó tharla $MA = MB$ sa $\Delta$ dronuilleach PAB, is eol dúinn gurb é M iomlár an $\Delta$ PAB, ionnus go bhfuil $MP = MB$ agus $M\hat{B}P = M$[...]

Dá bhrí sin is féidir cruthúnas na teoirme a bhunú ar chruthúnas na fírinne $M\hat{B}P = M\hat{P}B$.

Fágtar fá'n léitheoir an cruthúnas a chóiriú.

Tosach leathanach 104 sa LSS.

## 8.7 Anailís Geométreach

Nuair atá teoirim le cruthú (nó ceist le réiteach) againn, nach léir dúinn an cruthúnas roimh ré , is féidir teacht ar an réiteach go minic ar an gcionn seo a leanas.

Glac le fírinne na teoirme (nó cuir i gcás go gcóimhlíonann fioghair áirithe coinníollacha na ceiste) gan cruthúnas ar bith i dtosach. De thairbhe teoirmí eile atá ar eolas againn tig linn oibriú siar uaidh agus tátaill eile a ghnothú as. Má's féidir sa gcaoi sin tátall áirithe a fháil gurb eol a chrutúnas cheana agus gur leor fírinne an tátaill chun fírinne ne teoirme a dheimhniú , tig linn cruthúnas na teoirme a bhunú ar chruthúnas an tátaill.

## 8.8 Geometrical Analysis

When we have a theorem to prove (or a question to solve) and the proof is not clear to us to start with, it is often possible to arrive at the solution by proceeding as follows:

Assume the truth of the theorem (or assume that some figure satisfies the conditions of the question) without any proof to start with. By using other theorems that we know we may be able to work back from it and derive other conclusions from it. If in that way we find a conclusion whose proof was already known to us, and if the truth of that conclusion is enough to allow us to verify the truth of the theorem, then we can base the proof of the theorem on the proof of the conclusion.

### e.g. (1)

*Sa gceathairshleasán cóimhchiorcalach $ABCD$ tagann ne treasnáin le chéile go h-ingearach ag $P$. Cruthuigh go bhfuil líne cheangail $P$ le lár $AB$ ingearach le $CD$.*

Tá Fíoghair anseo sa LSS, leathanach 104.

Tugtar (1) gur ceathairshleasán i gciorcal é $ABCD$, (2) go bhfuil $\widehat{APB}= 90°$, (3) go bhfuil $AM = MB$.

Tá le cruthú go bhfuil $PED = 90°$.

### Anailís

De bhrí gur dronuille é $DPC$, chun go mbeadh $\widehat{PED}= 90°$, ba leor a chruthú go bhfuil $\widehat{DPE}= 90 - D = \widehat{PCE}$.

.i. go bhfuil $\widehat{MPB}=\widehat{MBP}$ (mar uilleacha sa teascán céanna is ea $\widehat{PCE}$ agus $\widehat{MBD}$).

Ach ó thárla $MA = MB$ sa $\Delta$ dronuilleach $PAB$, is eol dúinn gurb é $M$ iomlár an $\Delta PAB$, ionas go bhfuil $MP = MB$ agus $\widehat{MBP}=\widehat{MPB}$.

Dáaa bhrí sin is féidir cruthúnas na teoirme a bhunú ar chruthúnas na fírinne $\widehat{MBP}=\widehat{MPB}$.

Fágtar fá'n léitheoir an crutúnas a chóiriú .

## e.g. (1)

*In the cyclic quadrilateral $ABCD$ the diagonals meet at right angles at $P$. Prove that the line joining $P$ to the centre of $AB$ is perpendicular to $CD$.*

We are given (1) that $ABCD$ is a quadrilateral inscribed in a circle, (2) that $\widehat{APB} = 90°$, (3) that $AM = MB$.

We have to prove that $PED = 90°$.

## Analysis

Since $DPC$ is a right angle, in order for $\widehat{PED} = 90°$, it would be enough to prove that $\widehat{DPE} = 90 - D = \widehat{PCE}$.
.i. that $\widehat{MPB} = \widehat{MBP}$ (for the angles $\widehat{PCE}$ and $\widehat{MBD}$ are in the same segment).

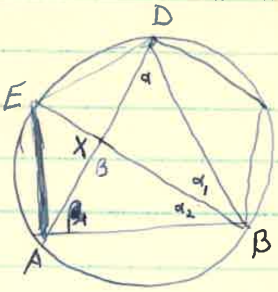
But since $MA = MB$ in the right-angle $\Delta\ PAB$, we know that $M$ is the circumcentre of the $\Delta PAB$, so that $MP = MB$ and $\widehat{MBP} = \widehat{MPB}$.

Therefore it is possible to base the proof of the theorem on the proof of the truth that $\widehat{MBP} = \widehat{MPB}$.

It is left to the reader to lay out the proof.

## e.g. (2) An Uille 36° a Thógáil le Compás agus Riail

Toisc gurb shin í an uille a ghabhas slios pentagóin rialta ag an imlíne an ciorcail (teoirim Caib III), is ceart dúinn tréithre na fioghrach sin a bhreithniú i dtosach.




### Anailís na Ceiste

Abair gur pentagón rialta é ABCDE, ionnus go bhfuil 36° san uillinn a ghabhas gach slios ag an imlíne. Ceangail DA, DB, BE.

Tá $\hat{\alpha} = 36° = \hat{\alpha}_1$, d'á réir, agus fágann sin $DX = XB$.

