There are surprisingly few prerequisites. Apart from a swallowable concession to operator algebras, substantially redeemed in the Appendix, the book is self-contained.

The authors have succeeded in achieving a declared objective of producing an interesting and substantial treatment of their subject at a level suitable for graduate students of mathematics and theoretical physics. The exposition is very good indeed. As well as being accessible to its intended audience, the book will be a valuable reference for everyone with an interest in Clifford $C^*$-algebras.

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## FUNCTION SPACES AND POTENTIAL THEORY
(Grundlehren der mathematischen Wissenschaften 314)

*By* David R. Adams and Lars Inge Hedberg: 366 pp., DM.148.–,
ISBN 3 540 57060 8 (Springer, 1996).

This book is one of a number of excellent recent accounts (Heinonen, Kilpeläinen and Martio [4], Maz'ya [6], Meyer [7], Ziemer [14]) of various different aspects of the theory of function spaces and potential theory. These books expose minimally-overlapping developments that have flowed from the ideas presented a generation ago in the enormously influential books of Carleson [1], Stein [12], Morrey [8], Federer [2], Peetre [9], Triebel [13] and Hörmander [5], and further developed in the interim in other books of Triebel, Stein, Tarkhanov, Rubio de Francia and Garcia-Cuerva, and among the schools of hard harmonic analysis, quasi-regular mappings, Cauchy integral estimates and PDEs. They also make available in book form the results of the classic papers of the same period, such as the work of Serrin, John, Nirenberg, Fefferman, Fuglede, Moser, Meyers, Havin, Jones and Wolff, besides those mentioned above, and many others. No library should be without them.

Do you need to buy this particular one? If you are interested in capacities and thinness concepts associated with Sobolev spaces, or traces (that is, 'restrictions') of Sobolev functions, or in $L^p$-norm approximation of solutions to elliptic partial differential equations, then it is an essential reference. One, not quite accurate, way to describe the book is as a leisurely proof of the quite comprehensive results now available about $L^p$ approximation by solutions of constant-coefficient elliptic equations (including approximation by holomorphic functions in one variable). Another way to describe it is as an introduction to potentials and capacities associated to Sobolev, Besov and Triebel–Lizorkin spaces, with applications to trace problems and to approximation theory.

The account assumes familiarity with the material of a good graduate course in real analysis, and the elements of functional analysis and complex analysis; for instance, the contents of Rudin's books [10, 11]. Also, the reader needs to be familiar with much of Stein's classic [12] on singular integrals. Also assumed at various points are some items from Ziemer's book [14], the full strength of the Fefferman–Stein maximal theorem [3], the elements of Suslin sets and Choquet's capacitability theorem. In view of the quite central role of the Fefferman–Stein maximal theorem in the atomic-decomposition theory, and hence in the whole book, one feels that it would have been more appropriate to have included a proof, instead of sending the reader to the *American Journal of Mathematics*. In one or two other cases, such as the fact that truncation works on $W^{1,p}$, it would have taken relatively little space to include a proof.

One regularly finds people who try to prove mathematical theorems with one hand tied behind their backs, metaphorically speaking. This can be regarded as a kind of virtuosity, but is probably just foolish. In function space problems, there is a tendency among old hands to try to work around the use of the Besov spaces $B^s_{p,q}$ and the Triebel–Lizorkin spaces $F^s_{p,q}$ (also known as atomic-decomposition spaces), if possible. In this book, several major results are proven twice. Typically, the non-atomic proof is longer and gives a less comprehensive or less precise result. This convinces me that if you are interested in Sobolev spaces, and you have not digested the elements of the atomic-decomposition technology, then it is high time you did. A major feature of the book is the exposition of penetrating atomic-based results of Netrusov, appearing here for the first time.

In general, this is a very well-written and meticulously proof-read account. The authors achieve a very high standard of expository writing. Proofs are broken down to expose the key ideas. Major results are proved by a deliberate process of gradual generalisation. This is usually helpful, although in one case (Theorem (3.3.3)) the result is a proof that is perhaps too scattered; this proof is spread over Chapters 3 and 4, and the details of the most general case are not actually given. Perhaps the organisation here is due to the distribution of tasks between the two authors, or to a relatively late decision to include Section 4.8 (due to Netrusov). The layout, careful cross-referencing and excellent indices make it very easy to find things. The book provides encyclopaedic coverage of its area, has substantial new results, and will be an essential reference for some time to come. It is a very fine book.

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