Mar an gcéanna tá $\hat{B}_1 = 72° = \hat{\alpha} + \hat{\alpha}_1 = \hat{\beta}$, ionnus go bhfuil $XB = BA$.

$\therefore$ Tá $DX = XB = BA$ ........................ (i)

De chairbhe $\hat{\alpha}_2 = 36° = \hat{\alpha}$, tadhlann AB ag B an ⊙ DXB, ionnus go bhfuil $DA.AX = AB^2$ (teoirim ) $= XD^2$, de réir (i).

Fágann sin $DA.AX = XD^2$ .i. déantar meán-teascadh ar DA ag X ........ (ii)

Nuair tugtar DA, tá X cinnteach de réir (ii), agus de bhrí go bhfuil B an fhad XD ó A is ó X de réir (i) socraíonn sé sin ionad B. Tig linn réiteach gonta a chóiriú anois mar seo.

### Réiteach

Déan líne ar bith DA a mheán-teascadh ag X ionnus go bhfuil $DA.AX = XD^2$; Le X agus A mar láir línigh ⊙ ar gach dóibh XD, a ghearras a chéile in B.

Tá $ADB = 36°$, agus tá $D\hat{A}B = D\hat{B}A = 72°$.

### Cruthúnas

De chairbhe $DA.AX = XD^2 = AB^2$, tadhlann AB an ⊙ DXB. $\therefore$ Tá $\hat{\alpha}_2 = \hat{\alpha}$.

Ach de bharr $XB = XD$, tá $\hat{\alpha}_1 = \alpha$, ionnus go bhfuil $D\hat{B}A = 2\hat{\alpha}$.

Mar an gcéanna de bharr $AB = XB$, tá $D\hat{A}B = \beta = \hat{\alpha} + \hat{\alpha}_1 = 2\hat{\alpha}$.

Ach tá suim uilleacha an $\Delta\, DAB = 180°$.

Fágann sin $5\alpha = 180°$, ionnus go bhfuil $\alpha = 36°$, $D\hat{A}B = D\hat{B}A = 72°$.

QED

Tosach leathanach 105 sa LSS.

## e.g. (2)

*An uille* 36° *a thógáil le Compás agus Riail.*

Toisc gurb shin í an uille a ghabhas slios pentagóin rialta ag imlíne an iomchiorcail (Teoirim Caib VII), is ceart dúinn tréithe na fiograch sin a bhreithniú i dtosach.

Tá Fíoghair anseo sa LSS, leathanach 105.

## Anailís na Ceiste

Abair gur pentagón rialta é $ABCDE$, ionas go bhfuil 36° san uillinn a ghabhas gach slios ag an imlíne. Ceangail $DA, DB, BE$.

Tá $\hat{\alpha} = 36° = \hat{\alpha}_1$ d'á réir, agus fágann sin $DX = XB$. Mar an gcéanna tá $\hat{\beta}_1 = 72° = \hat{\alpha} + \hat{\alpha}_1 = \hat{\beta}$, ionas go bhfuil$XB = BA$.

∴ Tá $DX = XB = BA$ — (i).

De thairbhe $\hat{\alpha}_2 = 36° = \hat{\alpha}$, tadhlann $AB$ ag $B$ an ⊙ $DXB$, ionas go bhfuil$DA{\cdot}AX = AB^2$ (teoirim ) $= XD^2$, de réir (i).

Fágann sin $DA \cdot AX = XD^2$ .i. déantar meán-teascadh ar $DA$ ag $X$ — (ii)

Nuair tugtar $DA$, tá $X$ cinnteach de réir (ii), agus de bhrí go bhfuil$B$ an fhad $XD$ ó $A$ is ó $X$ de réir (i) socraíonn sésin ionad $B$. Tig linn réiteach gonta a chóiriú anois mar seo:

*Réiteach:*

Déan líne ar bith $DA$ a mheán-teascadh ag $X$ ionas go bhfuil$OA \cdot AX = XD^2$. Le $X$ agus $A$ mar láir línigh ⊙ ar gatha dóibh $XD$, a ghearras a chéile in $B$.

Tá $ADB = 36°$, agus tá $\widehat{DAB} = \widehat{DBA} = 72°$.

*Cruthúnas:*

De thairbhe $DA \cdot AX = XD^2 = AB^2$, tadhlann $AB$ an ⊙$DXB$. ∴ Tá $\hat{\alpha}_2 = \hat{\alpha}$.

Ach de bharr $XB = XD$, tá $\hat{\alpha}_1 = \hat{\alpha}$, ionas go bhfuil$\widehat{DBA} = 2\hat{\alpha}$.

Mar an gcéanna de bharr $AB = XB$, tá $\widehat{DAB} = \beta = \hat{\alpha} + \hat{\alpha}_1 = 2\hat{\alpha}$. Ach tásuim uilleacha an $\Delta DAB = 180°$.

Fágann sin $5\alpha = 180°$, ionas go bhfuil$\alpha = 36°$, $\widehat{DAB} = \widehat{DBA} = 72°$. □

## e.g. (2)

*To construct the angle* 36° *with ruler and compass.*

In view of the fact that that is the angle subtended by a side of a regular pentagon at the perimeter of its circumcircle (Theorem in Chapter 7), we should first of all consider the properties of that figure.

## The Analysis of the Question

Suppose $ABCDE$ is a regular pentagon, so that 36° is the angle subtended by each side at the perimeter. Join $DA, DB, BE$.

We have $\hat{\alpha} = 36° = \hat{\alpha}_1$ accordingly, and it follows that $DX = XB$. Similarly, $\hat{\beta}_1 = 72° = \hat{\alpha} + \hat{\alpha}_1 = \hat{\beta}$, so that $XB = BA$.

$\therefore DX = XB = BA$ — (i).

On account of $\hat{\alpha}_2 = 36° = \hat{\alpha}$, $AB$ is tangent at $B$ to the $\odot DXB$, so that $DA \cdot AX = AB^2$ (Theorem ) $= XD^2$, by (i).

It follows that $DA \cdot AX = XD^2$ .i. $X$ makes the mean proportional cut on $DA$. — (ii)

When $DA$, is given, $X$ is determined by (ii), and since $B$ is a distance $XD$ from $A$ and from $X$ by (i) that determines the position of $B$. So we can arrange a quick solution like this:

*Solution:*

Cut any line $DA$ in mean proportion at $X$ so that $OA \cdot AX = XD^2$. With $X$ and $A$ as centres draw $\odot$ with radii $XD$, cutting one another at $B$.

We have $ADB = 36°$, and $\widehat{DAB} = \widehat{DBA} = 72°$.

*Proof:*

Since $DA \cdot AX = XD^2 = AB^2$, then $AB$ is tangent to the $\odot DXB$. $\therefore \hat{\alpha}_2 = \hat{\alpha}$.

But since $XB = XD$, we have $\hat{\alpha}_1 = \hat{\alpha}$, so that $\widehat{DBA} = 2\hat{\alpha}$.

Similarly, since $AB = XB$, we have $\widehat{DAB} = \beta = \hat{\alpha} + \hat{\alpha}_1 = 2\hat{\alpha}$. But the sum of the angles of the $\Delta DAB = 180°$.

It follows that $5\alpha = 180°$, so that $\alpha = 36°$, $\widehat{DAB} = \widehat{DBA} = 72°$. □

## Nóta

Má's in Y a ghearras síneadh DE an slios AB, gheofar amach go bhfuil $AB.BY = AY^2$ ∴ roinntear BA i meán-teascadh amuigh ag Y.

Tig linn an pentagón rialta a chógáil ar AB mar seo.
~~Roinn BA~~ Déan meán-teascadh amuigh ar BA ag Y, ionnas go mbeidh $AB.BY = AY^2$.
Le A agus B mar láir tarraing stuaghanna ar gatha dóibh AY.
Má's i bpointe D a thagas siad le chéile, faigh láir na E, C, mion-stuaghanna DA is DB sa gciorcal DAB. 'Sé ABCDE an pentagón rialta.

## Cleachtaithe

1) Roinn dronlíne dírithe i bpointe P amuigh chun go mbeidh $AP.PB = b^2$.

2) Faigh pointe P ar shíneadh AB chun go mbeidh (i) $AP^2 + PB^2 = b^2$; (ii) $AB^2 + BP^2 = 2AP.PB$.

3) Sa △ ABC is i bpointe X a ghearras comhroinnteoir na h-uilleann A an comhchiorcal, agus tagann AX is BC le chéile in Y. Cruthuigh $XA.XY = XB^2$.

4) Pointí ar láirlíne ⊙ AB is ea C is D agus iad comhfhada ó'n lár. Pointe ar bith san imlíne is ea X. Cruthuigh $XC^2 + XD^2 = AC^2 + AD^2$.

5) Má's pointe é X ar bhonn BC an △ chomhchosaigh ABC, cruthuigh $AB^2 - AX^2 = BX.XC$.

6) Roinneann P an líne AB istigh i gcaoi go bhfuil $AB.BP = AP^2$, agus pointe san líne is ea X ionnas go bhfuil $PX = PB$. Cruthuigh $AP.AX = XP^2$.

7) Faigh pointe X in AB istigh ionnas go mbeidh $AB^2 + BX^2 = 3AX^2$.

8) Sa △ comhchosach ABC gearrann parallél le BC na sleasa AB is AC sna pointí D, E. Cruthuigh $BE^2 - CE^2 = BC.DE$.

9) Sa △ comhchosach ABC, tá $\hat{B} = \hat{C} = 2\hat{A}$. Cruthuigh $\frac{BC}{AB} = \frac{1}{2}(\sqrt{5} - 1)$.

10) Má's é OX ga an ⊙ ABCDE atá ingearach leis an slios AB (áit gur pentagón rialta é ABCDE), cruthuigh go ndéanann BE meán-teascadh istigh ar an nga OX.

Má's in Y a ghearras BE agus OX, teaspáin go bhfuil XY cothrom le slios decagóin rialta a inscríobhtar sa gciorcal

11) Má's iad p, d sleasa an pentagóin agus an decagóin rialta a inscríobhtar i gciorcal gurb é R a ga, cruthuigh $p^2 + d^2 = R^2$.

Tosach leathanach 106 sa LSS.

### Nóta

Má's in $Y$ a ghearras síneadh $DE$ ar slios $AB$, gheofar amach go bhfuil$AB \cdot BY = AY^2$ .i. roinntear $BA$ i meán-teascadh *amuigh* ag $Y$.

Tig linn an pentagón rialta a thógáil ar $AB$ mar seo:

Déan meán-teascadh amuigh ar $BA$ ag $Y$, ionas go mbeidh $AB \cdot BY = AY^2$. Le $A$ agus $B$ mar láir tarraing stuanna ar gatha dóibh $AY$.

Má's i bpointe $D$ a thagas siad le chéile, faigh $E, C$, láir na mion-stuanna $DA$ is $BB$ sa gciorcal $DAB$. 'Sé $ABCDE$ an pentagón rialta.

### Note

If the extension of $DE$ cuts the side $AB$ at $Y$, one finds that $AB \cdot BY = AY^2$ .i. $Y$ cuts $BA$ in mean proportion *externally*.

We can construct the regular pentagon on $AB$ as follows:

Make the external mean proportion cut on $BA$ ag $Y$, so that $AB \cdot BY = AY^2$. With $A$ and $B$ as centres draw arcs with radius $AY$.

If they meets at a point $D$, find $E, C$, the centres of the minor arcs $DA$ and $BB$ in the circle $DAB$. Then $ABCDE$ is the regular pentagon.

### Cleachtaithe

1. Roinn dronlíne áirithe i bpointe $P$ amuigh chun go mbeidh
$$AP \cdot PB = b^2.$$

2. Faigh pointe $P$ ar síneadh $AB$ chun go mbeidh
(i) $AP^2 + PB^2 = b^2$;
(ii) $AB^2 + BP^2 = 2AP \cdot PB$.

3. Sa $\Delta ABC$ is i bpointe $X$ a ghearras cómhroinnteoir na h-uilleann $A$ an iomchiorcal, agus tagann $AX$ is $BC$ le chéile in $Y$. Cruthuigh
$$XA \cdot XY = XB^2.$$

4. Pointí ar láirlíne $\odot$ $AB$ is ea $C$ is $D$ agus iad cómhfhada ó'n lár. Pointe ar bith san imlíne is ea $X$. Cruthuigh
$$XC^2 + XD^2 = AC^2 + AD^2.$$

5. Má's pointe é $X$ ar bhonn $BC$ an $\Delta$ chómhchosaigh $ABC$, cruthuigh
$$AB^2 - AX^2 = BX \cdot XC.$$

6. Roinneann $P$ an líne $AB$ istigh i gcaoi go bhfuil $AB \cdot BP = AP^2$, agus pointe san líne is ea $X$ ionas go bhfuil$PX = PB$. Cruthuigh

$$AP \cdot AX = XP^2.$$

7. Faigh pointe $X$ in $AB$ istigh ionas go mbeidh

$$AB^2 + BX^2 = 3AX^2.$$

8. Sa $\Delta$ cómhchosach $ABC$ gearrann paralléil le $BC$ na sleasa $AB$ is $AC$ sna pointí $D, E$. Cruthuigh

$$BE^2 - CE^2 = BC \cdot DE.$$

9. Sa $\Delta$ cómhchosach $ABC$, tá $\hat{B} = \hat{C} = 2\hat{A}$. Cruthuigh

$$\frac{BC}{AB} = \frac{1}{2}(\sqrt{5} - 1).$$

10. Má sé $OX$ ga an $\odot ABCDE$ atá ingearach leis an slios $AB$ (áit gur pentagón rialta é $ABCDE$), cruthuigh go ndéanann $ABE$ meán-teascadh istigh ar an nga $OX$.

    Má's in $Y$ a ghearras $BE$ agus $OX$, teaspáin go bhfuil$XY$ cothrom le slios decagón rialta a inscríobhtar sa gciorcal.

11. Má 'siad $p, d$ sleasa an pentagóin agus an decagóin rialta a inscríobhtar i gciorcal gurb é $R$ a ga, cruthuigh

$$p^2 + d^2 = R^2.$$

## Exercises

1. Divide a given straight line at a point $P$ outside so that $AP \cdot PB = b^2$.

2. Find a point $P$ on the extension of $AB$ so that
   (i) $AP^2 + PB^2 = b^2$;
   (ii) $AB^2 + BP^2 = 2AP \cdot PB$.

3. In the $\Delta ABC$ the point $X$ is where the bisector of the angle $A$ cuts the circumcircle, and $AX$ meets $BC$ at $Y$. Prove that

$$XA \cdot XY = XB^2.$$

4. The points $C$ and $D$ lie on a diameter $AB$ of a $\odot$ and they are equidistant from the centre. The point $X$ is somewhere on the perimeter. Prove that

$$XC^2 + XD^2 = AC^2 + AD^2.$$

5. If $X$ is a point on the base $BC$ of the isosceles $\Delta\ ABC$, prove that
$$AB^2 - AX^2 = BX \cdot XC.$$

6. $P$ divides the line $AB$ internally so that $AB \cdot BP = AP^2$, and $X$ is a point on the line such that $PX = PB$. Prove that
$$AP \cdot AX = XP^2.$$

7. Find a point $X$ inside $AB$ such that $AB^2 + BX^2 = 3AX^2$.

8. In the isosceles $\Delta\ ABC$ a parallel to $BC$ cuts the sides $AB$ and $AC$ in the points $D, E$. Prove that
$$BE^2 - CE^2 = BC \cdot DE.$$

9. In the isosceles $\Delta\ ABC$, we have $\hat{B} = \hat{C} = 2\hat{A}$. Prove that
$$\frac{BC}{AB} = \frac{1}{2}(\sqrt{5} - 1).$$

10. If $OX$ is the radius of the $\odot ABCDE$ that is perpendicular to the side $AB$ (where $ABCDE$ is a regular pentagon), prove that $ABE$ cuts the radius $OX$ in internal mean proportion.

    If $Y$ is the common point of $BE$ and $OX$, show that $XY$ is equal to the side of a regular decagon inscribed in the circle.

11. If $p, d$ are sides of a regular pentagon and a regular decagon inscribed in a circle of radius $R$, prove that
$$p^2 + d^2 = R^2.$$

# Caibidil 9

# Notes

## 9.1 Exercises on chapter 1

I am providing geogebra files for the material below, at this book's page on the website logicpress.ie. These can be freely downloaded, and will run when loaded into geogebra. Geogebra is free, open-source software. You can download it for your computer from geogebra.org.

### Exercise I-3

In Figure 9.1, $A, B, C, X$ and $Y$ are placed at random, and then determine $Z, R, S$ and $T$. Loading the figure I-3.ggb into geogebra, you can pull the data $A, B, C, X, Y$ around and observe how the figure changes shape while $R, S, T$ remain collinear (on the blue line).

This exercise is an interesting pedagogical innovation. The procedure is completely elementary, and will not tax the abilities of any pupil. On the face of it, it just exercises the student in the use of the terminology and of the ruler and pencil. If a whole class of students tackle it independently, there should be a variety of figures resulting. It may easily happen that some of $R, S, T$ lie off the pupil's page. This is not without interest in itself, for various reasons, but initially pupils might be encouraged to vary the data until they manage to get all three on their page. This should result in a sheaf of variant diagrams, in all of which $R, S, T$ are collinear. The same effect can be produced nowadays by using geogebra (or any equivalent graphical aid).

For the student blessed with some curiosity, the outcome of the exercise might well cause wonder. What is going on? If the student, then or later, begins to ask *why this works*, then we are on the way: a mind has been opened to mathematics. The talented student is left with a want.

### Exercise I-4

This exercise is in the same spirit as I-3. I have provided a geogebra file I-4.ggb. A typical diagram is shown in Figure 9.2. You can pull the data $A, B, C, L, N, M$ around and observe the result that the (blue) line through $R$ and $S$ always passes through $T$. The explanation

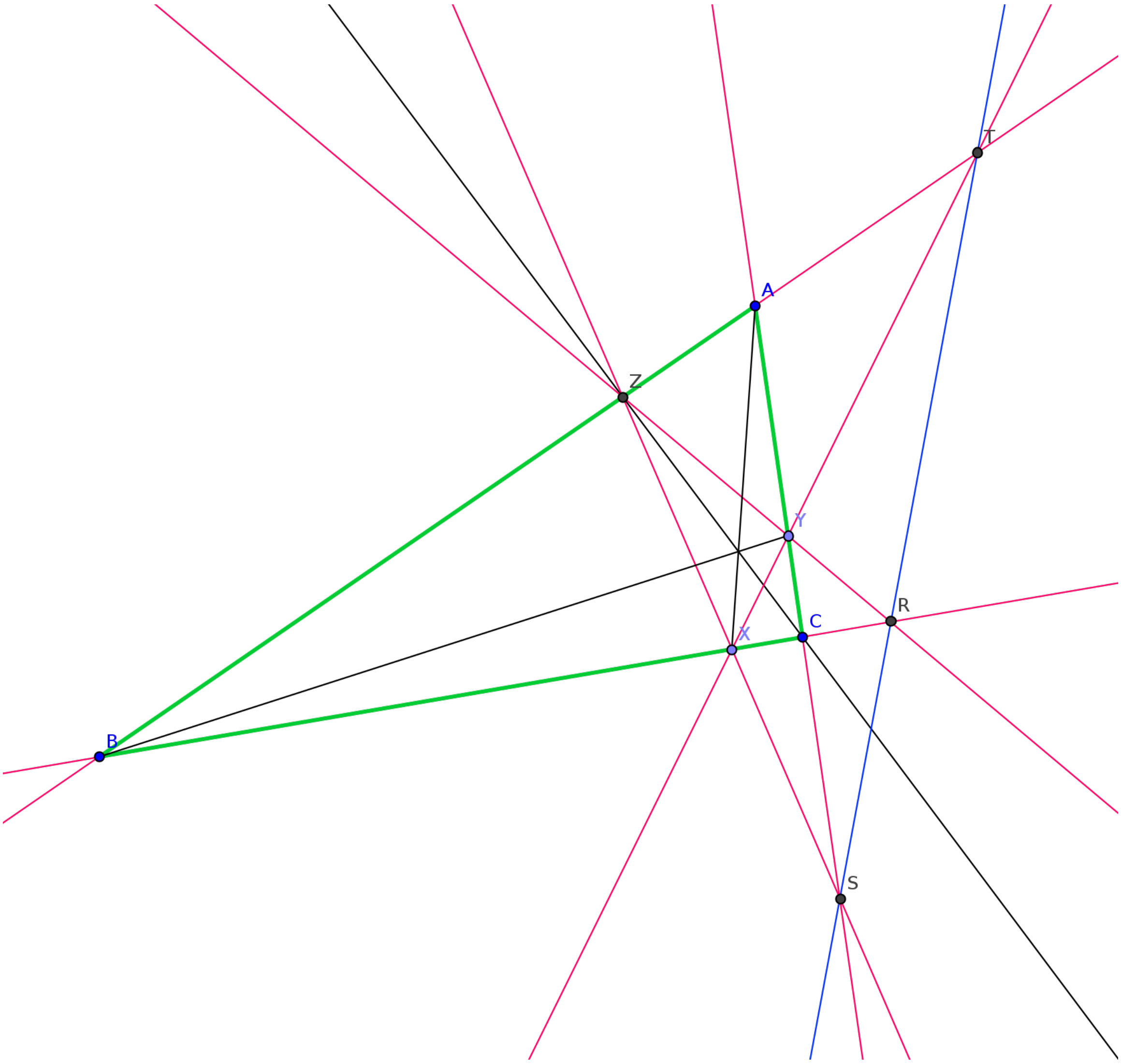



Figure 9.1: Exercise I-3

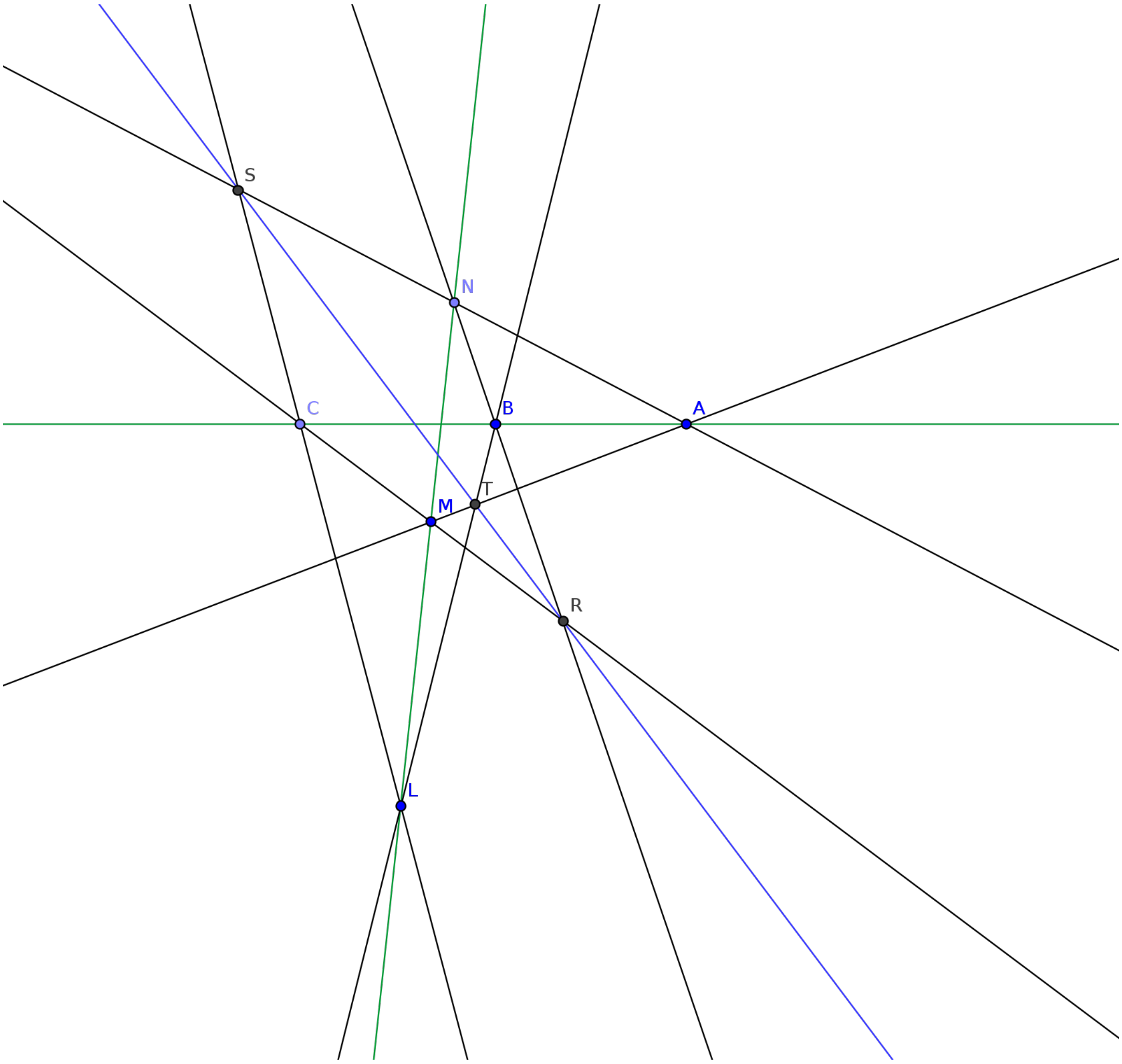



Figure 9.2: Exercise I-4

of the phenomenon that the students will observe lies in the fact that a pair of straight lines constitute a (degenerate) conic, so the collinearity of $R, S, T$ is a case of a theorem about a hexagon inscribed in a conic.

The person who wants to understand this has a motive to find out about Pascal and Brianchon. The teacher or elder guiding a young person will have to judge when it might be appropriate to drop some hints about the wonders of algebraic geometry. For people of Newell's generation, the route into this paradise was Salmon's *Conic Sections*[7], leading on to the magic of the Italian masters of the nineteenth century. By my time, the work of Emmy Noether had established the organic connection between the abstract theory of rings and algebraic geometry, and I was motivated to view the subject through the lens of sheaf theory.

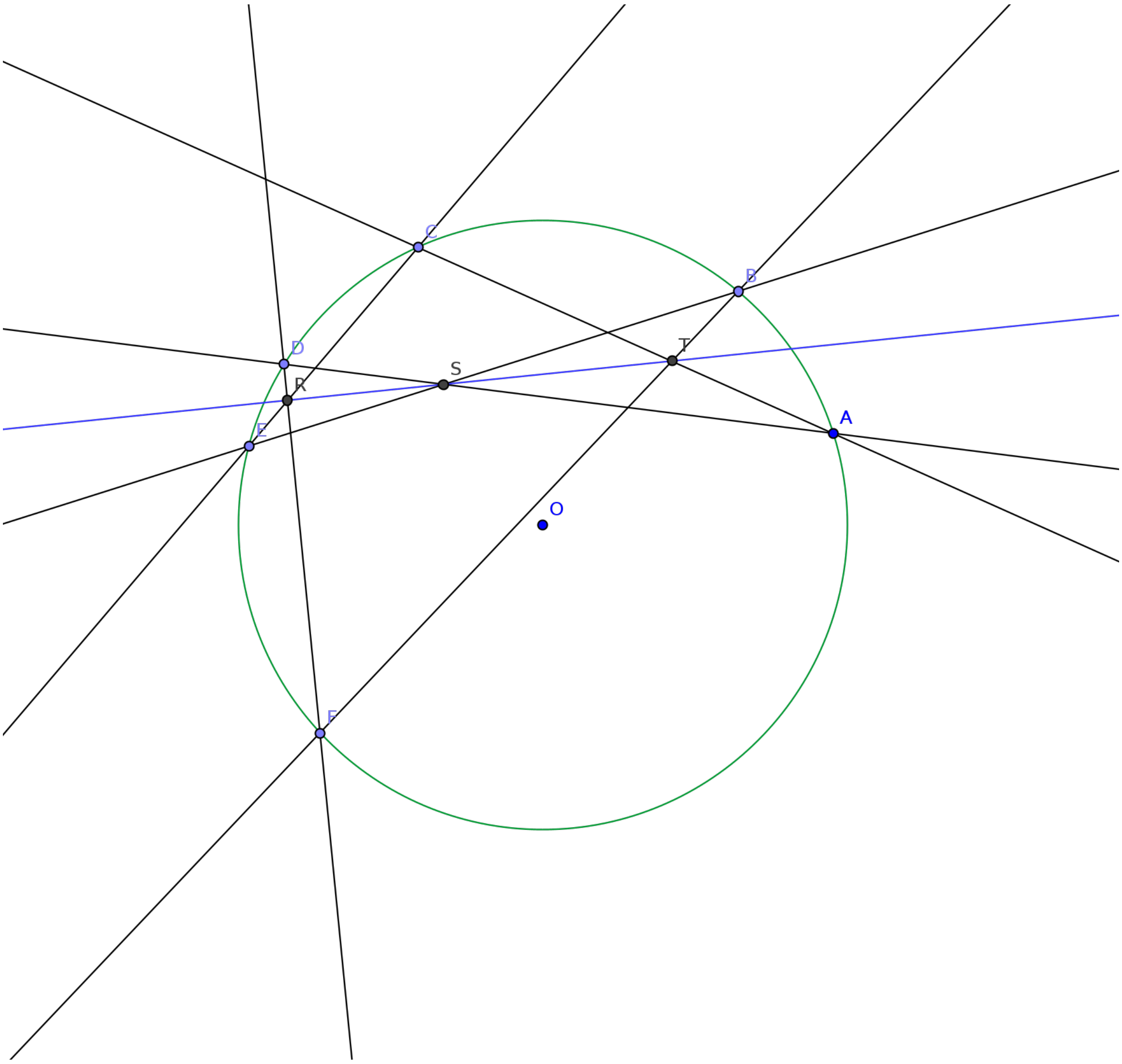



Figure 9.3: Exercise I-5

## Exercise I-5

The geogebra file is I-5.ggb, and a typical diagram is in Figure 9.3. This time, we are looking at the case where the conic is a circle. I gather that Pascal first noticed this case, and then observed that, since the statements are all affine-invariant, the result must hold for a hexagon inscribed in any ellipse. The general principle of invariance of functional relationships then tells us that it holds for generic conics, and continuity in projective space tells us that it holds even for singular conics. I learned the result in 1967 from Richard Timoney (Sr). He proved it using the algebraic properties of plane cubics. Specifically, a (complex, projective) plane cubic is usually determined by nine points, but each cubic through eight given points will pass through a particular ninth point. The general principle in the background was Bezout's theorem that in the (complex projective) plane a curve of degree $m$ will meet a curve of degree $n$ is exactly $mn$ points, counting multiplicities appropriately. This in turn rests on the fundamental theorem of algebra, via the use of resolvents.

These matters were common knowledge among educated Irishmen, back in the days before the reforms of the seventies. Those days now now appear as a golden age of Irish geometry.

## 9.2 Background

### Some quotations

Here are some quotations from Susan Mac Donald's *Euclid Transmogrified*, a book about the history of the school geometry syllabus.

She reports on a lecture on *Motion Geometry* by MOT to the IMTA Galway branch in February 1965:

> Dr Newell first aroused our interest by outlining the history of Geometry and then dealt at length with the Klein conception of Geometry. Finally, his treatment of the Geometry of Euclid, from a Motion point of view, proved most inspiring and made clear the importance and vital need of his work, which he began in the early 1950's. [3, p.190]:

On a lecture by David Simms, and an inspector's reaction:

> In Ireland during the academic year 1967/1968, two lecture courses in modern mathematics for schoolteachers were provided by the Mathematics Department of Trinity College Dublin. One of these courses was a ten-lecture course based on Choquet's *L'enseignement de la géométrie* given by Professor David Simms of Trinity College Dublin, Simms states that he was 'quite unaware of Papy' at this stage; and that on announcing this series of lectures, Conchiúir Ó Caoimh came to him (Simms) and 'indicated that he [Ó Caoimh] was unhappy that I was proposing to follow Choquet and not Papy'. On the other hand, Ó Caoimh described Dr Martin J. Newell of University College Galway, who lectured on motion geometry to the IMTA in 1965, as a 'great supporter' of Papy. [3, p.189].

On a course given Bishop John Kirby:

> Quoting from a set of notes that formed the basis of a series of lectures that he gave to secondary and vocational teachers in St Peter's Convent, Athlone in July 1966, Kirby stated: 'The lectures were given to teachers and thus the notes are intended for teachers, they are of little or no value to pupils. The notes are based largely on the work of Papy and Choquet as found in *Mathématique Moderne* by Papy, and an essay in the Synopses for

Modern Secondary School Mathematics published by the OEEC, by Choquet'. Kirby quotes from sections concerned with the approach and content of his course:

> The approach we refer to here is the axiomatic, deductive approach, where from a certain number of undefined concepts and unproven statements we make other statements and other definitions and gradually build up a course of geometry on solid foundations. First, we study those geometrical ideas and properties which do not involve measurement and in this section we will not use such terms as length, size or distance. This makes up what is called affine geometry. Then in the second stage, we study the geometrical properties of figures which depend on distance and size of angles. This is called metrical geometry. The combination of these two distinct parts gives us the traditional geometry of Euclid. The distinction between affine and metrical properties of figures is important and helps to clarify the basic ideas for the pupil. This approach was first outlined as far back as 1872 in a speech by the famous geometer, Felix Klein. It was later elaborated on by a secondary teacher, Herman Grassmann, and is now in colleges on the Continent. ... Composition of central symmetries, central symmetry, the medians of a triangle, the image of a line under central symmetry, central symmetry, co-ordinating the line, Thales' theorem ... homothety, what in Euclidean geometry is called similar figures, similar triangles, composition of homotheties, dilations, metrical geometry, composition of orthogonal symmetries, isometries, orthogonal projections, scalar product of vectors.

Commenting on the content, Kirby states: 'I'm protecting myself from its obtuseness, early on, by saying it was intended for teachers, and it wasn't a pupils' text' [3, pp190-191].

## Wider context

We see that MOT's book can be viewed as a contribution to the active international debate about the best way to present geometry in secondary schools. This debate was intense in the fifties and sixties of the last century.

I have stated views on this matter, and refer to my paper [6] and my paper with the late Paddy Barry [2]. My opinion is that a secondary school course in geometry should exist on three levels: level 1 (completely rigorous), level 2 (accessible to teachers and strong students) and textbook. At present, in 2024, the Irish course is based on the level 1 account in Barry's book [1] *Geometry with Trigonometry* and the level 2 account in [5] *Geometry course for post-primary school mathematics*. Its success as a programme depends on the availability of suitable textbooks and on teacher engagement.

MOT's book is intended as a textbook, and its use in schools would require the existence of level 1 and 2 versions of the geometry it presents. These versions would differ significantly from those based on Barry's book, or on Birkhoff's geometry, or, indeed on Euclid. From the remarks quoted earlier, it may be that MOT's text relies on level 1 and 2 accounts by Choquet or Papy. But Newell, as a researcher, was well-regarded as a proficient expert on Lie groups. Murnaghan wrote in 1957 [4] about group representations:

> This theory is now better understood than it was when I wrote, some twenty years ago, my book *The Theory of Group Representations*; and its exposition in the present lectures is considerably simpler than that in my book. I may mention, for instrance, the treatment of the modification rules for the rotation, symplectic and orthogonal groups, in which I have been able to use with great profit the ideas of Professor M.J.Newell.

So it is not altogether clear that Newell did not have his own unique take on the ideal structure for a level 1 account, and it would be interesting to see such an account in place as a foundation for his text.

### Note to Teachers

It would not be appropriate to have students preparing for the Irish Junior Certificate examinations learn the proofs in MOT's text. The national examinations are set in the context of the currently-prescribed syllabus, and the geometry in that syllabus is structured in a different way. So the proofs in *Nuachúsa Céimseatan* are not valid proofs in that context.

## 9.3 Caibidil 1

Initial education in geometry, which I have categorised as level 3, must involve visual and tactile experience with geometrical aspects of human experience. This has to preceed any reasoning about geometry, and should continue in tandem with reasoning. Caibidil 1 begins at this level, and then moves to introduce the basic abstractions of elementary geometry: solid, surface, curve and point, straight line and circle.

### Length

The concept of length is not defined, but is silently taken for granted. Ditto measurement and dimension. We are told that a line has length but has no other measurements (dimensions). We are not told *what kind of thing this length is*, but in exercises we are invited to measure it with a ruler.

I infer that the underlying level 1 account incorporates the real number system, and a metric on space.

We are told that the straight line $AB$ is the shortest line between $A$ and $B$. In other words, straight lines are geodesics for the metric.

We are told that there is exactly one straight line through each two given points. This has to be an axiom at level 1.

(There is reference to sliding a line along itself. This preceeds any discussion of planar motions, and only seems to make sense if the context is a one-dimensional manifold. The naive reader may understand this discussion in terms of rigid translation of the whole planar surface in the direction of the line. It seems a bit premature here.)

Circles are defined in terms of length.

## Planes

A plane is defined as a surface that contains the whole of each straight line as soon as it meets it in two different points.

## Alternative Worlds

It would be interesting the see how three readers would get on with his book:

- Eithne, who lives in a surface of constant curvature $+1$,
- Fíona, who lives in a surface of constant curvature 0, and
- Hypatia, who lives in a surface of constant curvature $-1$.

At what point, if any, would each reader begin to smell a rat?

Newell's whole approach is based on the view of the plane as a homogeneous space under the action of its isometry group. This is what lies behind all this stuff about sliding a solid cyclinder inside a cyclindrical sleeve, sliding a solid sphere inside a ball socket, and sliding a line along itself.

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